Theory of Machines and Mechanisms

(机械原理)

Chapter 7

GEAR TRAINS

7.1 Gear Trains (轮系) and Their Classification

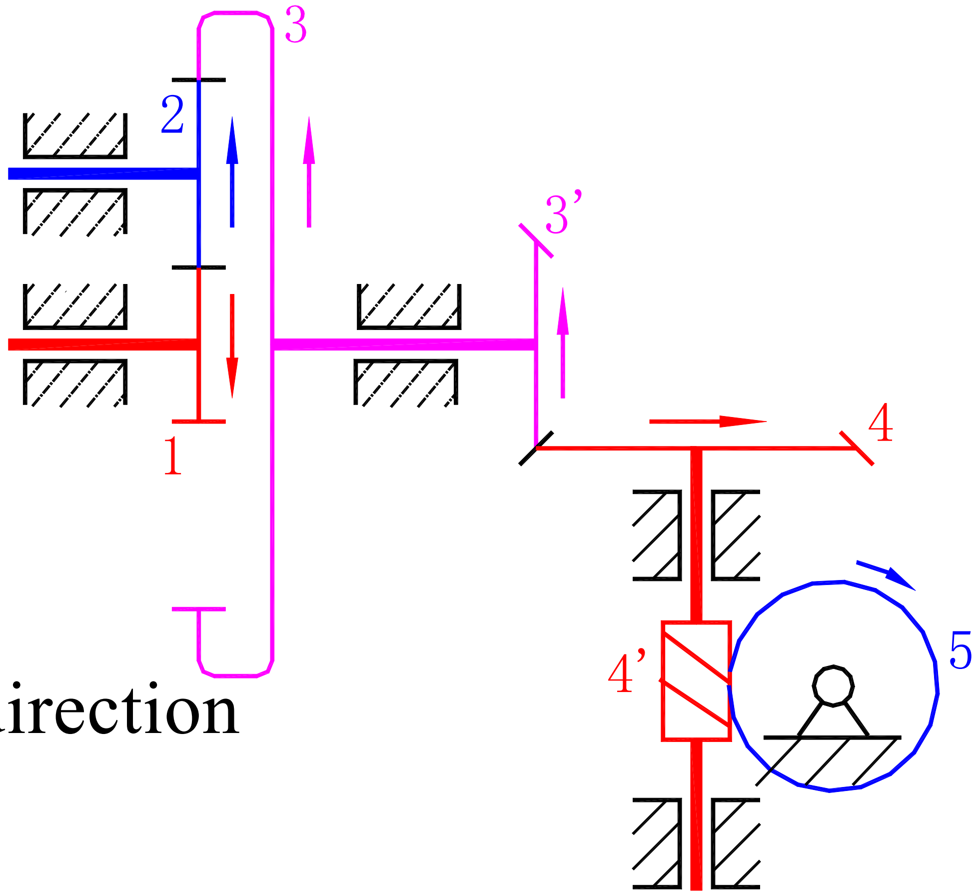
Gear Train ——A transmission system by more than one pair of gears.

Gear Train
- Gear train with fixed axes (定轴轮系) (Ordinary gear train)
- Elementary Epicyclic gear train(周转轮系)
  - Planetary gear train (行星轮系)
  - Differential gear train (差动轮系)
- Combined gear trains (复合轮系)

7.2 Train Ratio of a Gear Train with Fixed Axes

The train ratio of a gear train(轮系传动比):

$$i = \frac{\omega_{input}}{\omega_{output}}$$



Two factors:

1.The magnitude;

2.The relative rotating direction of the two members.

7.2.1 Absolute Value of Train Ratio

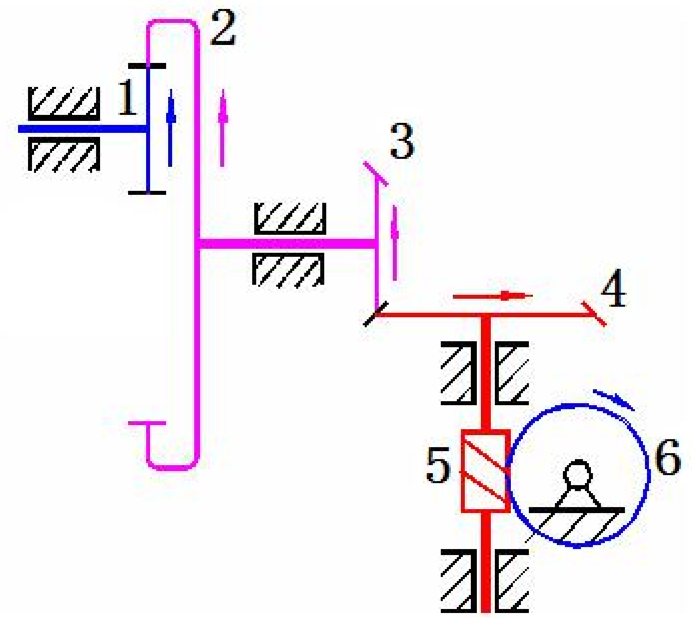
1——Driving Wheel. Input member

6——Driven Wheel. output member

$$|i_{12}| = \left|\frac{\omega_1}{\omega_2}\right| = \frac{Z_2}{Z_1}$$

$$|i_{34}| = \left|\frac{\omega_3}{\omega_4}\right| = \frac{Z_4}{Z_3}$$

$$|i_{56}| = \left|\frac{\omega_5}{\omega_6}\right| = \frac{Z_6}{Z_5}$$



$$|i_{12} i_{34} i_{56}| = \left|\frac{\omega_1}{\omega_2}\frac{\omega_3}{\omega_4}\frac{\omega_5}{\omega_6}\right| = \left|\frac{\omega_1}{\omega_6}\right| = |i_{16}| = \frac{Z_2 Z_4 Z_6}{Z_1 Z_3 Z_5}$$

i_{1n} 从1传动到n的传动比，从1传动到n来判断主动与从动。

$$i_{1n} = \frac{product\ of\ \ tooth\ \ numbers\ of\ \ all\ \ the\ \ driven\ \ gears}{product\ of\ \ tooth\ \ numbers\ of\ \ all\ \ the\ \ driving\ \ gears}$$

7.2.2 Relative rotating directions of gears

(1) Gear trains with fixed parallel axes:

\+ ——输入轴和输出轴的方向相同

\- ——输入轴和输出轴的方向相反

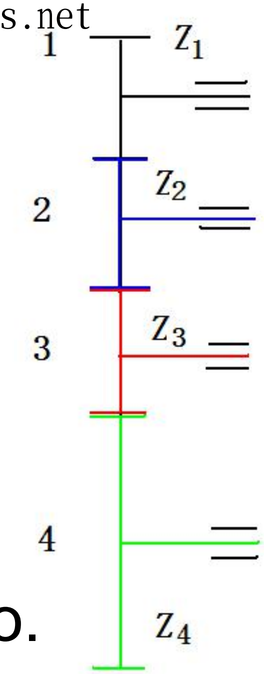
Planar ordinary gear train (平面定轴轮系) → $(-1)^m$,m——the number of external gear pairs

Spatial ordinary gear train (空间定轴轮系) → Determined by drawing arrows, "+" or "–" must be added before the ratio.

(2) Gear trains with fixed non-parallel axes.

Drawing arrows in figure.

Example1: Calculate the Ratio i_{14}

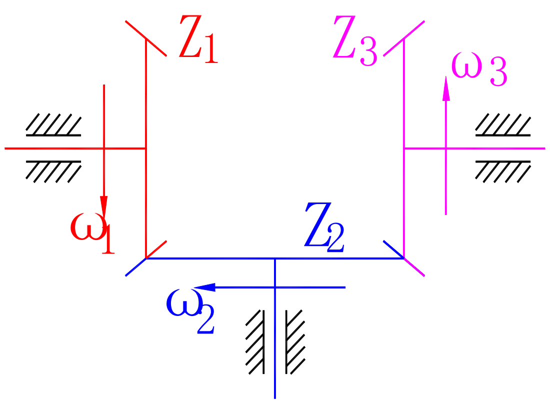


$$i_{14} = (-1)^3 \frac{Z_2 Z_3 Z_4}{Z_1 Z_2 Z_3} = -\frac{Z_4}{Z_1}$$

Attention: Gear 2 and 3 are **idle wheels**, which change the rotating direction of the output gear ,not the absolute value of the train ratio.

Example2: Calculate the Ratio i_{13}

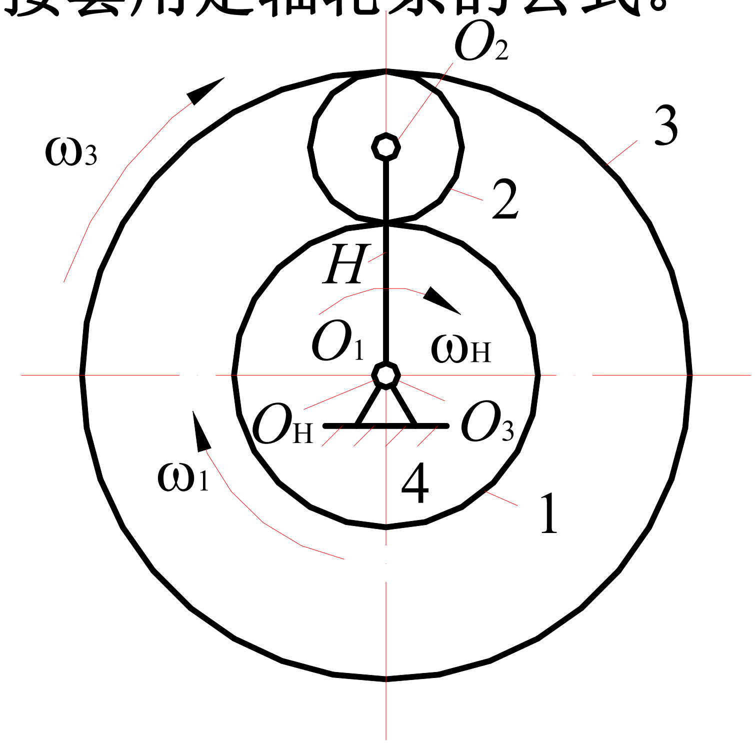
$$i_{13} = -\frac{Z_2 Z_3}{Z_1 Z_2} = -\frac{Z_3}{Z_1}$$

