



Theory of Machines and Mechanisms

(机械原理)



Chapter1 Introduction

Chapter2 Structural Analysis of Planar Mechanisms

Chapter3 Kinematic Analysis of Mechanisms

Chapter4 Planar Linkage Mechanisms

Chapter 5 Cam Mechanisms

Chapter 6 Gear Mechanisms

Chapter 7 Gear Trains

Chapter 8 Other Mechanisms in Common Use

Chapter 9 Balancing of Machinery

Chapter 10 Motion of Mechanical Systems and Its Regulation

Chapter 11 Efficiency of Machine



Chapter 4

Planar Linkage Mechanisms



本章重点： 平面四杆机构的类型、
基本知识和设计。

本章难点： 平面四杆机构的设计。



4.1 Characteristics of Planar Linkage Mechanisms

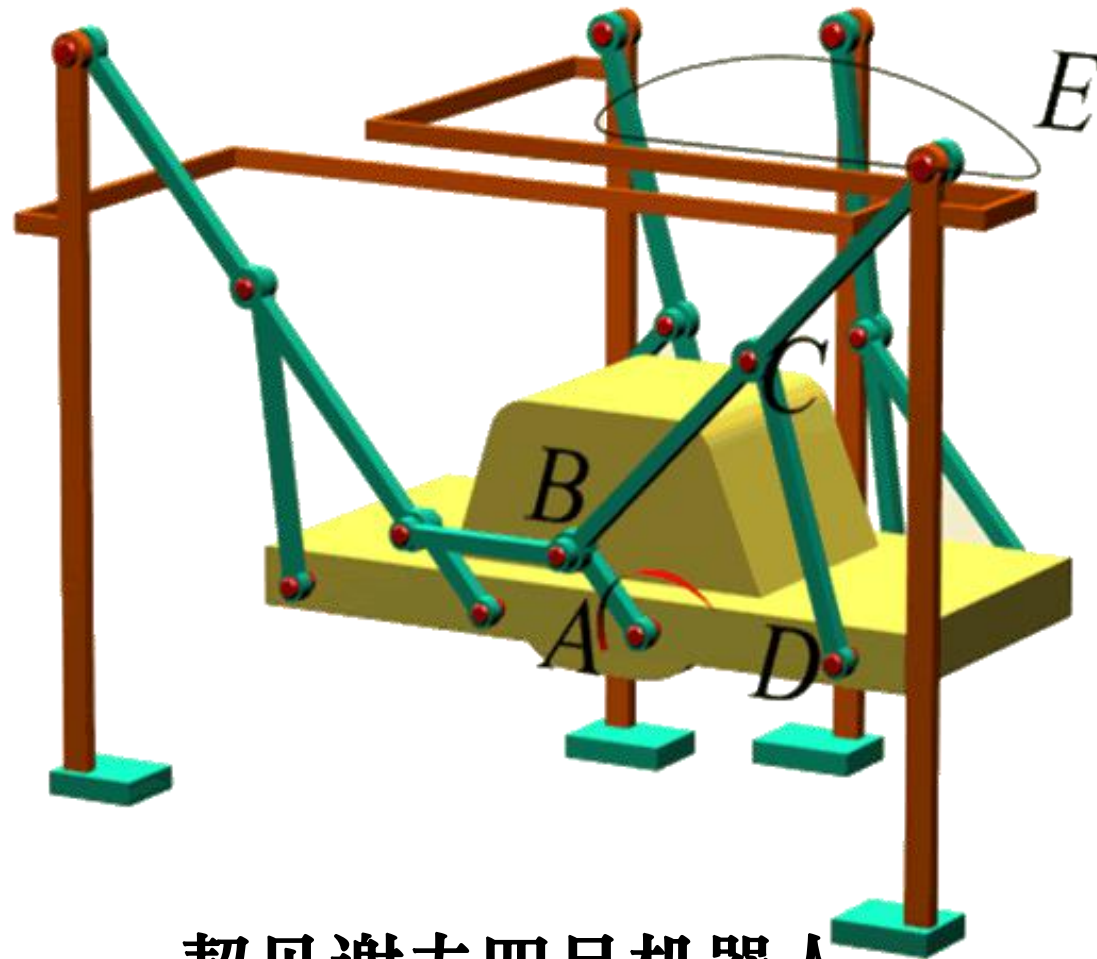
4.2 The Types of Four-bar Linkages

4.3 Characteristics Analysis of Four-bar Linkages

4.4 Dimensional Synthesis of Four-bar Linkages

4.1 Characteristics of Planar Linkage Mechanisms

all kinematic pairs → lower pairs



契贝谢夫四足机器人



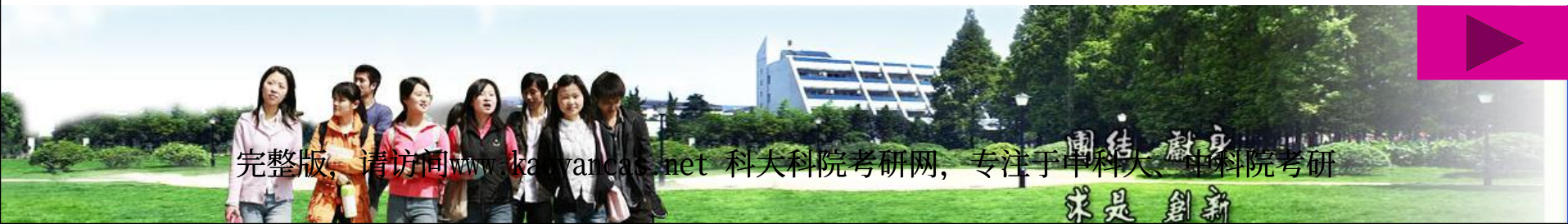
Advantages:

- 1.The main advantage is their better ability to trap lubricant between enveloping surface.
(便于润滑、寿命长)The contact pressure is lower.**
- 2.The lower pairs are easy to manufacture.**

Disadvantages:

- 1.It is difficult to perform precision motion**
- 2.Low speed**
- 3.Difficult to balance**

So,the linkage is preferred for low speed and heavy load situation.



4.2 The Types of Four-bar Linkages

4.2.1 The Basic Form of Four-bar Mechanism





Revolute Four-bar Mechanism 铰链四杆机构

所有运动副均为转动副的机构

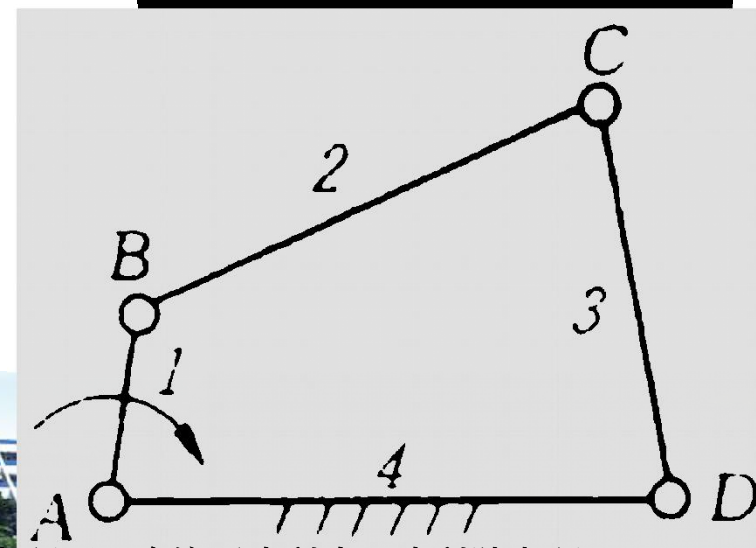
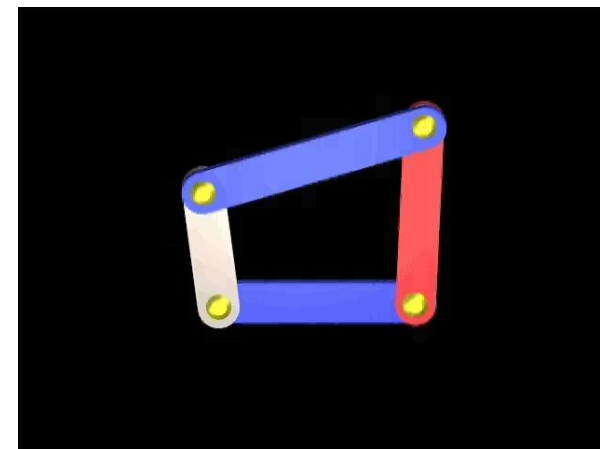


side links(连架杆)——1 & 3

coupler (连杆) -----2

crank (曲柄) 1

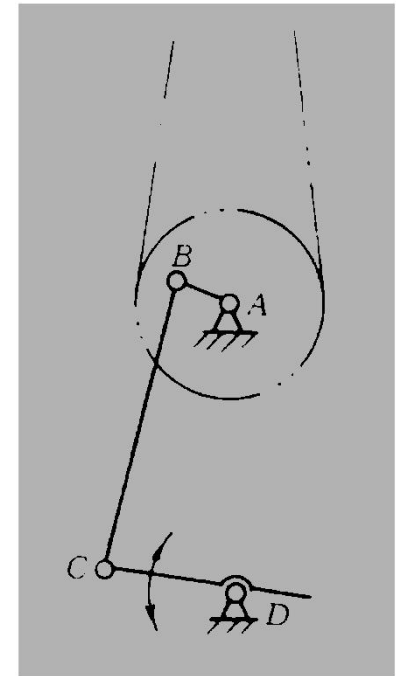
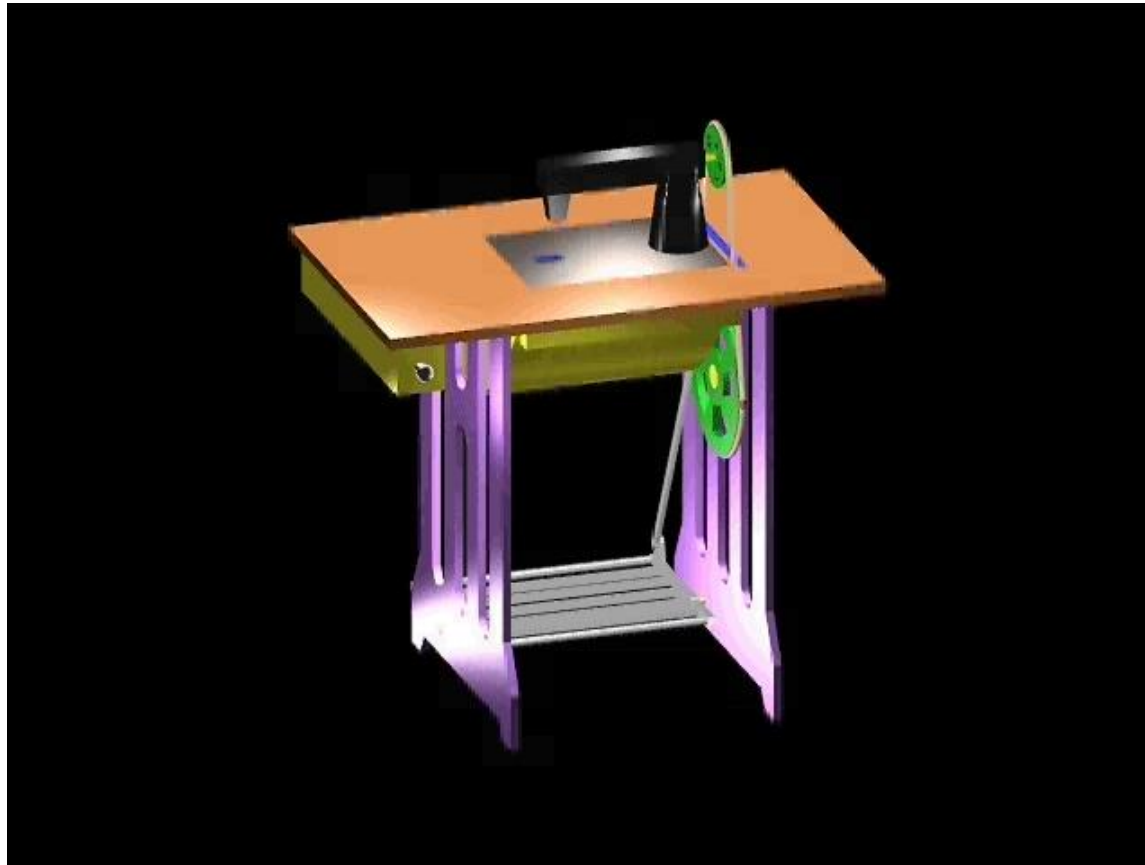
rocker (摇杆) 3



