

1, (20 分)

解：开始时系统波函数为

$$\begin{aligned}\psi(x, 0) &= \psi(x) = A \left(1 + b \cos \frac{\pi x}{a}\right) \sin \frac{\pi x}{a} = A \left(\sin \frac{\pi x}{a} + \frac{b}{2} \sin \frac{2\pi x}{a}\right) \\ &= \frac{1}{\sqrt{1 + |b/2|^2}} \psi_1(x) + \frac{b/2}{\sqrt{1 + |b/2|^2}} \psi_2(x) = c_1 \psi_1 + c_2 \psi_2 \\ c_1 &= \frac{1}{\sqrt{1 + |b/2|^2}}, c_2 = \frac{b/2}{\sqrt{1 + |b/2|^2}}\end{aligned}$$

这里波函数 $\psi_n(x)$ 为无限高势阱中粒子能量的本征态

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}, \quad n=1, 2, 3, \dots$$

已归一化，相应能级为

$$E_n = \frac{\pi^2 \hbar^2}{2ma^2} n^2, \quad n=1, 2, 3, \dots$$

(1) t 时刻系统的状态

$$\begin{aligned}\psi(x, t) &= e^{-i\hat{H}t/\hbar} \psi(x, 0) = c_1 \psi_1(x) e^{-iE_1 t/\hbar} + c_2 \psi_2(x) e^{-iE_2 t/\hbar} \\ &\equiv c_1(t) \psi_1(x) + c_2(t) \psi_2(x)\end{aligned}$$

(2) t 时刻，能量平均值

$$\begin{aligned}\bar{E} &= E_1 |c_1(t)|^2 + E_2 |c_2(t)|^2 \\ &= (1 + |b|^2) E_1 / (1 + |b|^2 / 4)\end{aligned}$$

(3) 动量平均值

$$\begin{aligned}\bar{p}(t) &= (\psi(t), \hat{p} \psi(t)) = 2 \operatorname{Re}[c_1^*(t) c_2(t) (\psi_1, \hat{p} \psi_2)] \\ &= 2c_1 c_2 \frac{8\hbar}{3a} \sin \omega t = \frac{|b|^2}{4 + |b|^2} \frac{8\hbar}{3a} \sin \omega t \\ \omega &= (E_2 - E_1) / \hbar\end{aligned}$$

考试科目：量子力学

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