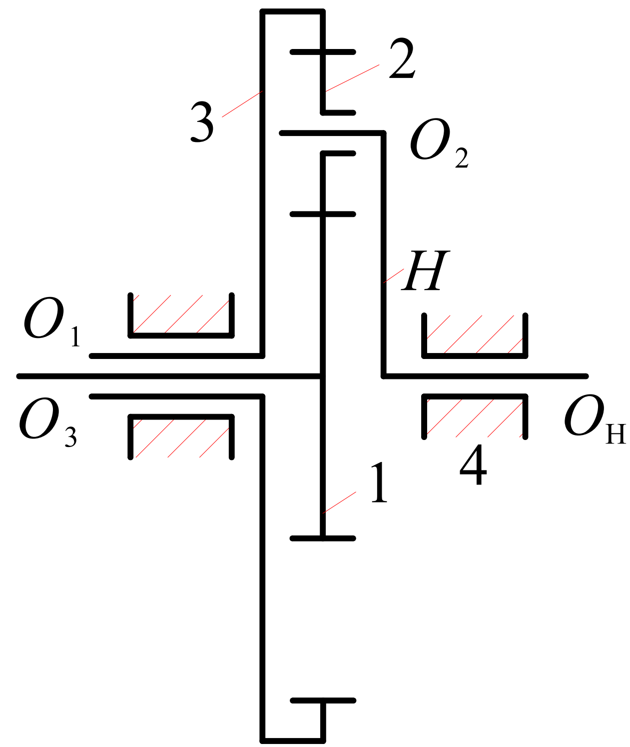


Attention:Rotating directions of gears in this gear trains with fixed non-parallel axes can not be determined by $(-1)^m$.

7.3 Train Ratio of Elementary Gear Train

周转轮系和定轴轮系的根本区别在于：在周转轮系中存在几何轴线运动的行星轮。因此，周转轮系的传动比不能直接套用定轴轮系的公式。





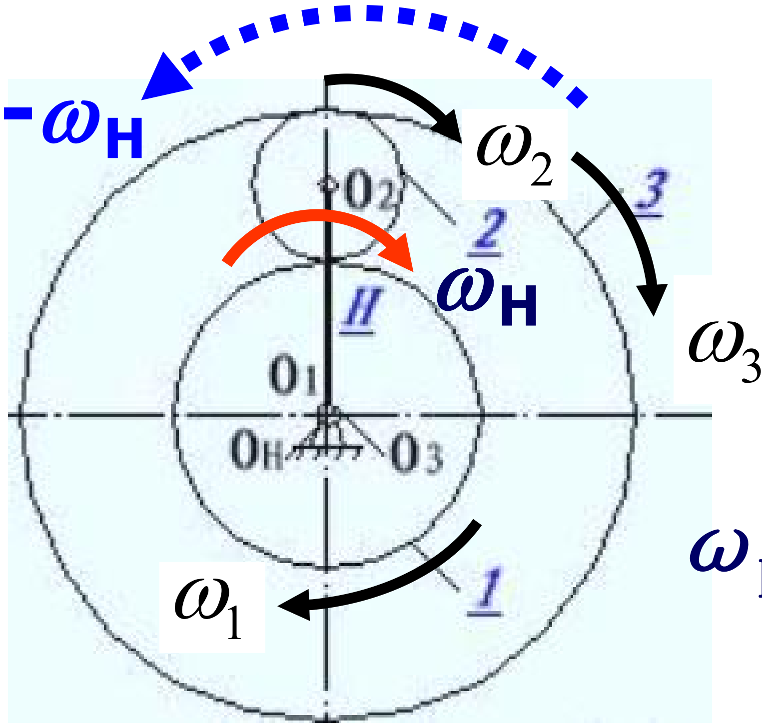
Add an angular velocity "-w_H" to the whole gear train

→converted gear train （转化轮系）

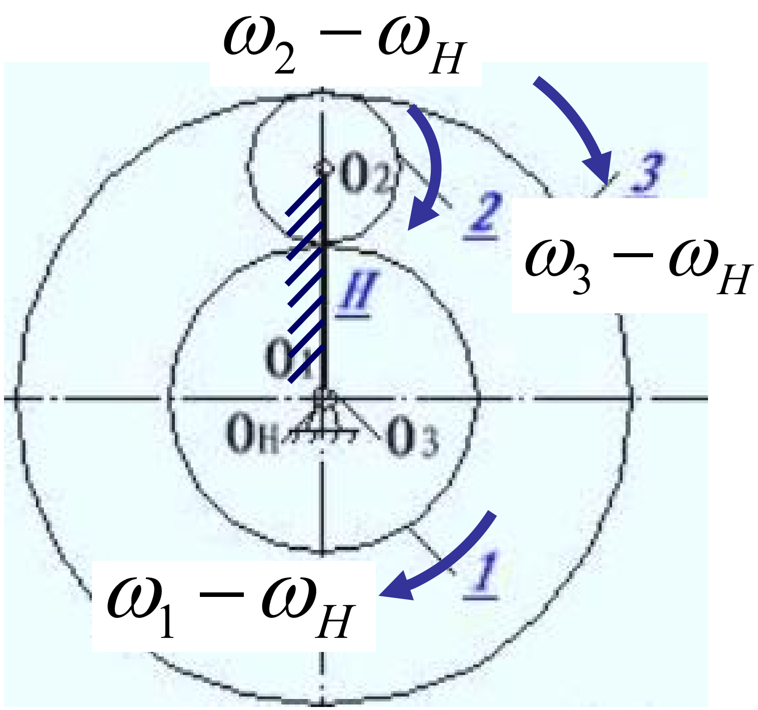
行星架的负角速度

Original elementary epicyclic gear train
原周转轮系

converted gear train
转化轮系



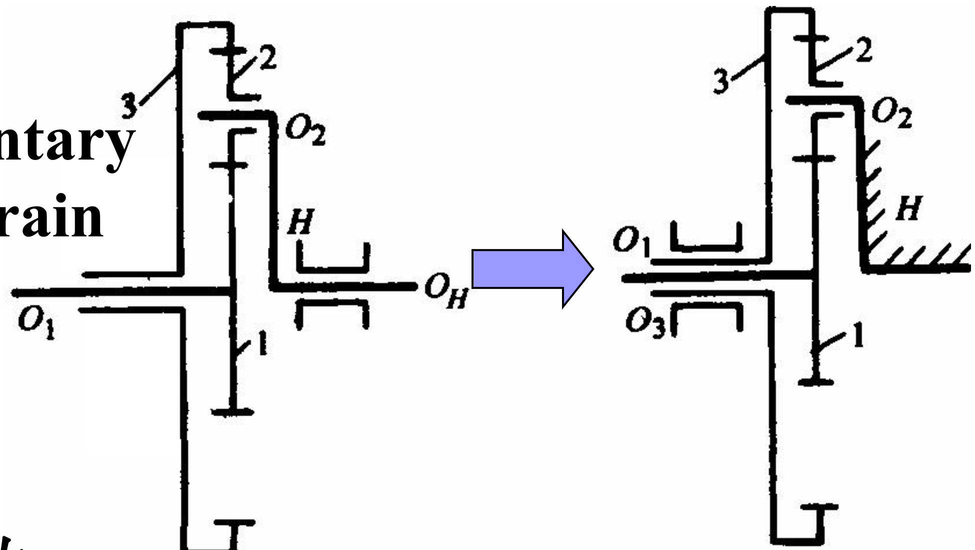
$\omega_H - \omega_H = 0$



Imaginary ordinary gear train
假想定轴轮系

构件	原周转轮系角速度	转化轮系各构件的角速度
1	ω_1	$\omega_1^H=\omega_1-\omega_H$
2	ω_2	$\omega_2^H=\omega_2-\omega_H$
3	ω_3	$\omega_3^H=\omega_3-\omega_H$
H	ω_H	$\omega_H^H=\omega_H-\omega_H=0$

Original elementary epicyclic gear train
原周转轮系



converted gear train
转化轮系
↓
Ordinary gear train
定轴轮系

转化轮系的传动比按定轴轮系计算：

$$i_{13}^H=\frac{\omega_1^H}{\omega_3^H}=\frac{\omega_1-\omega_H}{\omega_3-\omega_H}=(-1)^1\frac{z_2\cdot z_3}{z_1\cdot z_2}=-\frac{z_3}{z_1}$$

推广到一般情况：

任意周转轮系的两中心轮A、B及行星架H的关系：

$$i_{AB}^{H}=\frac{\omega_A^H}{\omega_B^H}=\frac{\omega_A-\omega_H}{\omega_B-\omega_H}=f(z)$$

1. 该公式适用于齿轮A、B及行星架H三者轴线重合的机构。

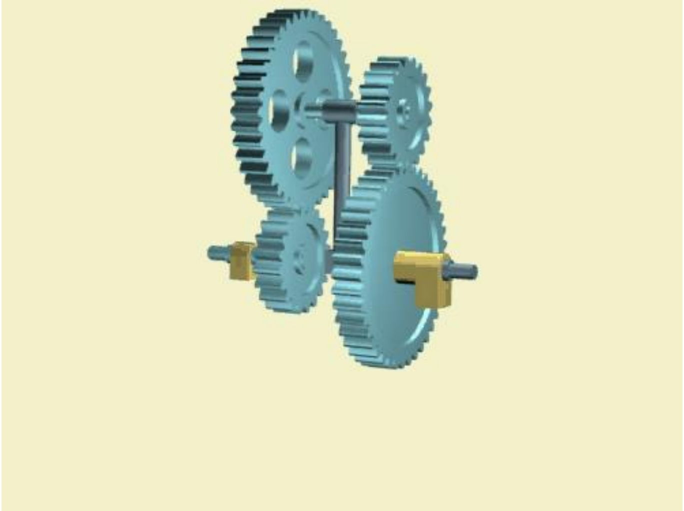
2. 对于差动轮系，若已知的两个构件转速相反，代入公式时，规定一个方向为正，另一方向就代以负值，求出的第三个构件的转速根据其符号来确定转向。

3. 周转轮系的传动比正负由计算求得，不需判断。

4. f(z)由定轴轮系的方法求得，f(z)的正负号反映转化轮系中A,B轮间转向关系。

例1：Given: $Z_1=100$, $Z_2=101$, $Z_{2'}=100$, $Z_3=99$.