(a) Crank-rocker mechanism

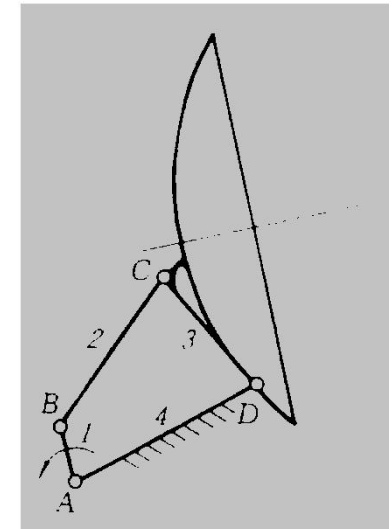
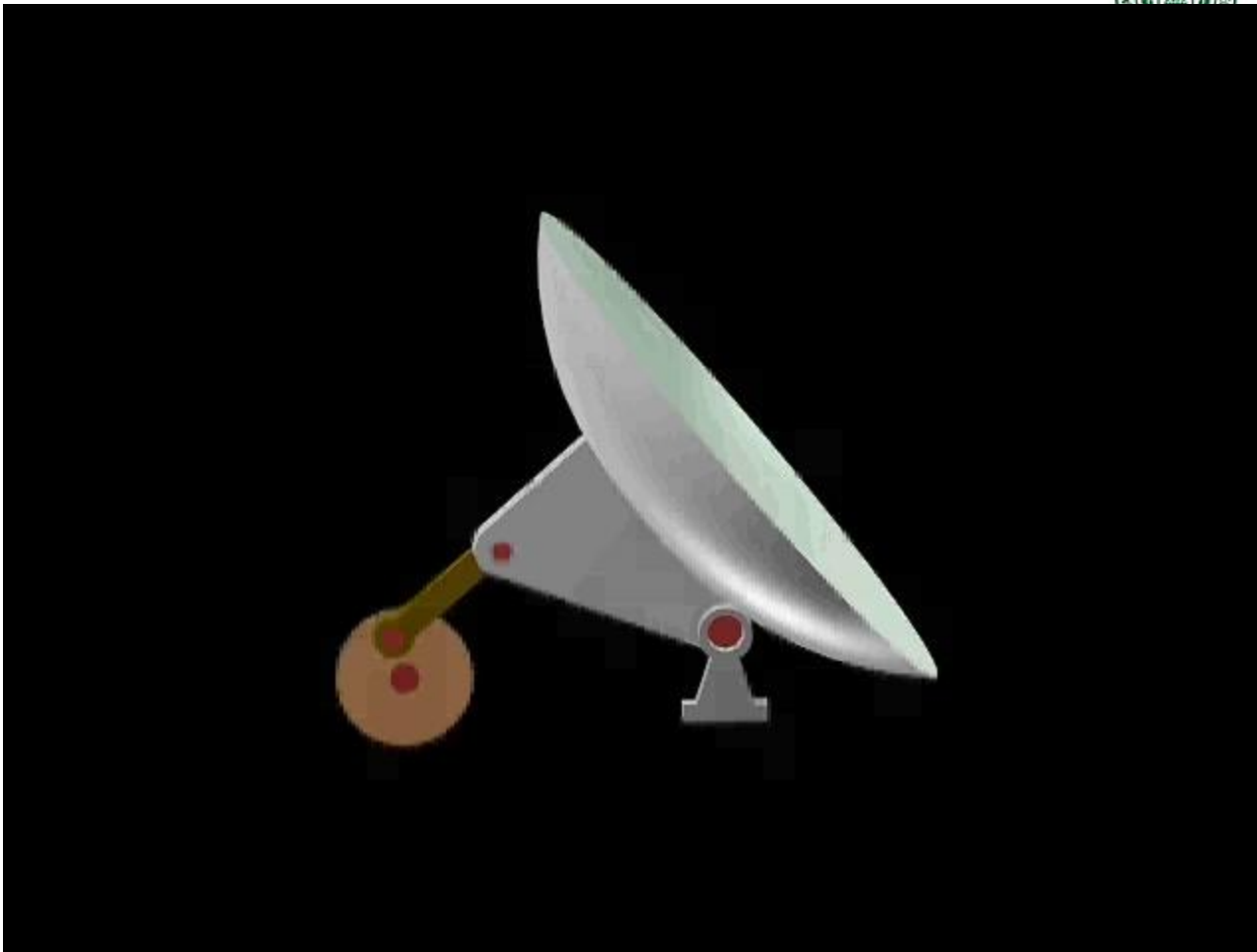


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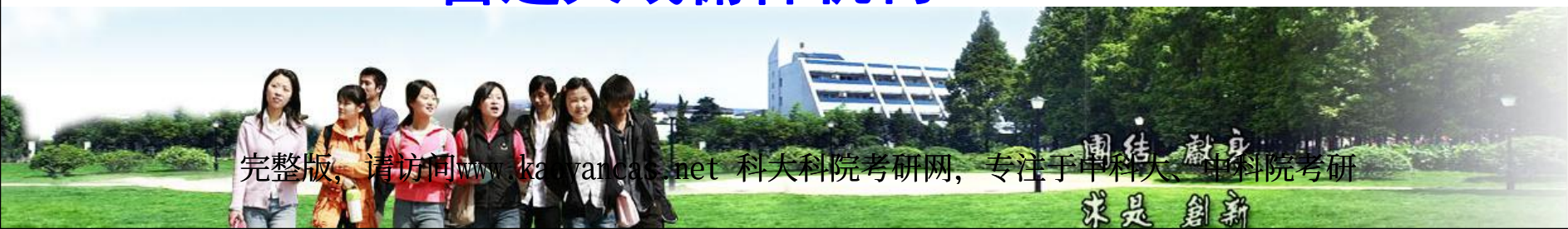


Foot operated sewing machine





雷达天线俯仰机构

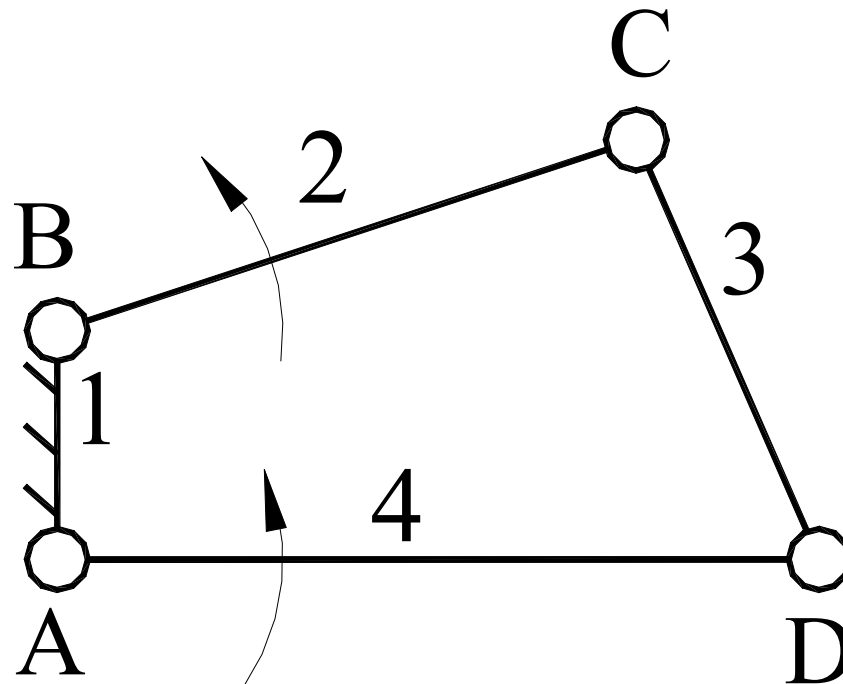




(b) Double-crank mechanism

If one crank rotates at a constant speed, the other crank will rotate in the same direction at a varying(变化的) speed.

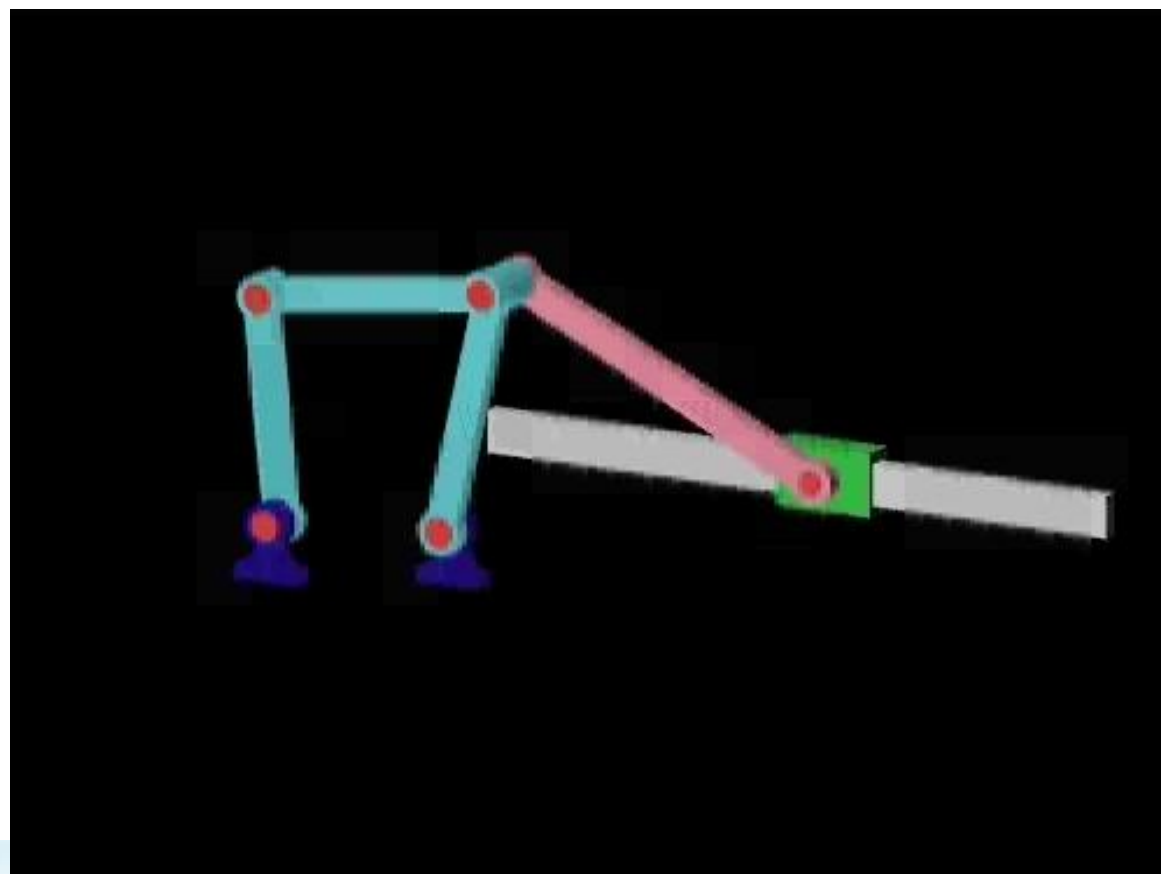
当主动曲柄匀速转动时，从动曲柄作变速转动。





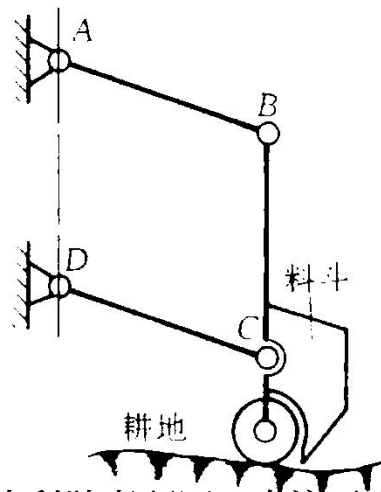
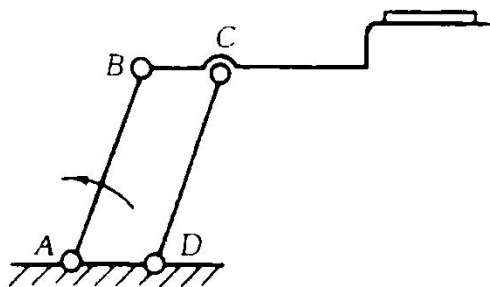
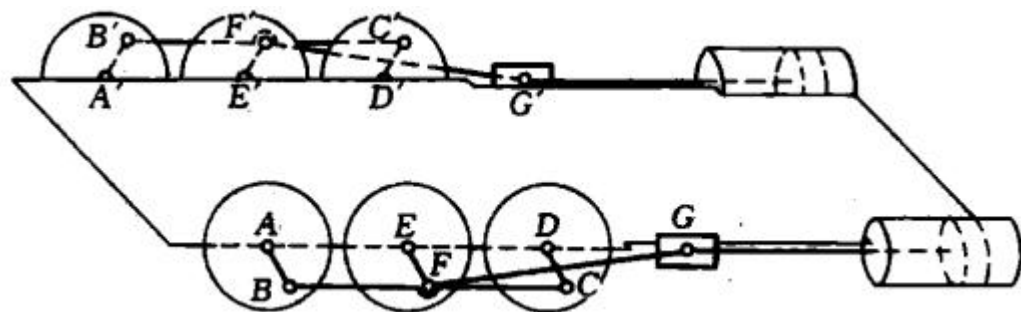
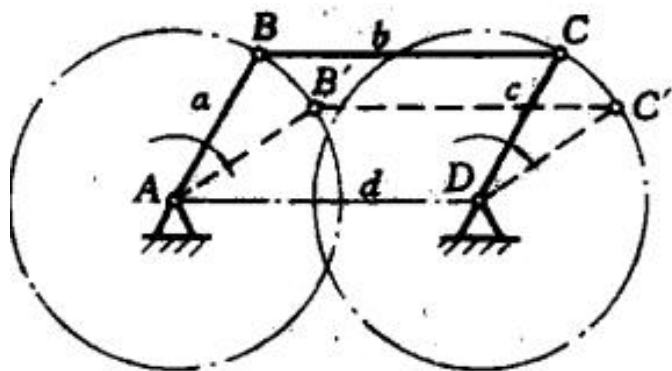
Unbalanced throw screen mechanism

惯性筛机构



平行四边形机构：Parallel-crank Mechanism

正平行四边形机构如天平称、机车联动机构、摄影平台升降机构和播种料斗机构等。两曲柄等速转动



Antiparallel-crank Mechanism

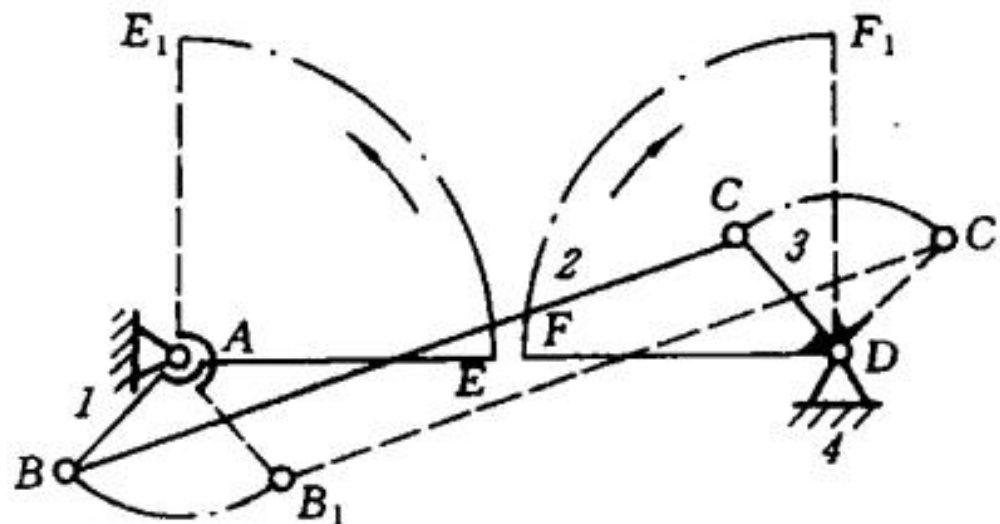


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反平行四边形机构：两对杆长度相等，但不平行。

当以长边为机架时，两曲柄等速反向转动。用于车门开闭机构，如图所示。



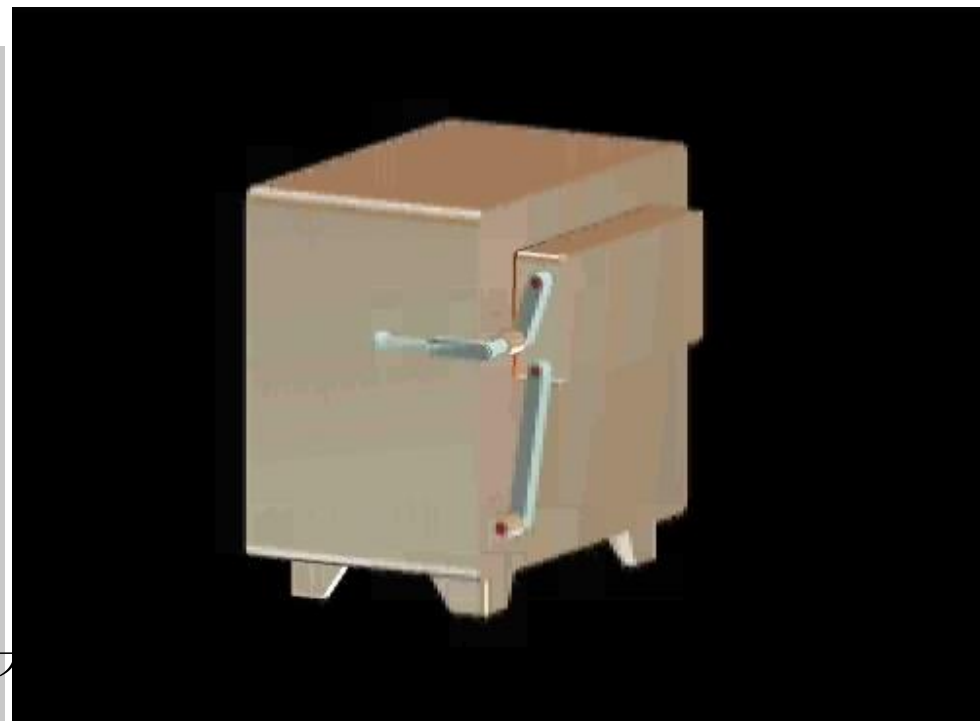
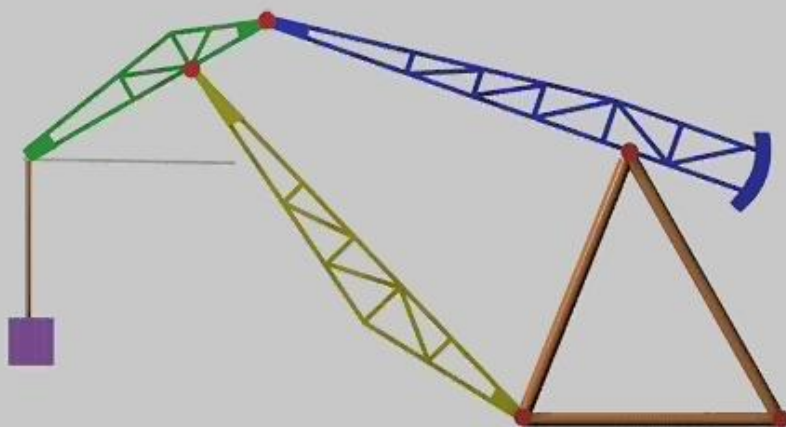
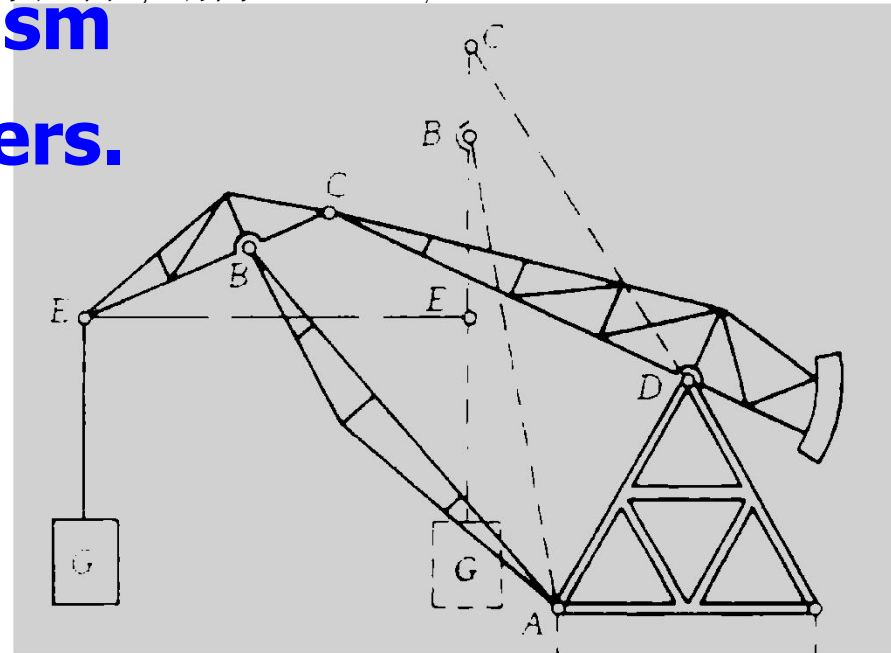
车门开闭机构动画

(c) Double-rocker mechanism

Both side links are rockers.