Find the train ratio i_{H1}



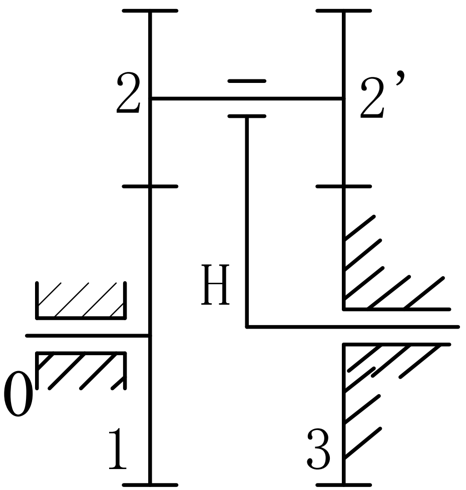
Solutions

$$\begin{cases} i_{13}^H = \dfrac{\omega_1^H}{\omega_3^H} = \dfrac{\omega_1 - \omega_H}{\omega_3 - \omega_H} = (-1)^2 \dfrac{Z_3 Z_2}{Z_1 Z_{2'}} \\ \omega_3 = 0 \end{cases}$$

$$\frac{\omega_1 - \omega_H}{0 - \omega_H} = \frac{Z_3 \cdot Z_2}{Z_1 \cdot Z_{2'}}$$

$$-\frac{\omega_1}{\omega_H} + 1 = \frac{Z_3 \cdot Z_2}{Z_1 \cdot Z_{2'}}$$

$$\therefore \quad i_{H1} = \frac{\omega_H}{\omega_1} = \frac{1}{1 - \dfrac{Z_3 \cdot Z_2}{Z_1 \cdot Z_{2'}}} = \frac{1}{1 - \dfrac{99 \times 101}{100 \times 100}} = 10000$$

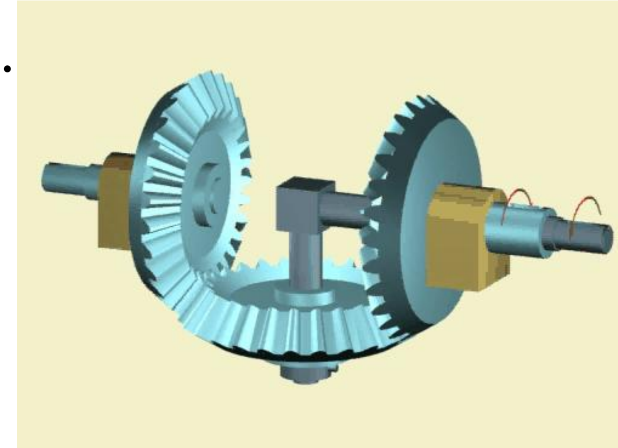


周转轮系的传动比正负由计算求得，不需判断。

若$Z_1=99$，$i_{H1} = -100$

例 2： Given: $Z_1 = Z_3$ ，$\omega_3 = 12 rad/s$ ，$\omega_H = 8 rad/s$

Their directions are shown in the Fig.



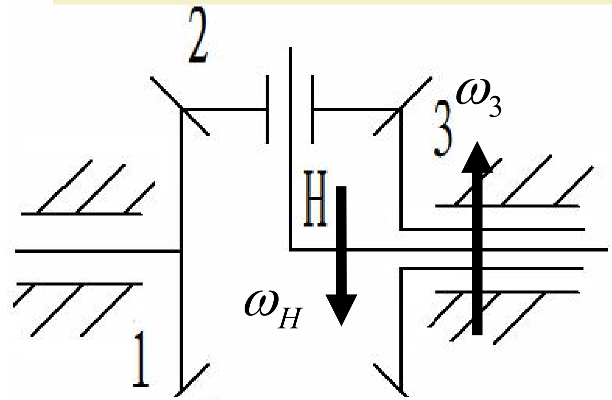
Find: ω_1

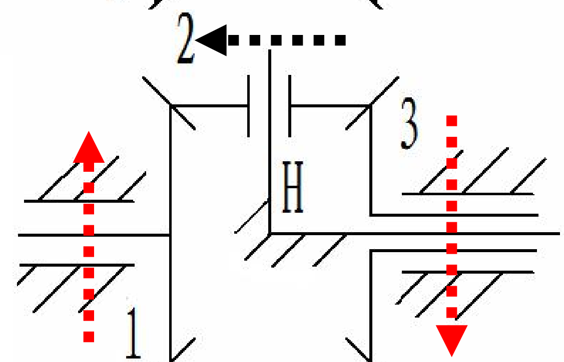
Sol： $i_{13}^{H} = \dfrac{\omega_1^H}{\omega_3^H}$

$$= \frac{\omega_1 - \omega_H}{\omega_3 - \omega_H}$$

$$= -\frac{Z_2 Z_3}{Z_1 Z_2}$$

$$= -1$$





一正一负带入 ω_H和ω_3

$$\frac{\omega_1 - 8}{-12 - 8} = -1 \quad \therefore \quad \omega_1 = 28 \text{rad/s}$$

转向与 ω_H 相同

7.4 Train Ratio of a Combined Gear Train

Steps：

Find Sequence(顺序): Planet gear → planet carriers → sun gear

1. Divide the combined gear train into several **epicyclic gear trains** and ordinary gear trains

2. Derive(推导) their ratio equations independently

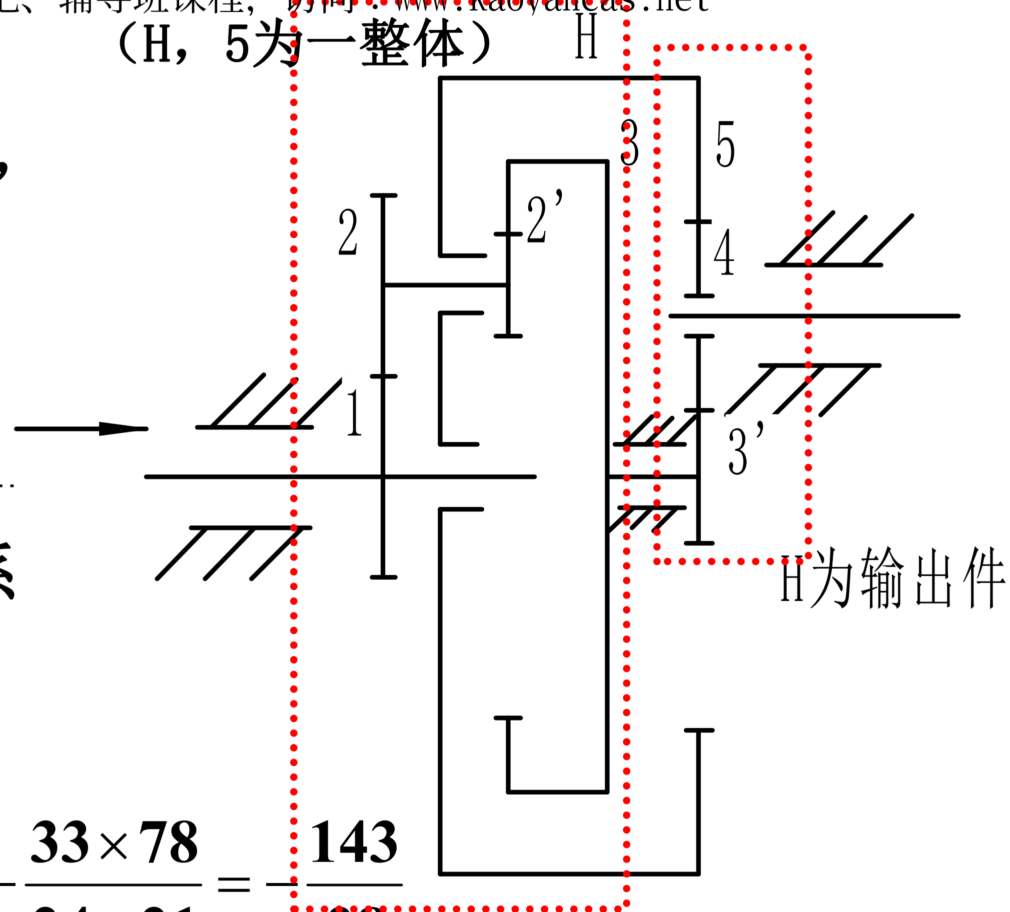
3. Find the relationship between several gear trains

4. Solve the equations simultaneously （联立方程求解）

Example:

Given: $Z_1=24$, $Z_2=33$, $Z_{2'}=21$, $Z_3=78$, $Z_{3'}=18$, $Z_4=30$, $Z_5=78$, $n_1=1500r/min$.

Find: n_5 ?