两连架杆都是摇杆。

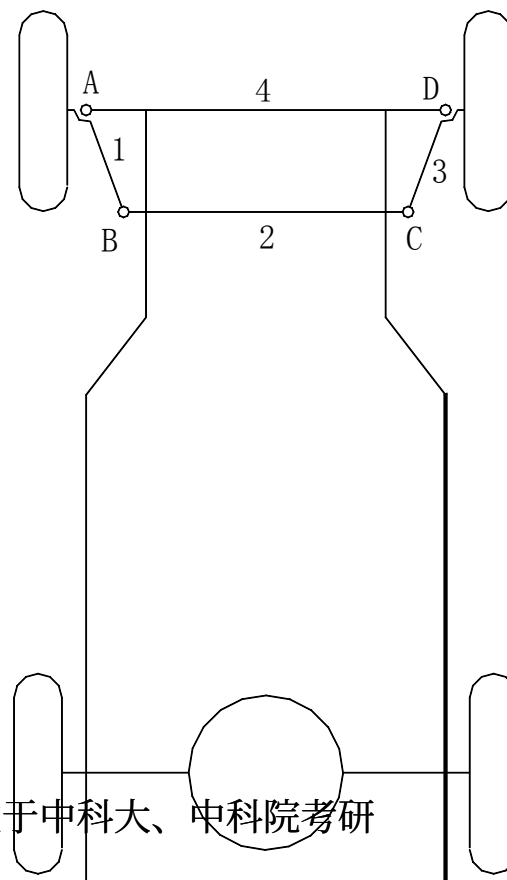
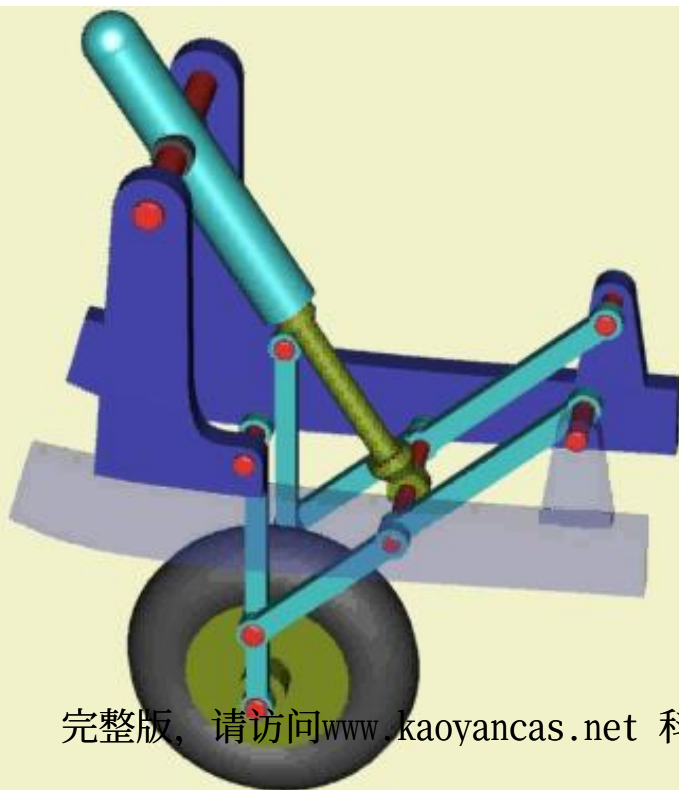
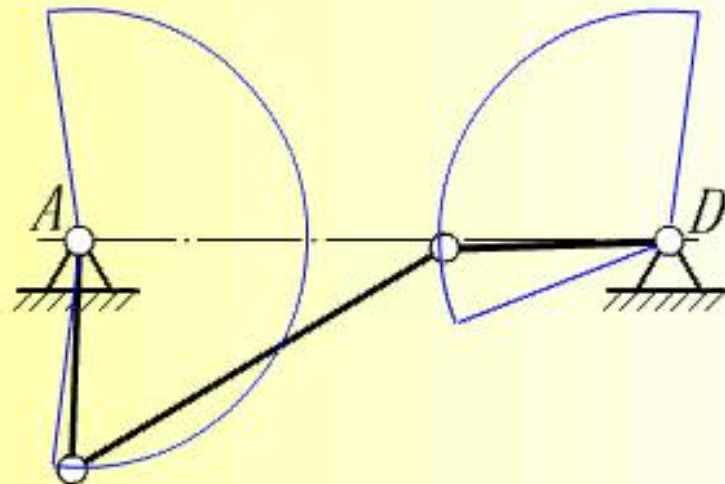
**用于鹤式起重机和炉
门开关机构等。**



等腰梯形机构也是双摇杆机构

应用：

转向机构——车辆的前轮转向
飞机起落架



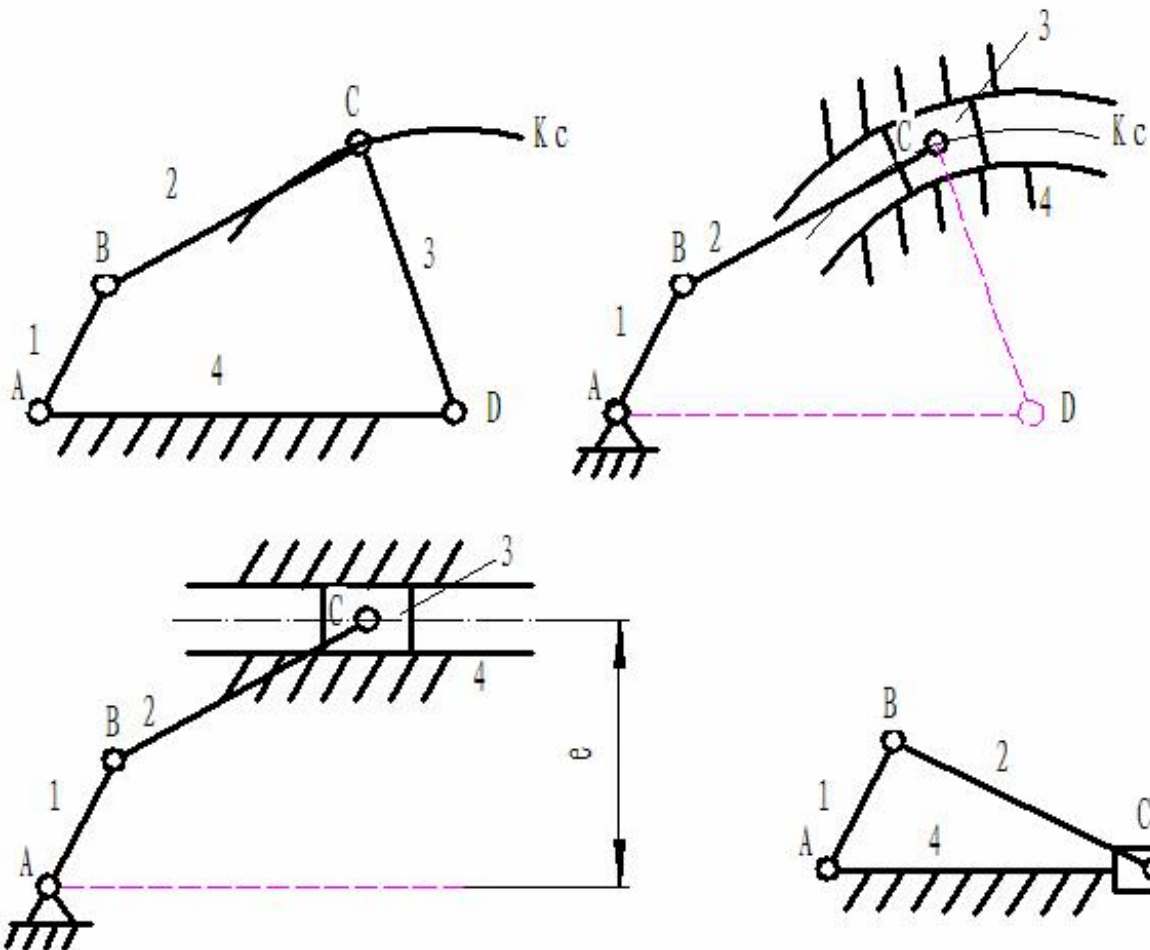


4.2.2 Variation (变异) of Revolute Four-bar Mechanisms

((1)) Replacing a revolute pair with a sliding pair

Slider-crank mechanism

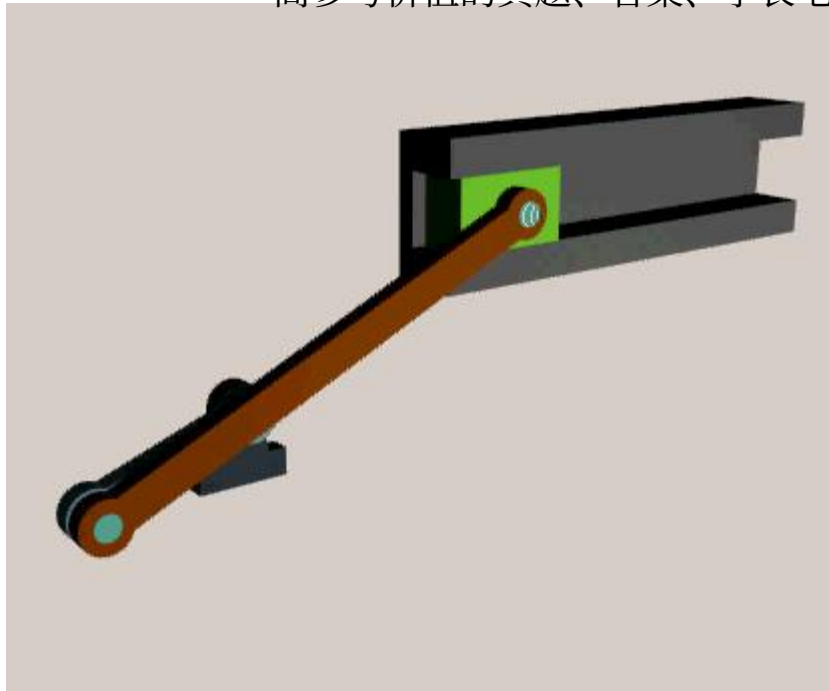
曲柄滑块机构 (偏距 e)





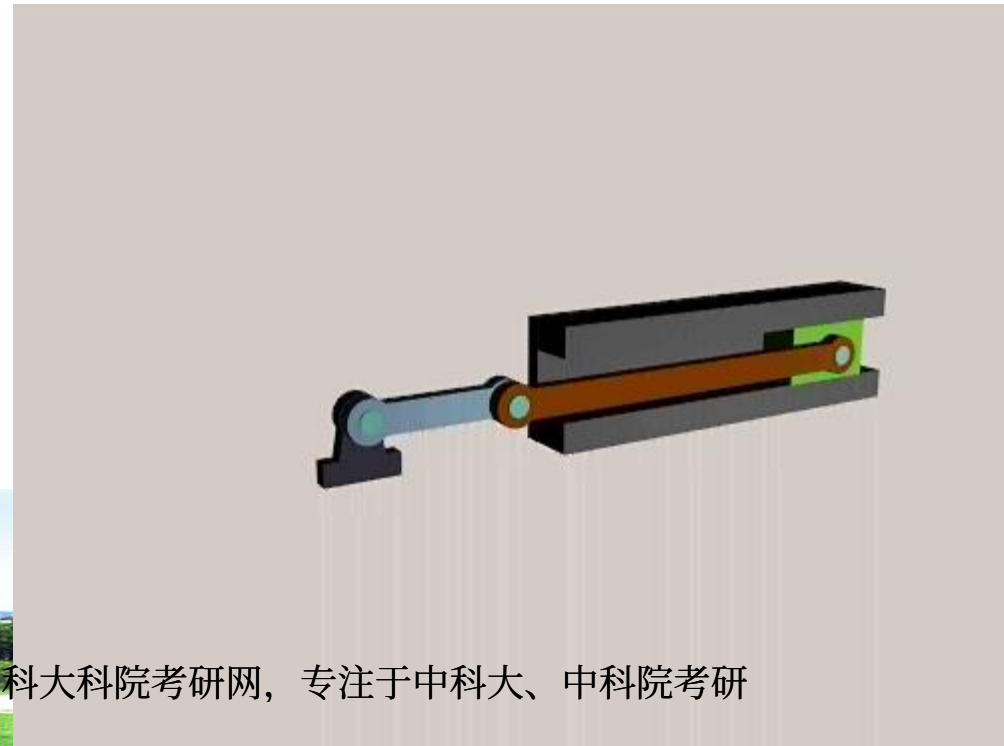
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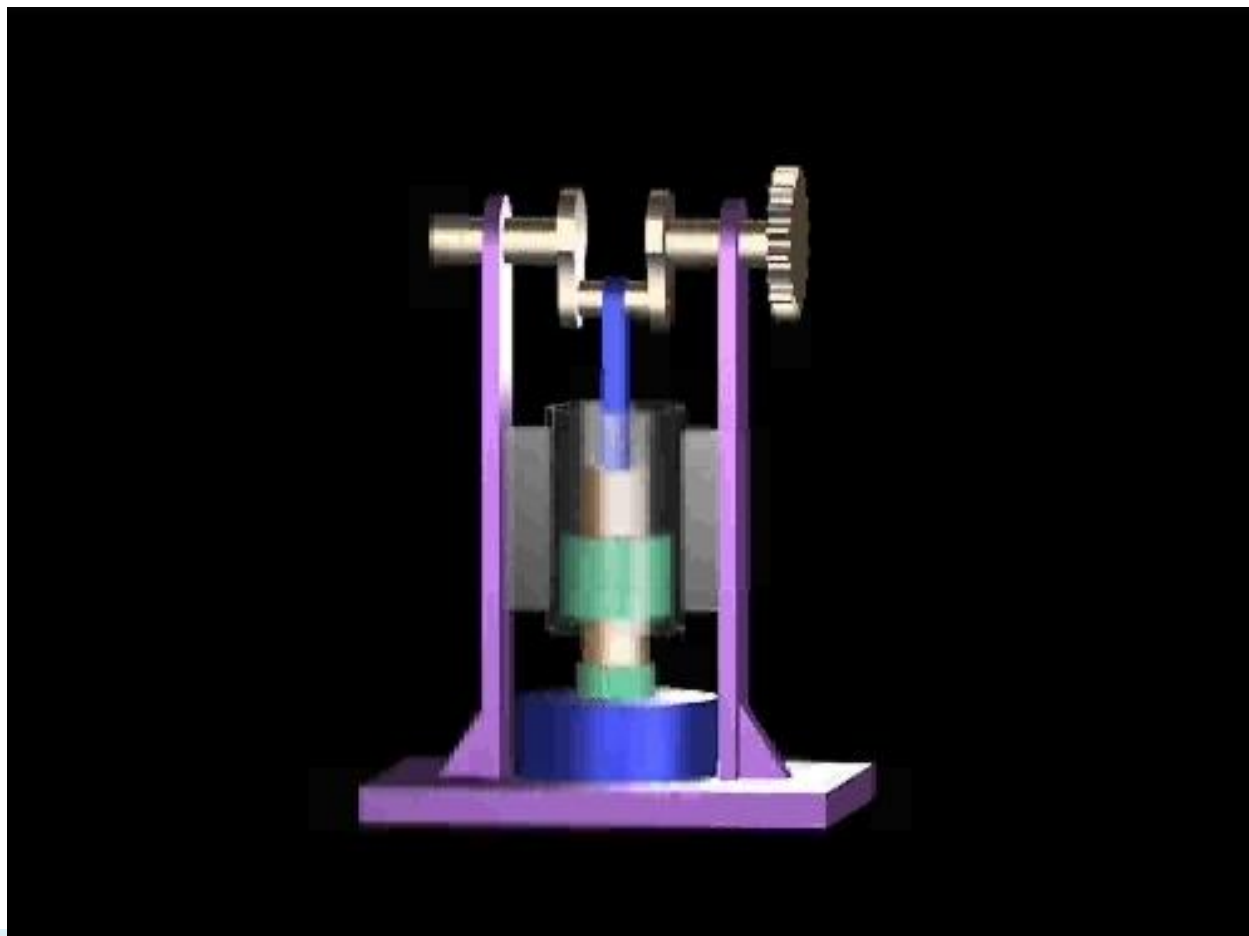
$e \neq 0$,
**Offset slider-crank
mechanism**
偏置曲柄滑块机构

$e=0$,
**Centric slider-crank
mechanism**
对心曲柄滑块机构





用于冲床、内燃机、空压机、曲柄压力机等机械中。



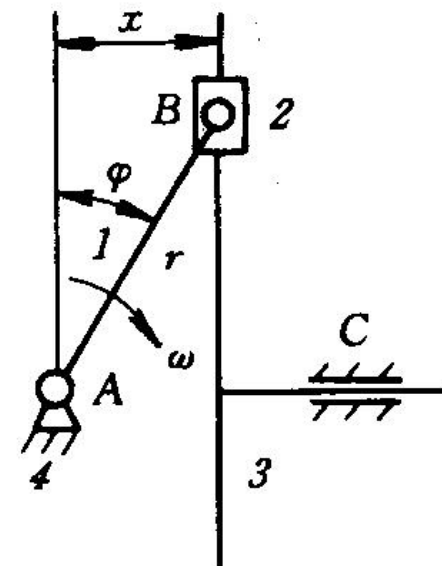
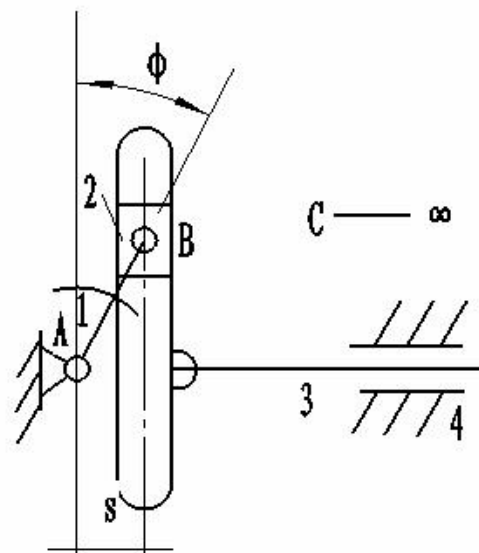
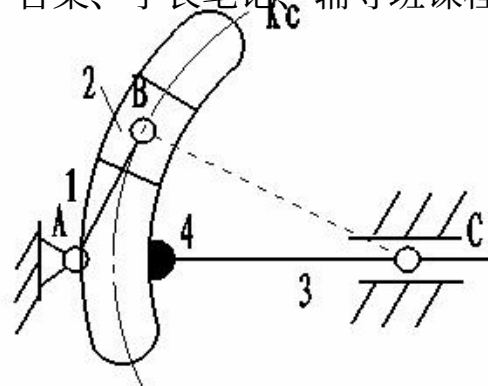
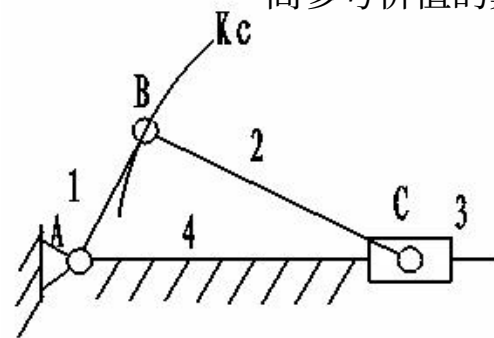
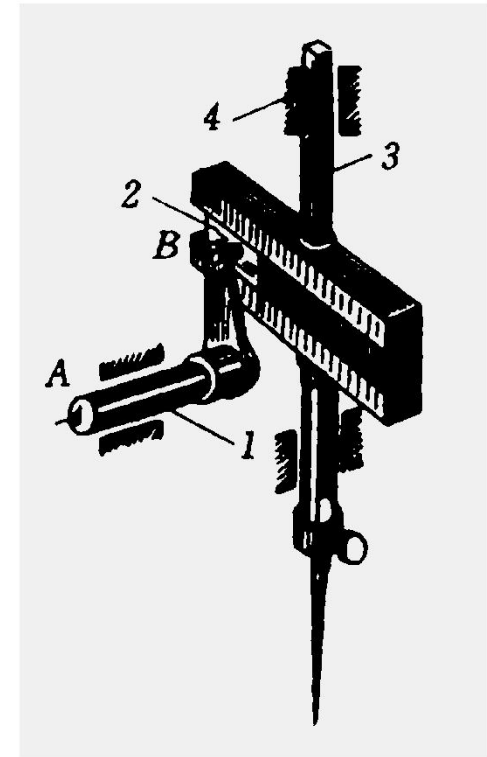
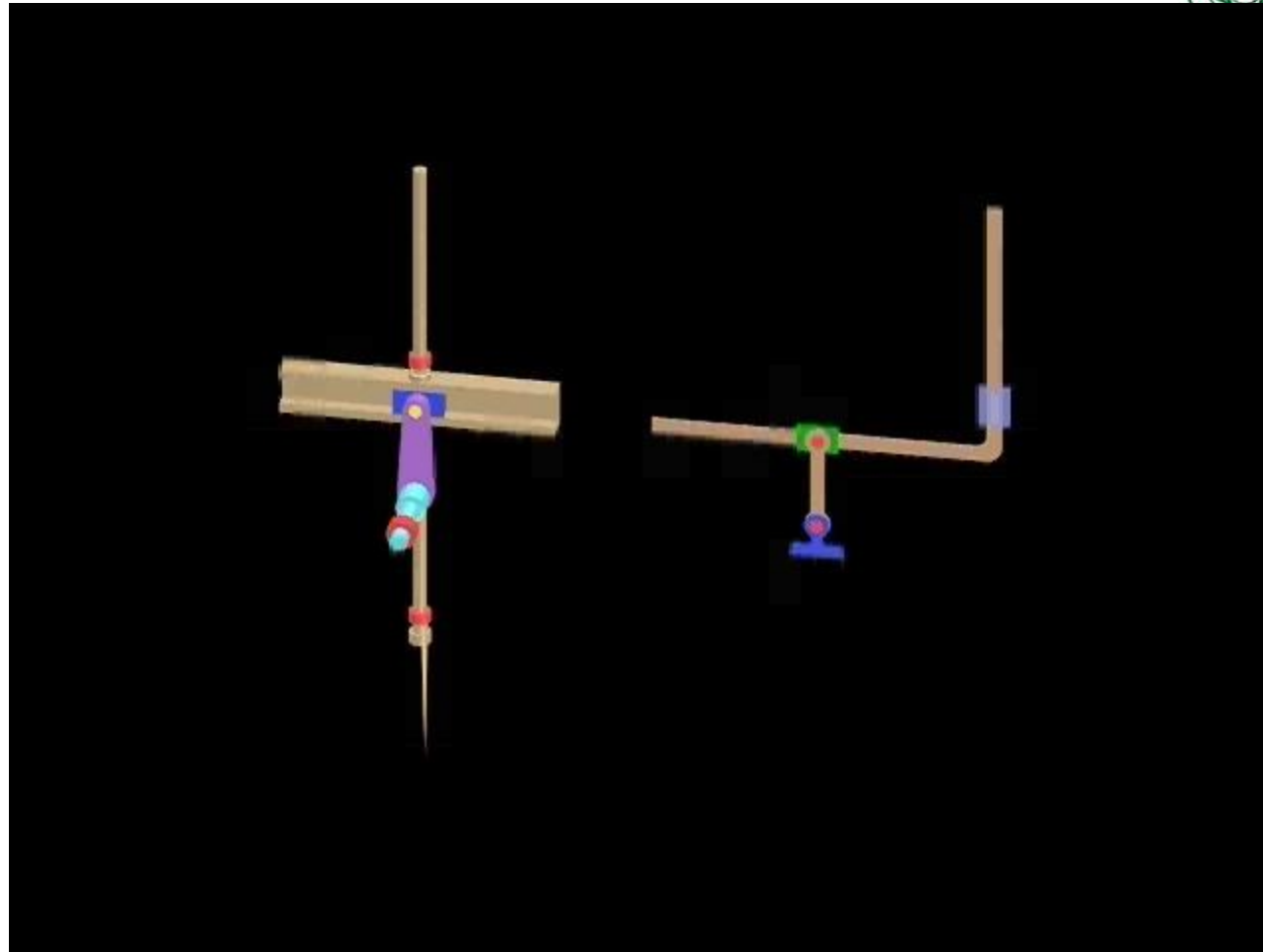
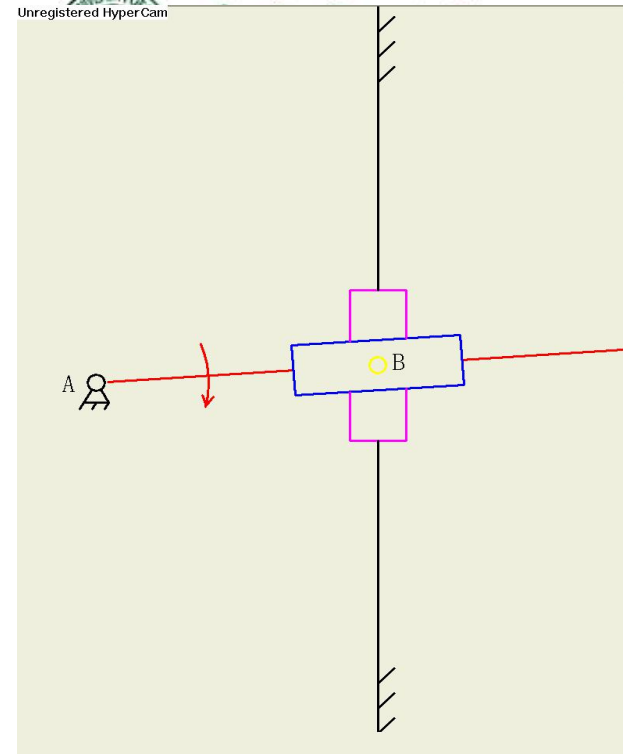
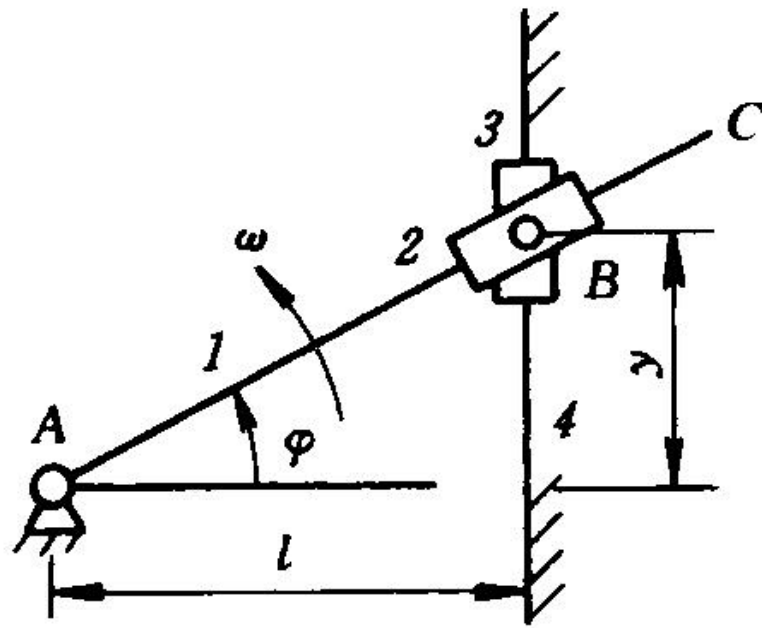