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(1) 1，2-2'，3，H——周转轮系

3'，4，5——定轴轮系

(2)
$$i_{13}^{H}=\frac{\omega_1-\omega_H}{\omega_3-\omega_H}=(-1)^1\frac{Z_2Z_3}{Z_1Z_{2'}}=-\frac{33\times78}{24\times21}=-\frac{143}{28}$$

$$i_{3'5}=\frac{\omega_{3'}}{\omega_5}=-\frac{Z_5Z_4}{Z_4Z_{3'}}=-\frac{78}{18}=-\frac{13}{3}$$

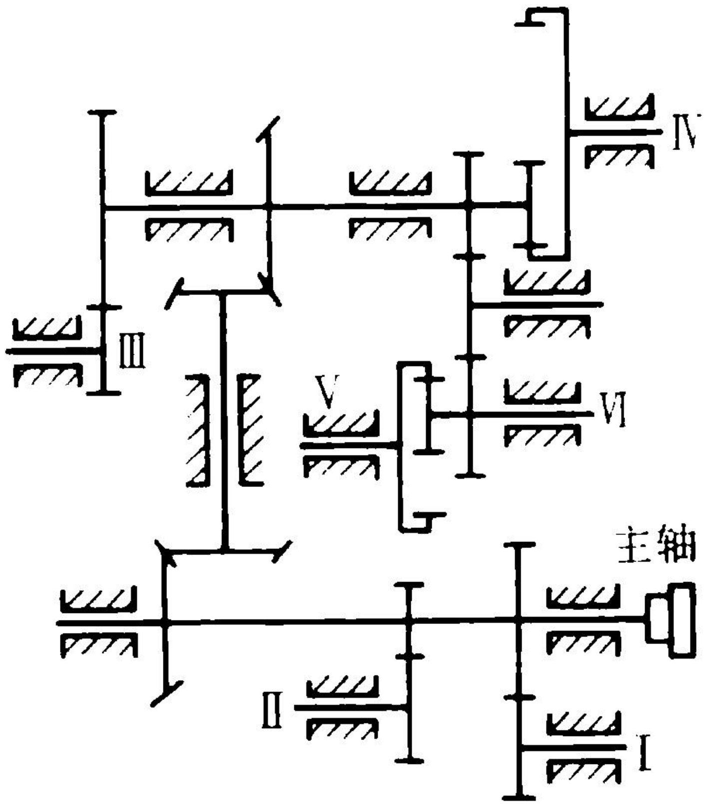
n_5与n_1转向相同。

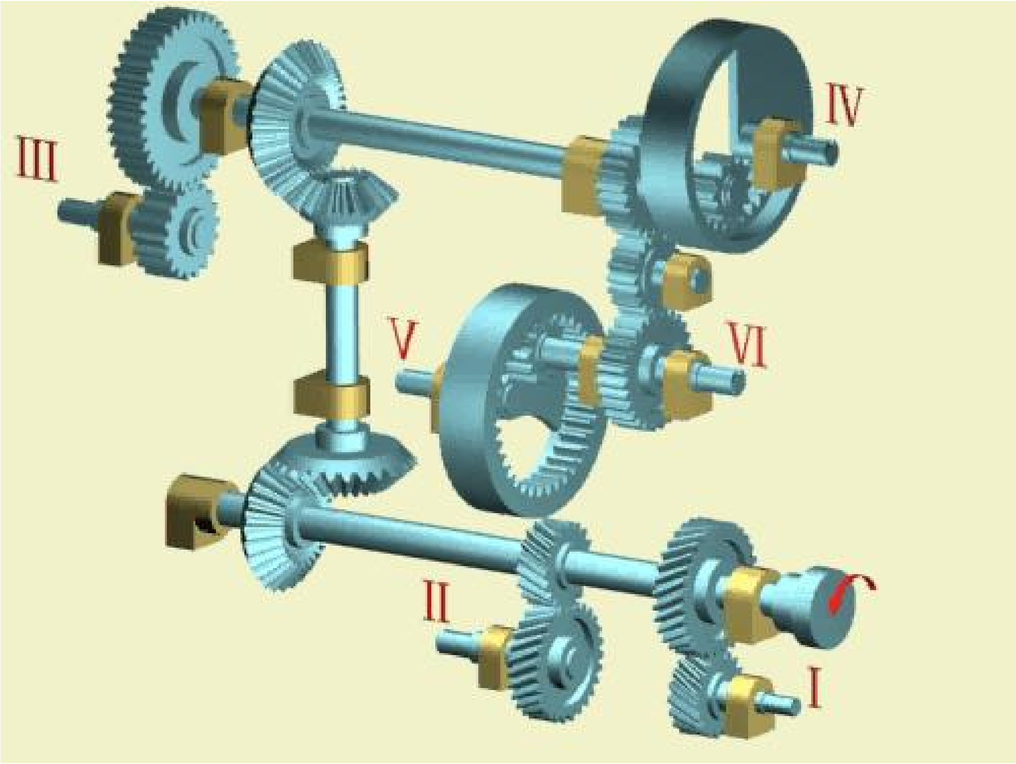
(3)
$$\left.\begin{aligned}\omega_3&=\omega_{3'}\\ \omega_H&=\omega_5\end{aligned}\right\}$$

(4) $i_{1H}=i_{15}=28.24$ $\quad n_5=\frac{n_1}{i_{15}}=\frac{1500}{28.24}\approx53.12r/\min$