Fig. 4-6

$$S = l_{AB} \sin \phi \longrightarrow \text{Sinusoid Generator 正弦机构}$$



缝纫机的刺布机构



$y = l \tan \varphi \longrightarrow$ Tangent generator 正切机构



(2) Taking different links as frame.-----inversions

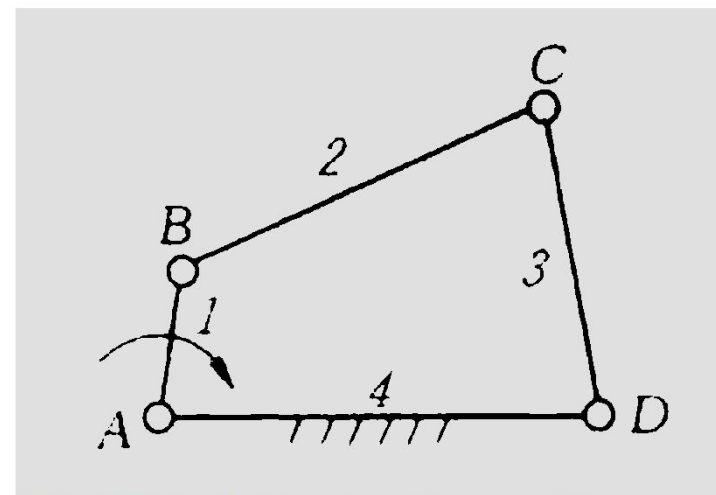
机架变换

Fully rotating revolute:

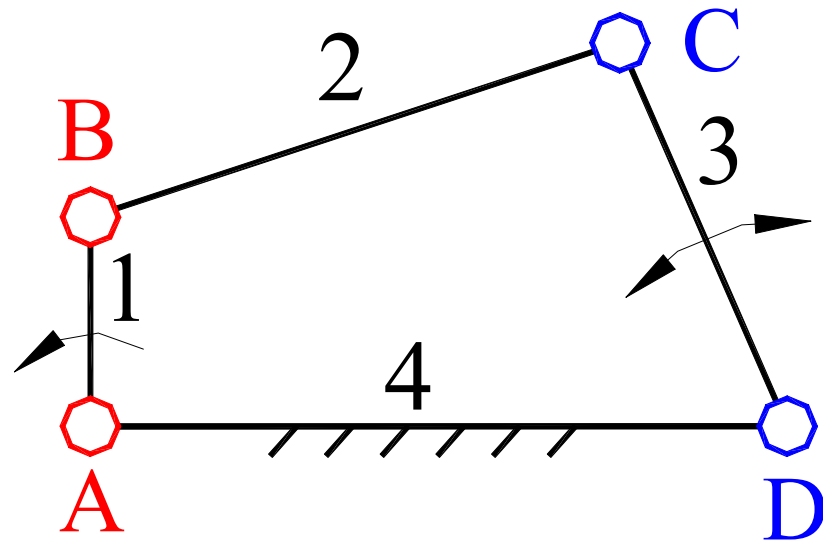
A & B (整转副)

Partially rotating

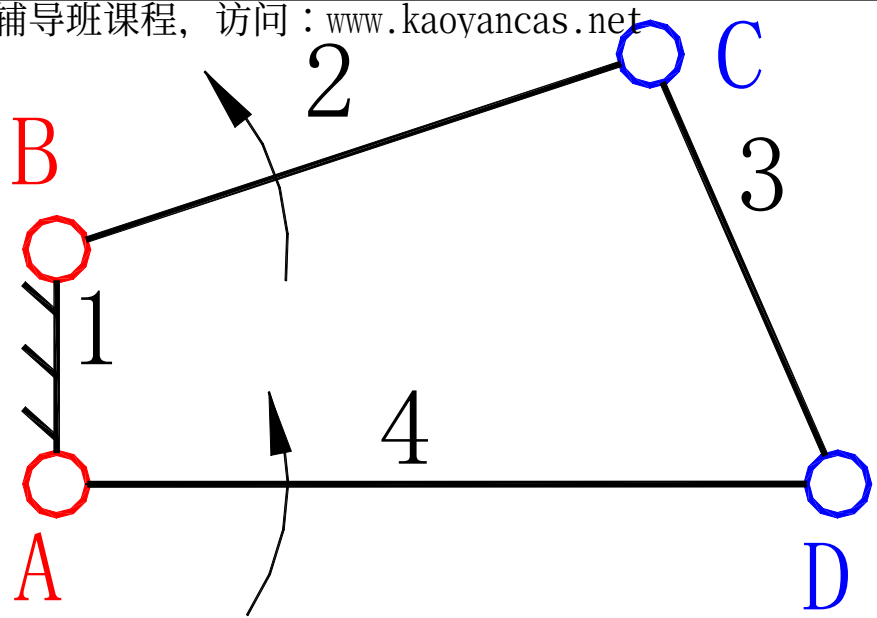
revolute :C&D (摆动副)



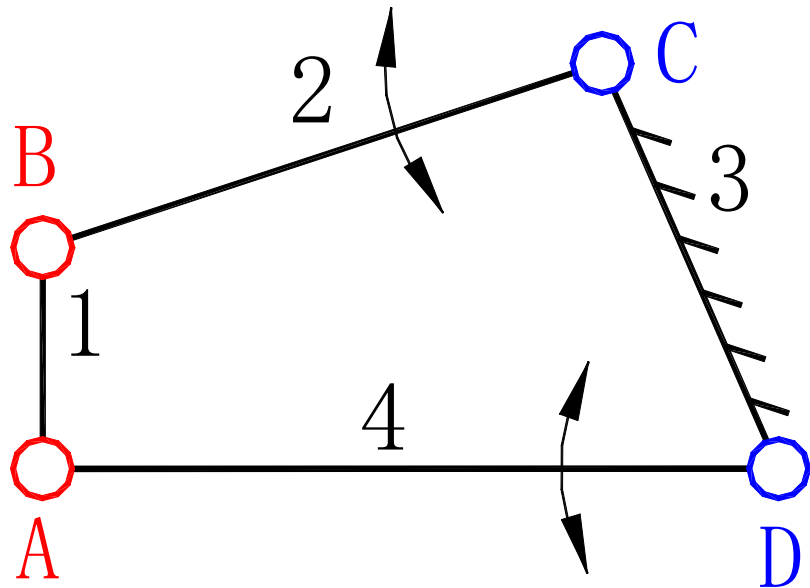
机构中任意两构件间的相对运动关系不因其中哪个是机架而变化。So, inversion of a mechanism in no way changes the type of revolute and the relative motion between its links.