7.5 Applications of Gear Trains

1) Branching transmission 多支路输出

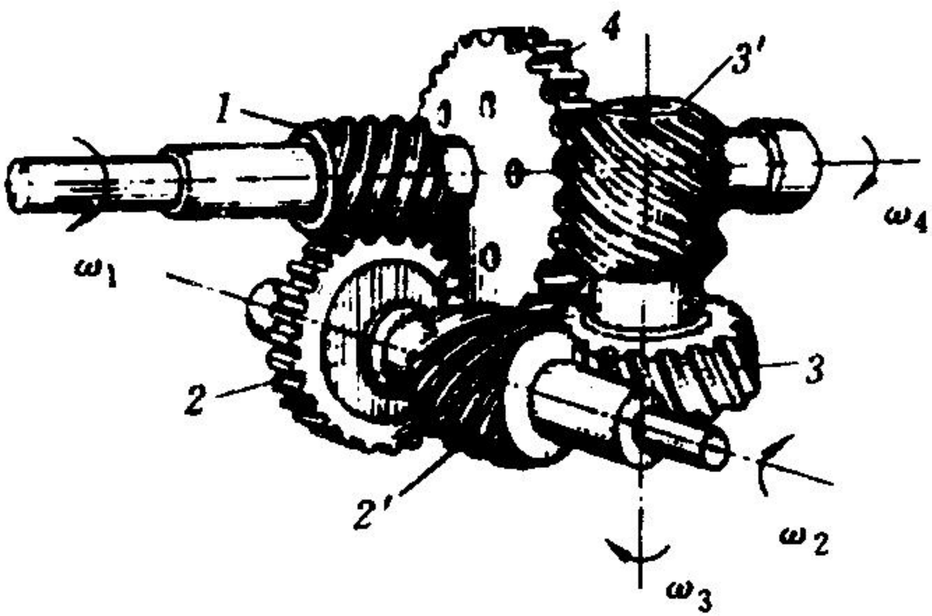


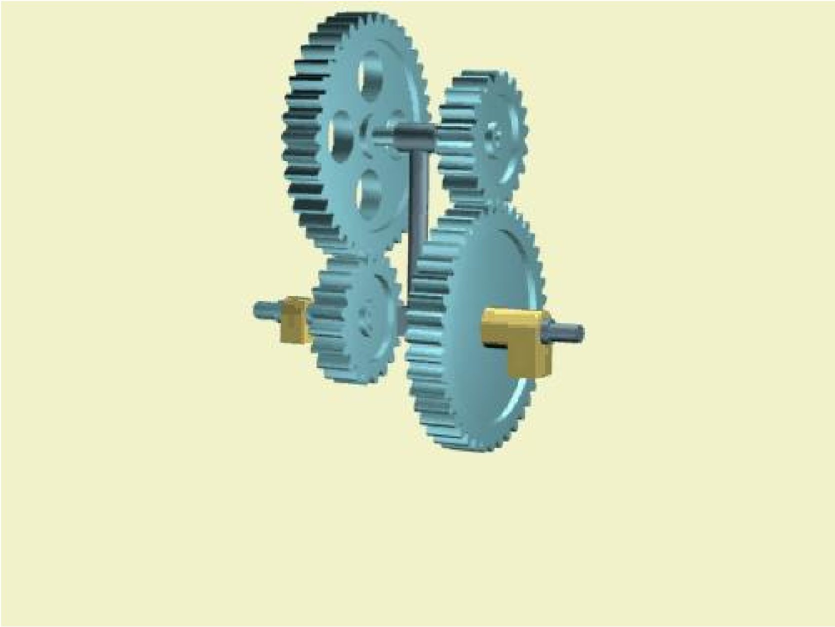


7.5 Applications of Gear Trains

1) Branching transmission 多支路输出

2) To get a large train ratio





如图中定轴轮系的传动比为8000

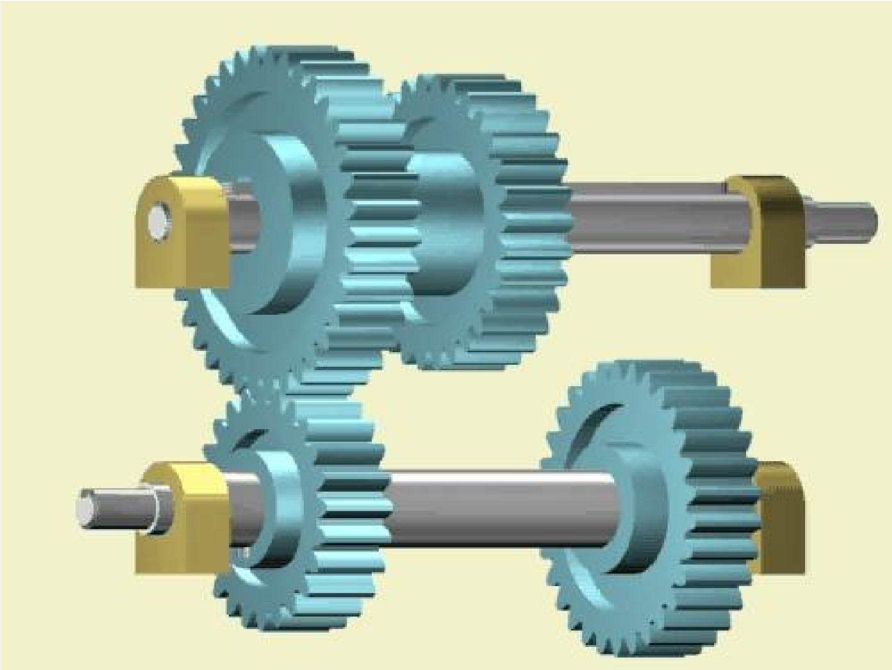
如图中行星轮系的传动比为10000

7.5 Applications of Gear Trains

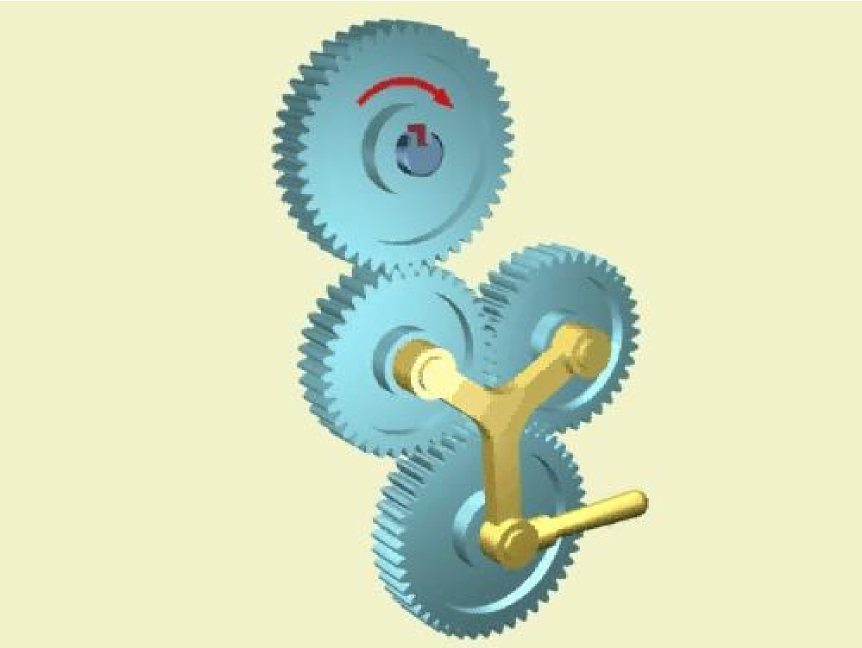
1) Branching transmission 多支路输出

2) To get a large train ratio

3) To change the speed of rotation



变速传动



换向机构

7.5 Applications of Gear Trains

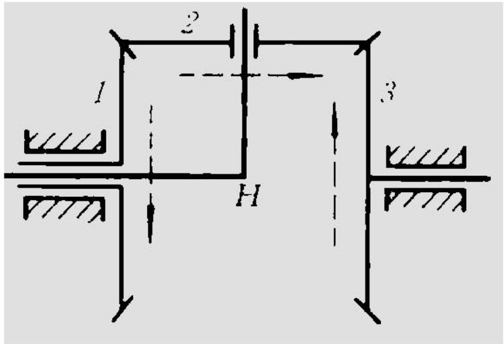
1) Branching transmission 多支路输出

2) To get a large train ratio

3) To change the speed of rotation

4) To combine or resolve the motion

利用差动轮系的双自由度特点，可把两个运动合成为一个运动。图示的差动轮系就常被用来进行运动的合成。



对图所示的差动轮系，若$Z_3 = Z_1$，则

$$i_{13}^{H} = \frac{n_1 - n_H}{n_3 - n_H} = -\frac{z_3}{z_1} = -1 \Rightarrow n_H = 0.5(n_1 + n_3)$$

此轮系可作机械式计算机中的和差运算机构。

Rear wheel differential gear train of an automobile

7.5 Applications of Gear Trains

1) Branching transmission 多支路输出

2) To get a large train ratio

3) To change the speed of rotation

4) To combine or resolve the motion

7.6 Mechanical Efficiency of Planetary Gear trains

7.7 Tooth number of Gear and Number of planet Gears

7.8 Introduction to Other Kinds of Planetary Gear Trains

Chapter 7
Attentions

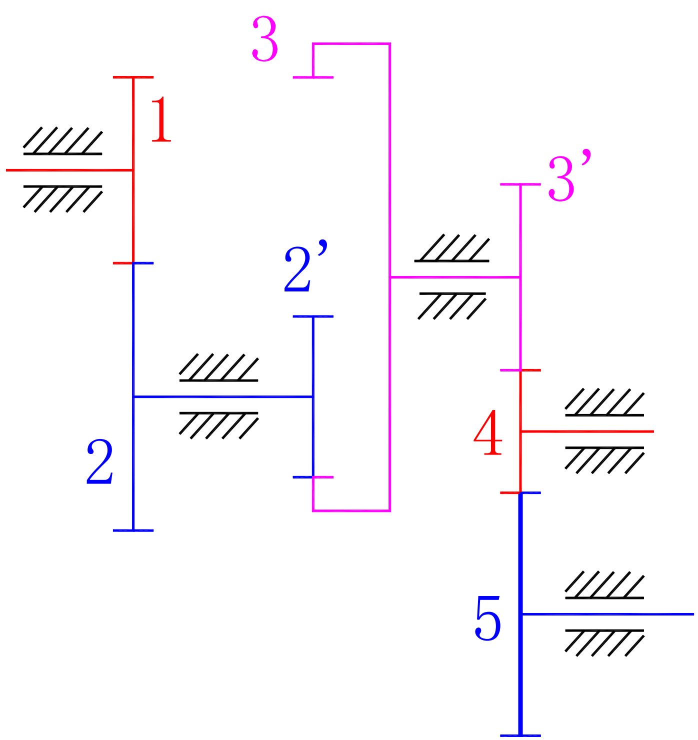
1. Calculate the ratio of Elementary Epicyclic gear train.

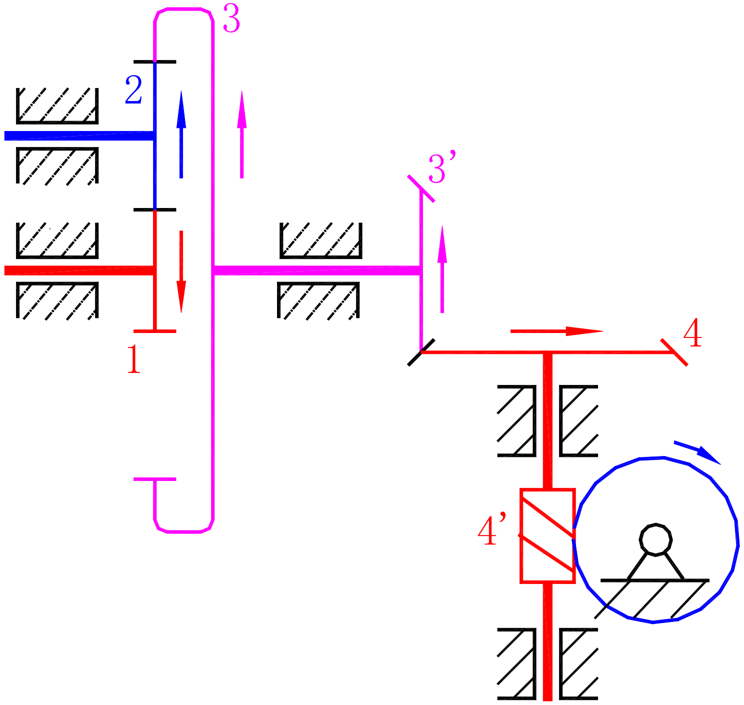
2. Calculate the ratio of Combined gear trains .

Homework: 7-9,7-11,7-13

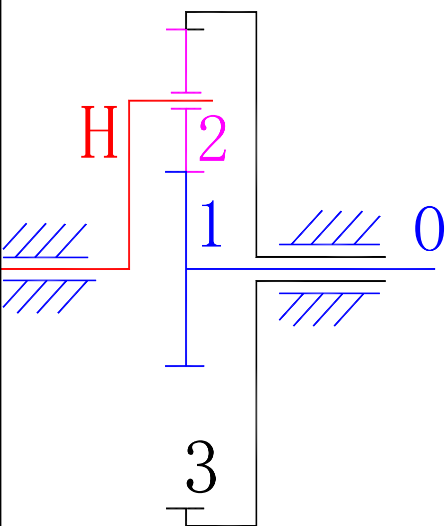
Gear train with fixed axes(定轴轮系) (ordinary gear train)

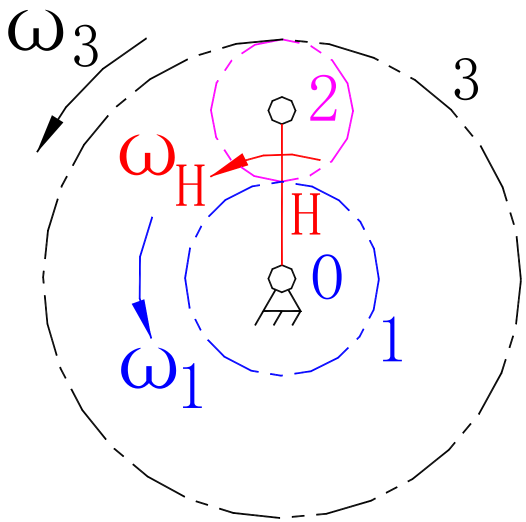
The positions of all gear axes in a gear train are fixed.