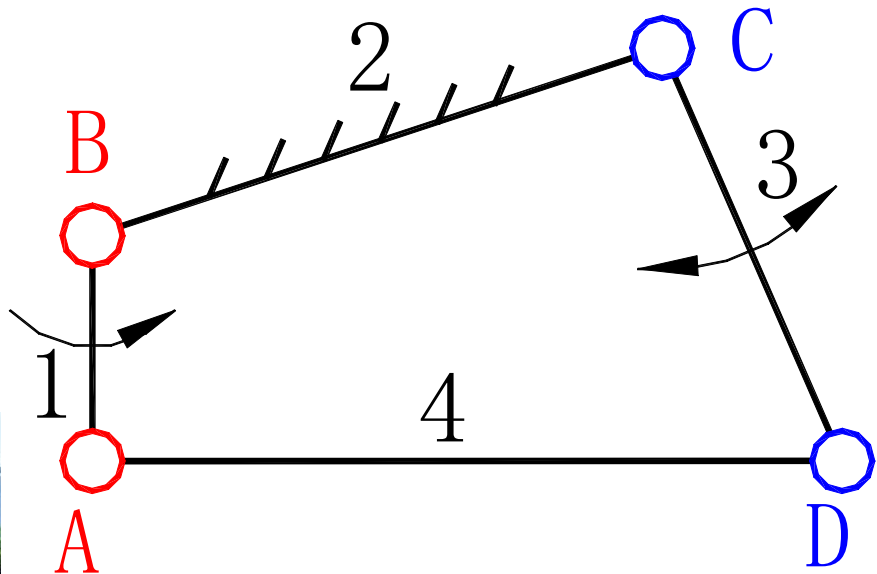
Crank-rocker



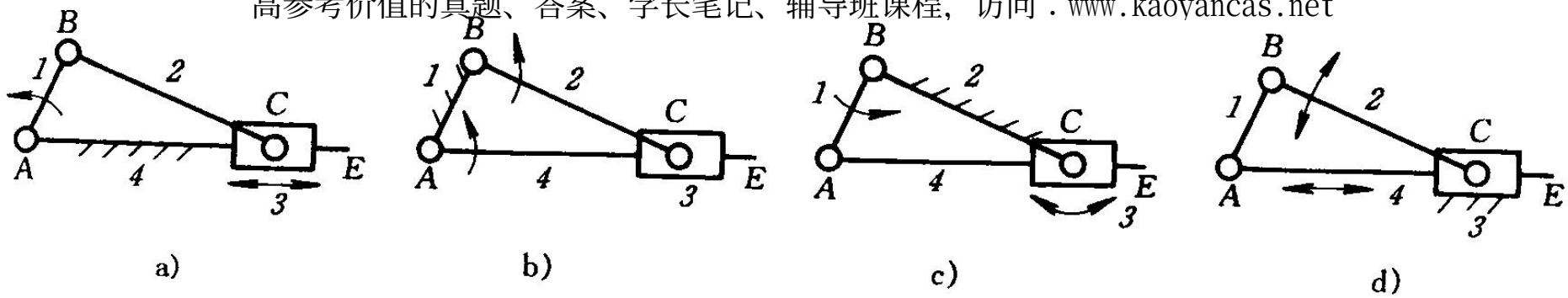
Double-crank



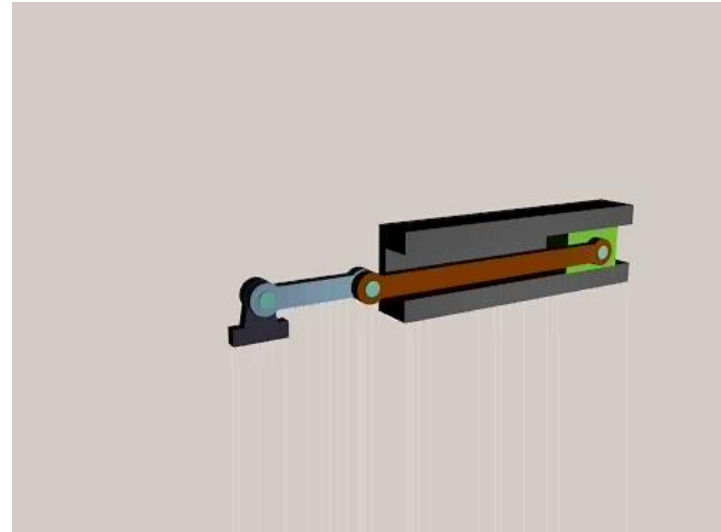
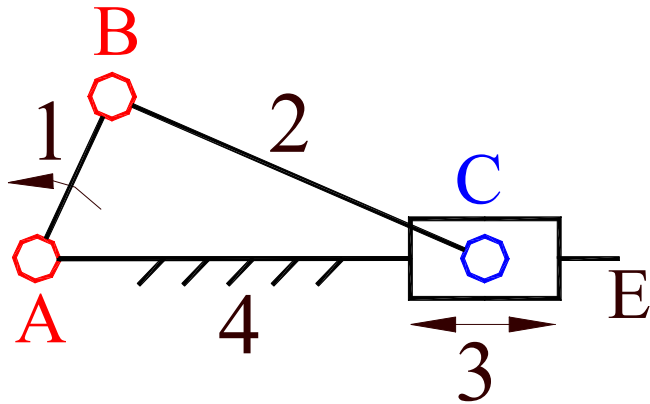
Double-rocker

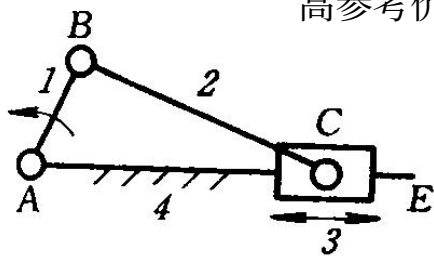


Crank-rocker

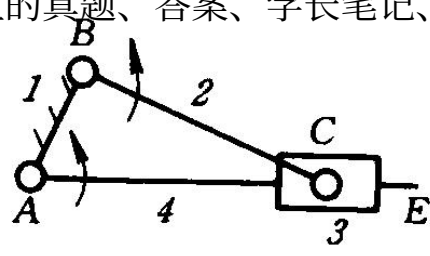


a) slider-crank mechanism 曲柄滑块机构

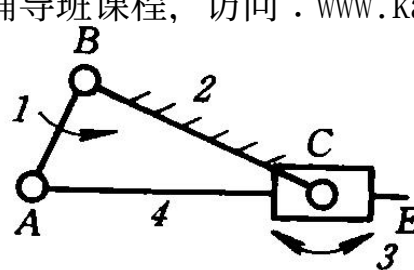




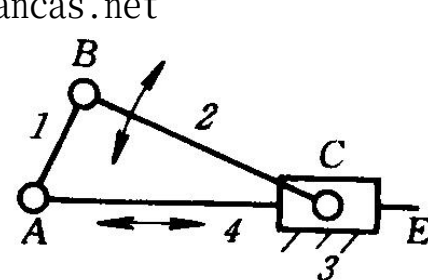
a)



b)

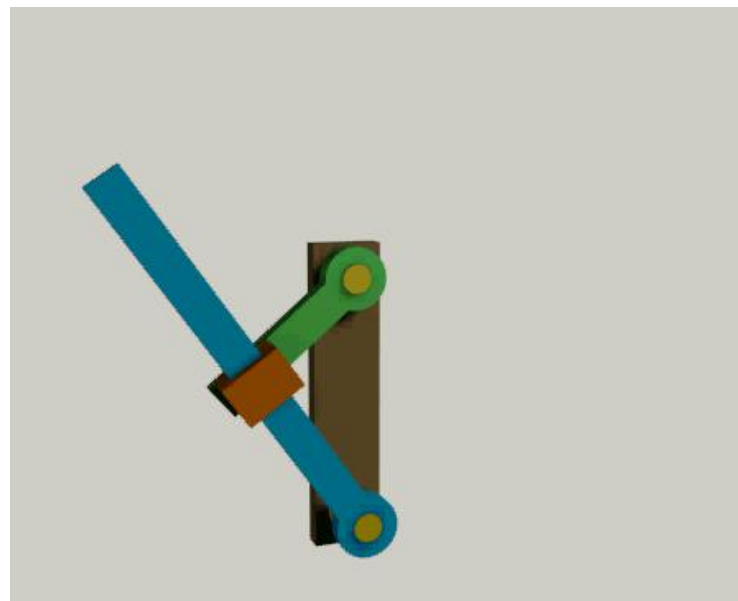
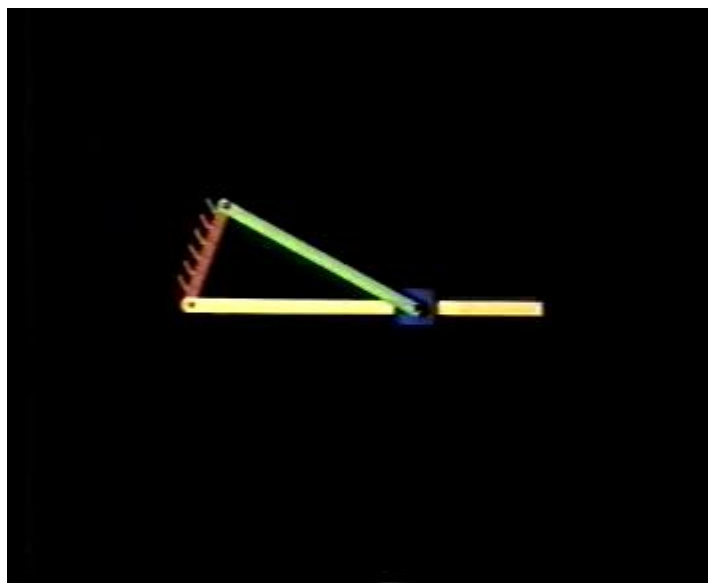


c)



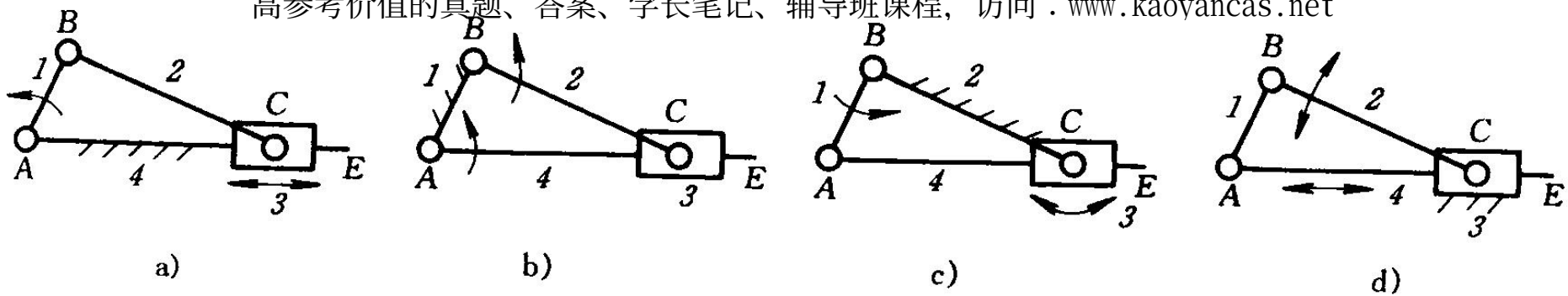
d)

b) guide-bar mechanism 导杆机构

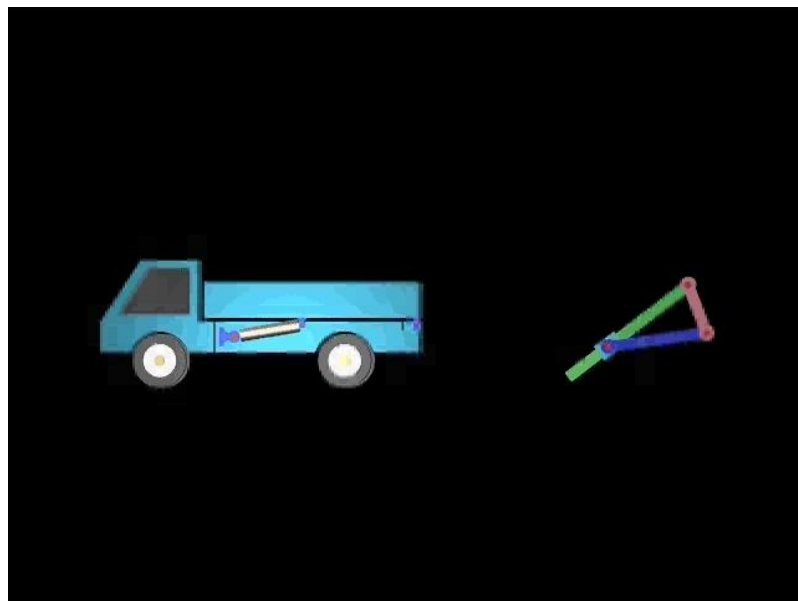
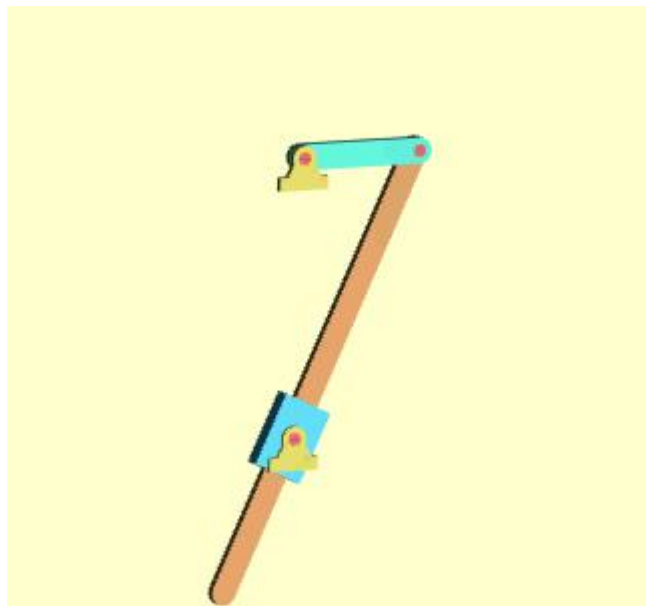