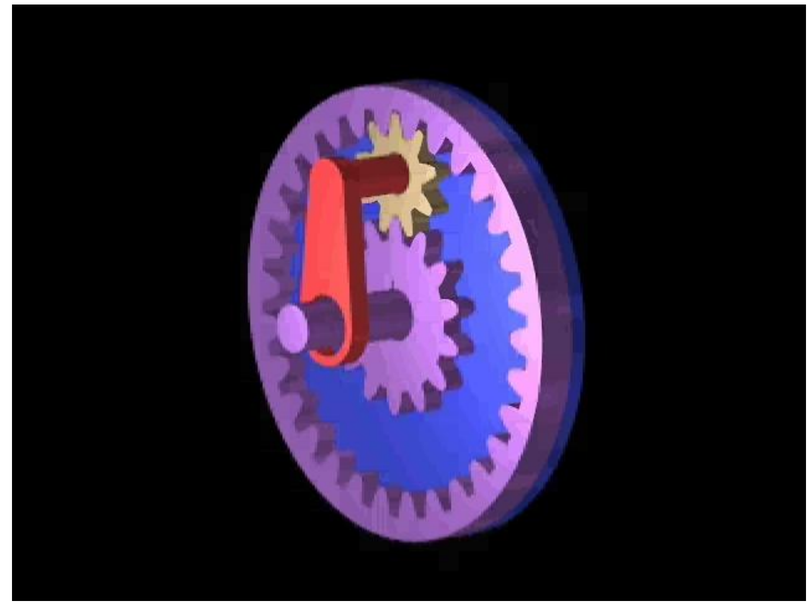




Epicyclic gear train(周转轮系)---- at least one gear axis rotates around the other axis.







Planet gear (行星轮) —— The gear the axis of which rotates around the fixed axis. 2

Planet carrier(行星架,系杆) —— The link carrying planet gears, denoted by H.

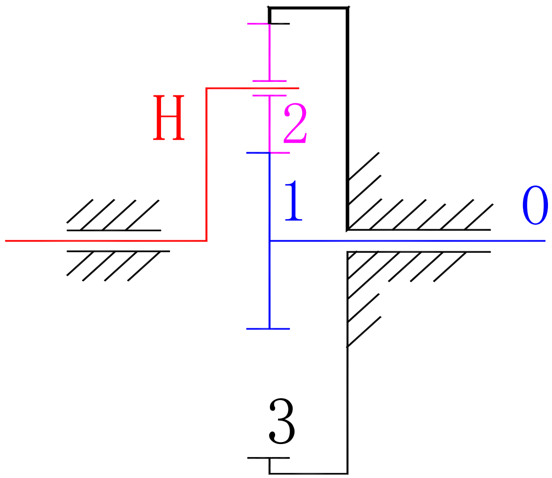
Sun gears(太阳轮,中心轮) —— Engage with planet gears and rotate around the fixed axis. 1,3

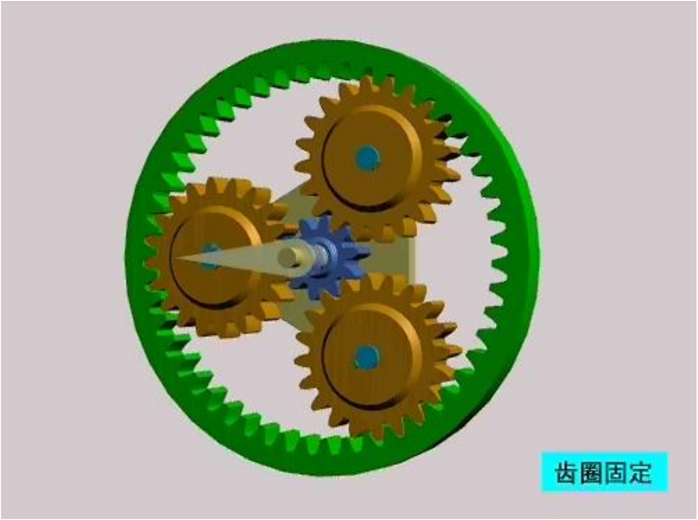
fundamental members

Planetary gear train(行星轮系)

One of sun gears is fixed with the frame.

DOF: $F=3n-2P_L-P_h=3x3-2x3-2=1$





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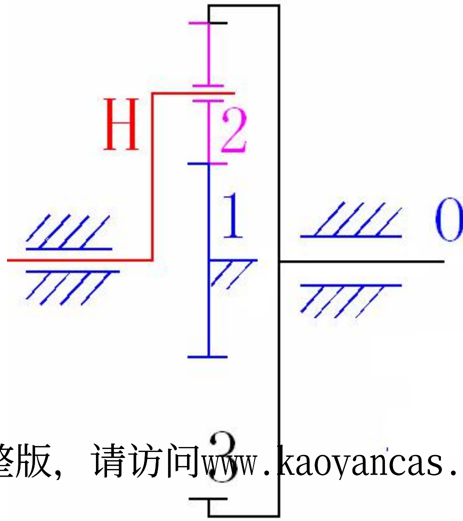
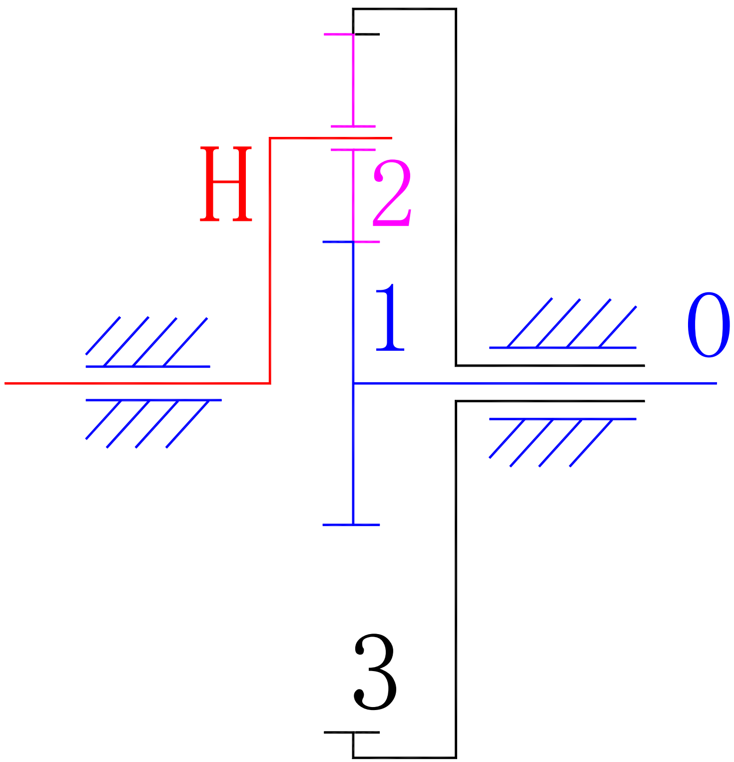
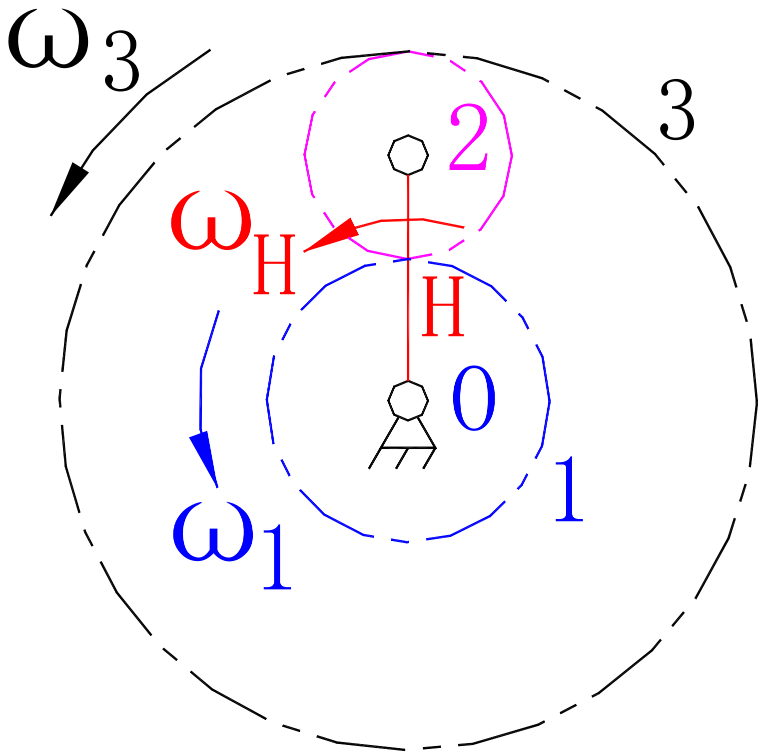


Differential gear train(差动轮系)

Neither of the two sun gears is fixed.

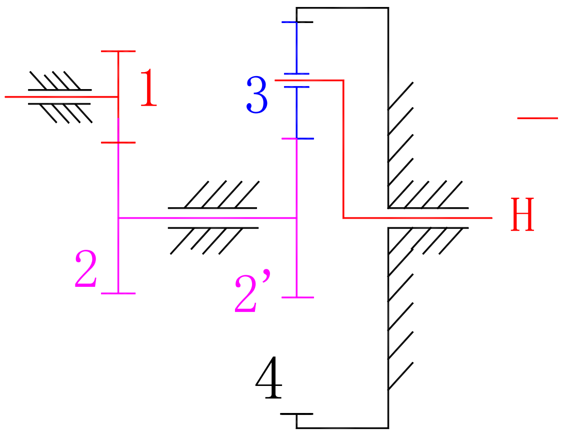
DOF： $F=3n-2P_L-P_h=3x4-2x4-2=2$

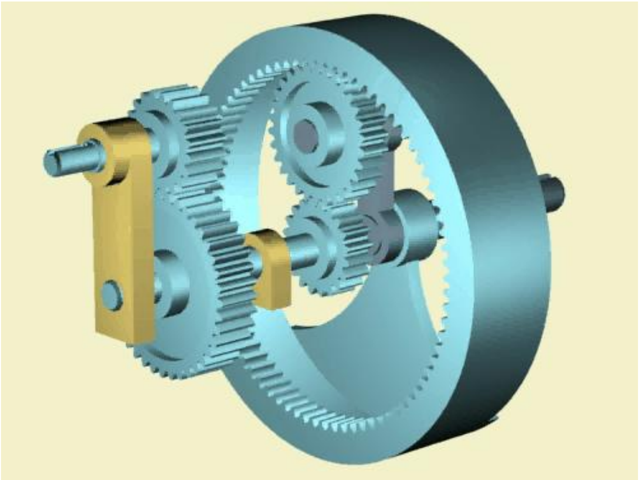


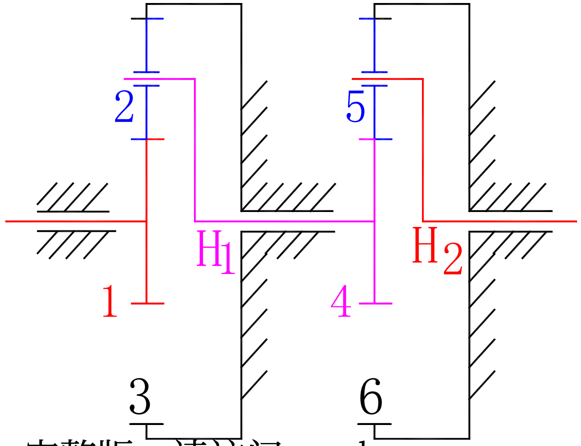


Combined gear trains(复合轮系)

combination of at least one ele**men**tary epi**cy**clic gear train with at least one ordinary gear train or another ele**men**tary epi**cy**clic gear train.

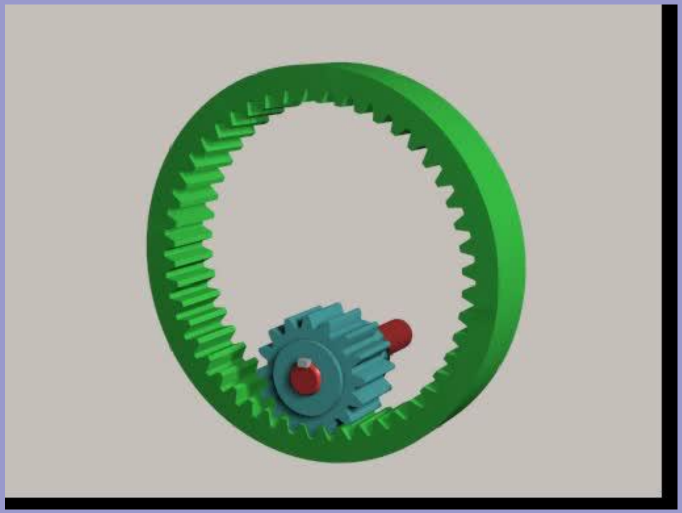


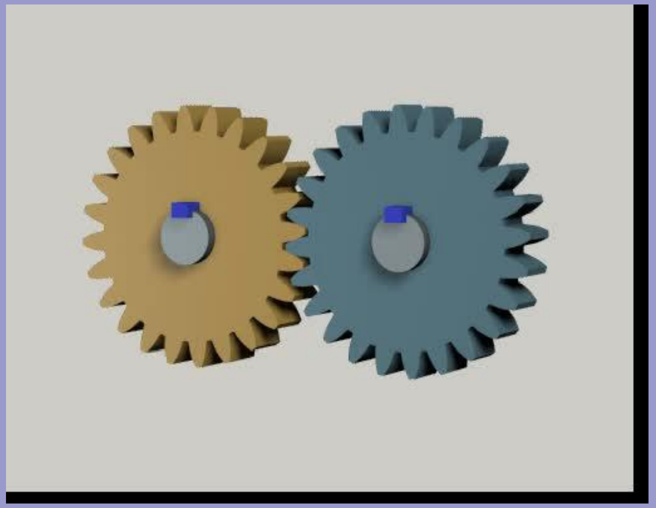




Planar ordinary gear train （平面定轴轮系）

内啮合转向相同 外啮合转向相反



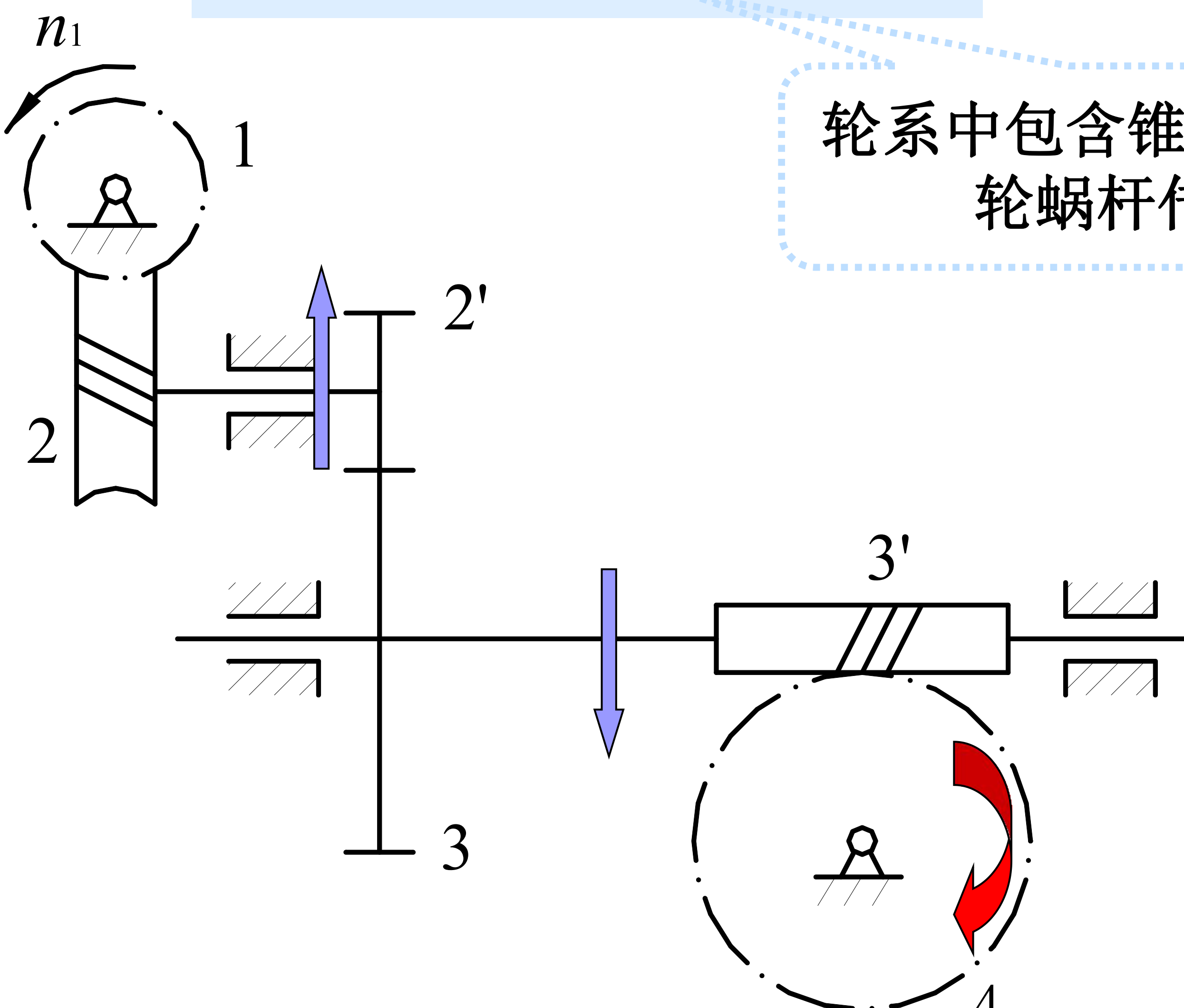


所以，平面定轴轮系的传动比为：

$$i = \frac{\omega_1}{\omega_5} = (-1)^m \frac{\text{所有从动轮齿数的乘积}}{\text{所有主动轮齿数的乘积}}$$

m——外啮合的次数

Spatial ordinary gear train
（空间定轴轮系）

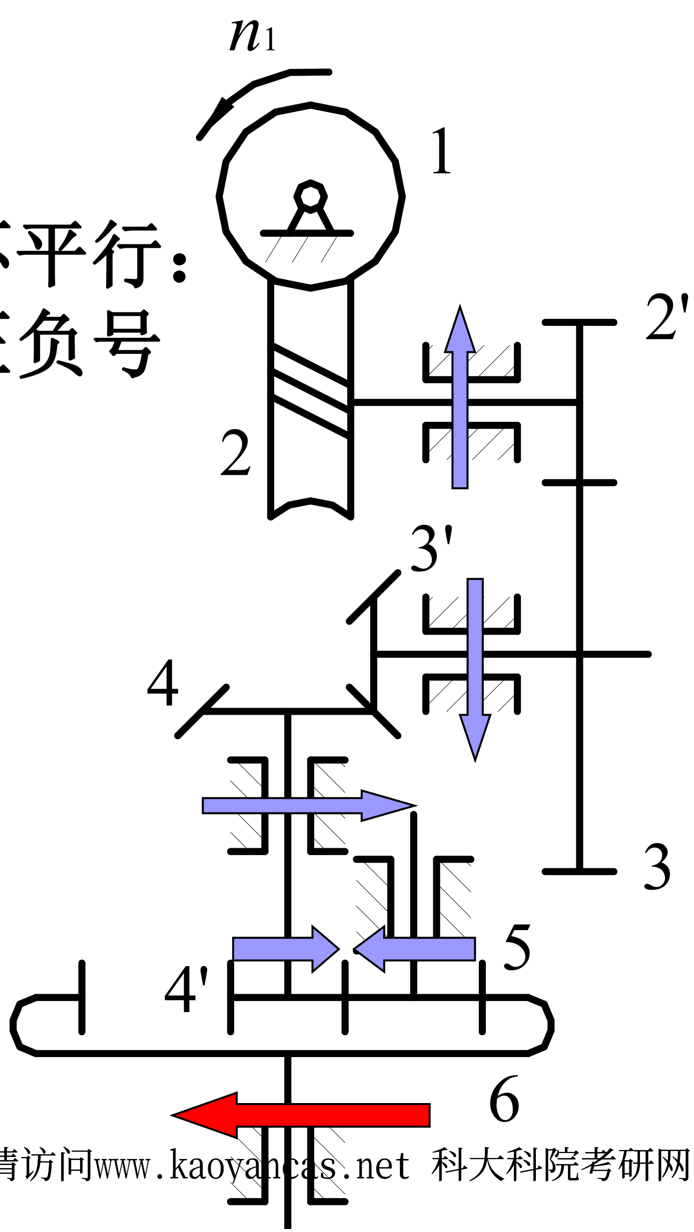


轮系中包含锥齿轮、蜗轮蜗杆传动

确定传动比正负：在图上画箭头判别，再添加正负号。

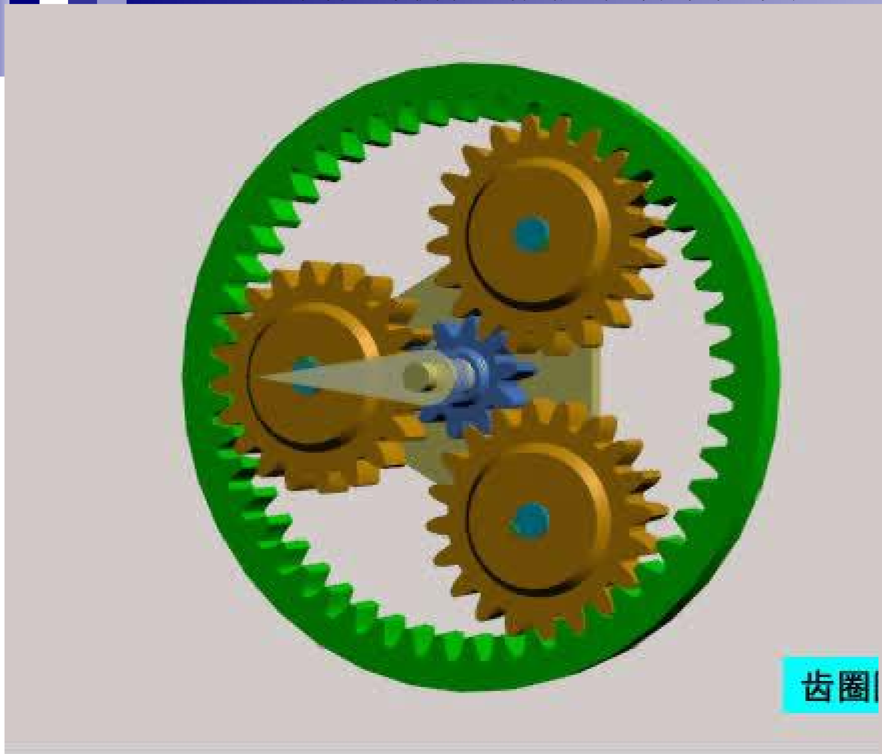
Gear trains with fixed non-parallel axes.

输入输出轴不平行：方向不能用正负号来表示。



传动比方向表示：

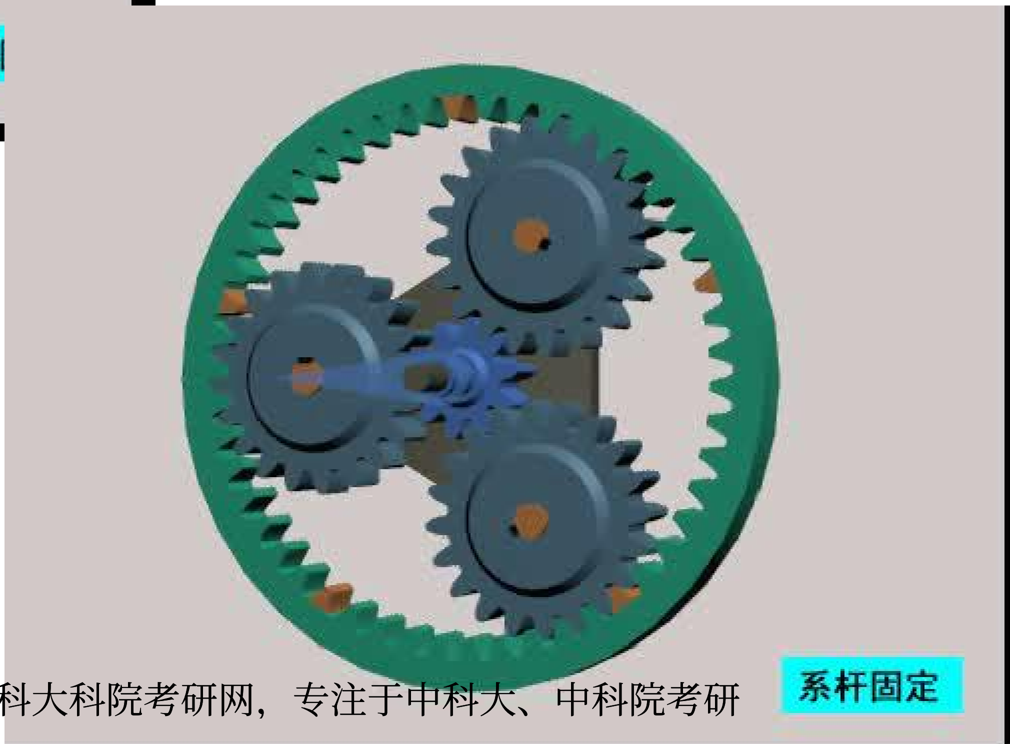
从输入轴到输出轴各轮轴逐一画箭头表示。



原周转轮系

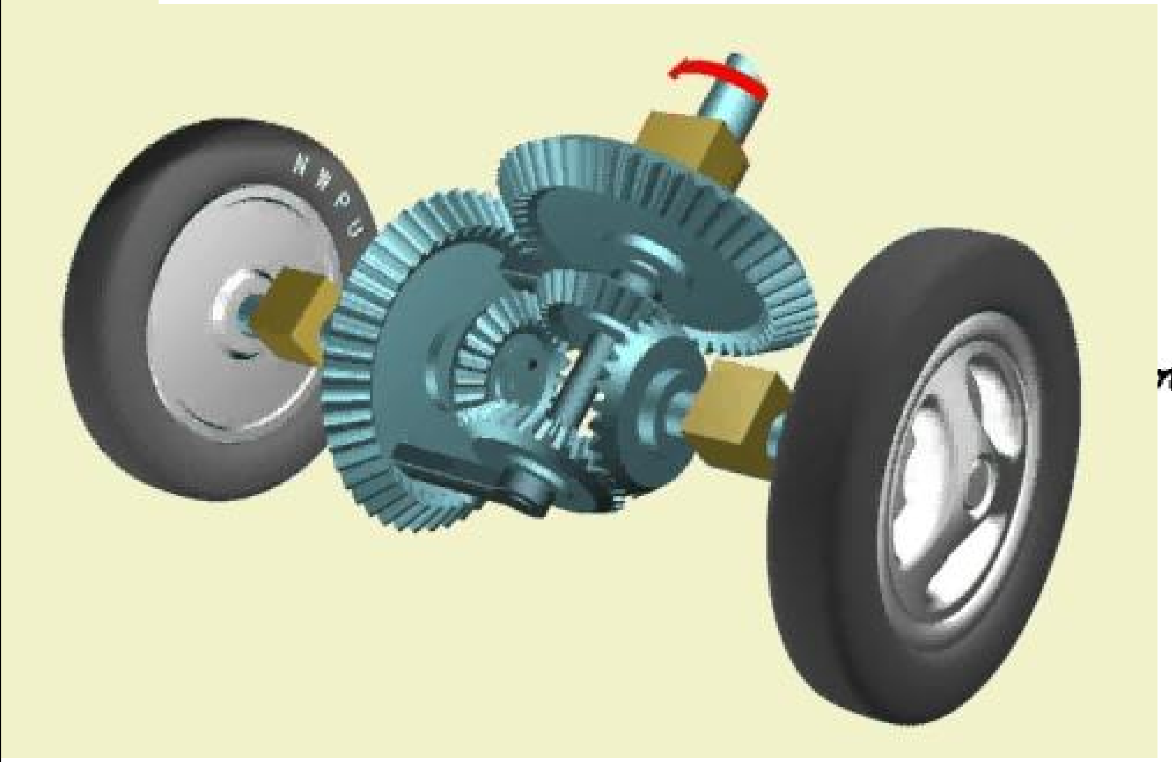
转化轮系

定轴轮系



Rear wheel differential gear train

汽车后桥的差速器



差动轮系不仅能将两个独立的运动合成为一个，而且还可将一个基本构件的主动转动按所需比例分解成另两个基本构件的不同运动。

整个轮系是由定轴轮系(轮5和轮4)和差动轮系(轮1、2、3、和4)组成的一个复合轮系。

当直线行驶时，$n_1= n_3$：齿轮2和齿轮1（或3）之间没有相对运动，轮2不绕自已的轴线转动，轮1、2、3和4如同一体。

当转向行驶时，$n1\neq n3$：轮2不仅随轮4作公转，而且还绕自身轴线自转，是行星轮，轮4为系杆，而轮1和3为二个中心轮。

对差动轮系：1、2、3、4(H)，因$Z_1 = Z_3$，可知：

$$i_{13}^{4} = \frac{n_1 - n_4}{n_3 - n_4} = -\frac{z_3}{z_1} = -1 \longrightarrow n_4 = \frac{1}{2}(n_1 + n_3)$$

根据轮胎和地面之间作纯滚动的要求：n_1和n_3应与弯道半径成正比，可得左转时：

$$\frac{n_1}{n_3} = \frac{r - L}{r + L}$$