If $l_{BC} < l_{AB}$

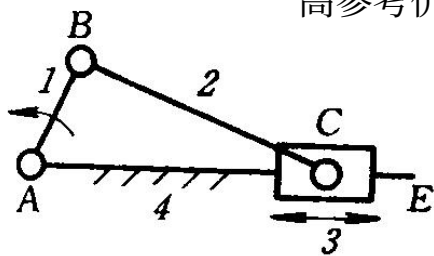
Rotating guide-bar mechanism 转动导杆机构 **Oscillating guide-bar mechanism 摆动导杆机构**



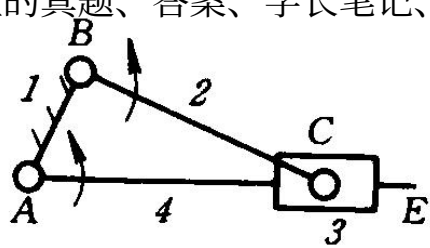
c) crank and oscillating block mechanism 摇块机构



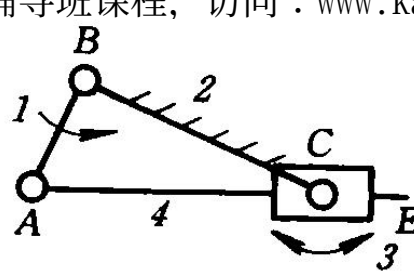
Self-tipping vehicle 自卸车辆



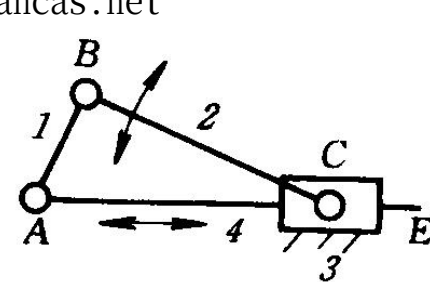
a)



b)



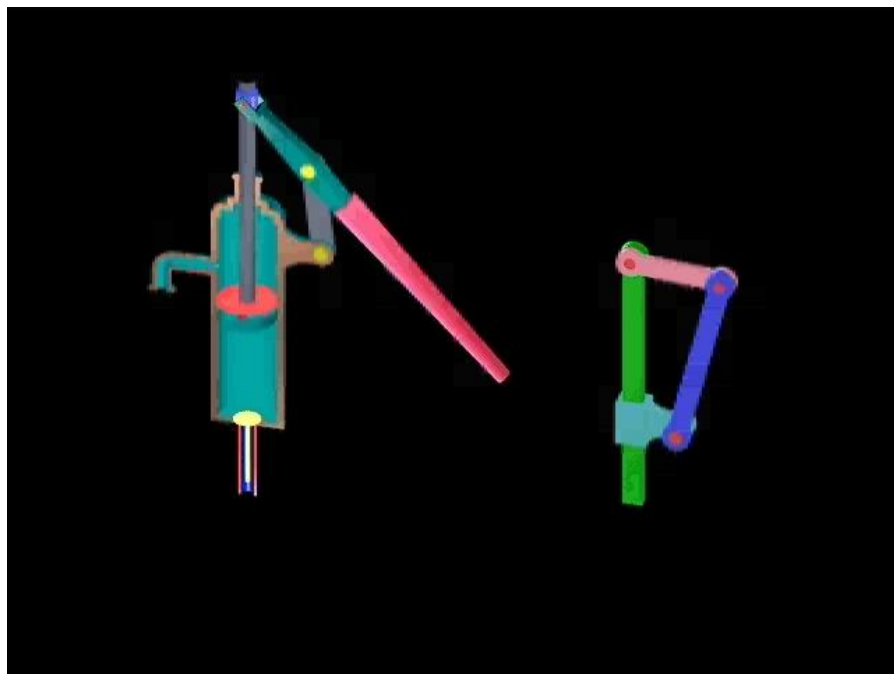
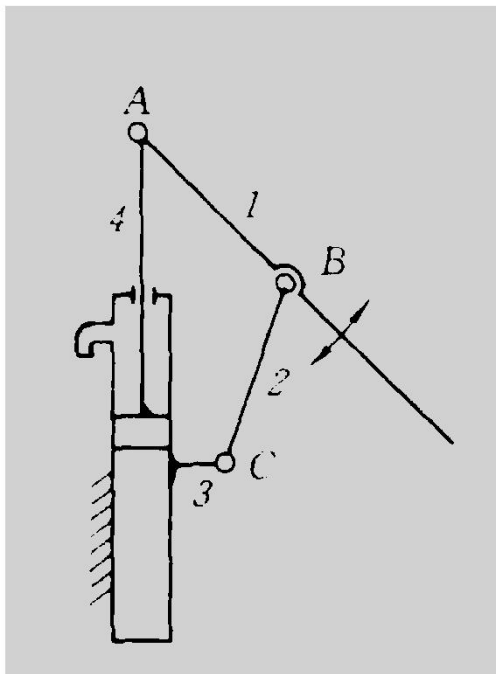
c)



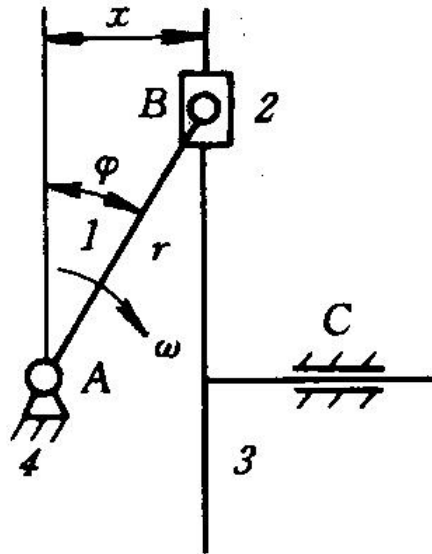
d)

d)translating sliding-rod mechanism

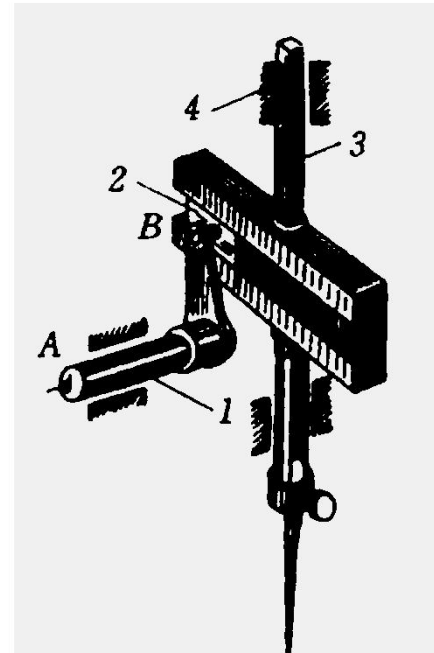
移动导杆机构或定块机构



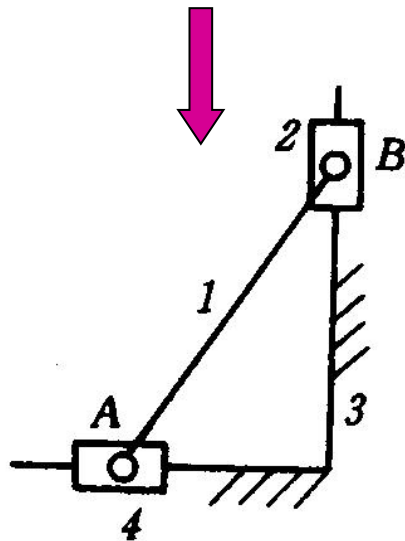
含有两个移动副的情况：



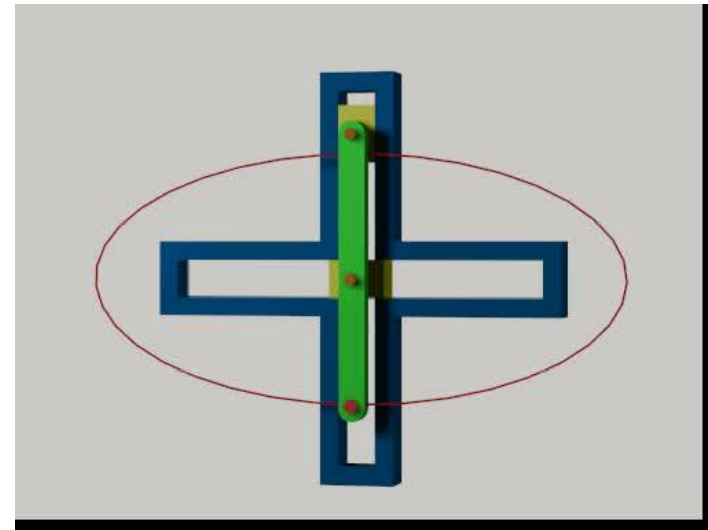
正弦机构



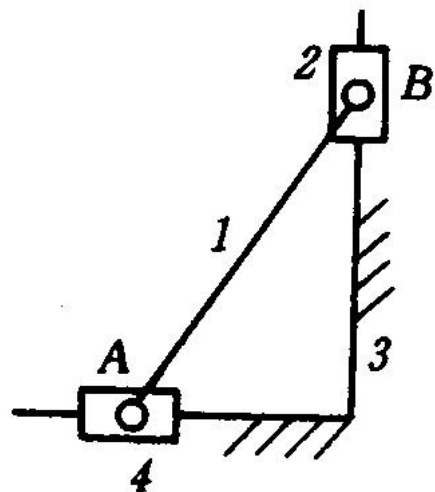
刺布机构



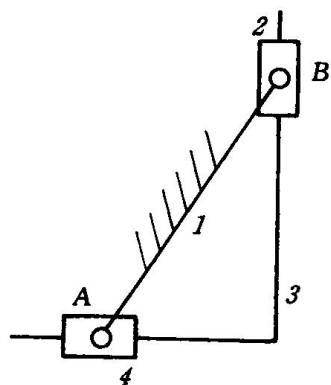
双滑块机构



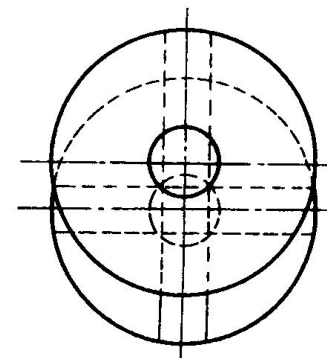
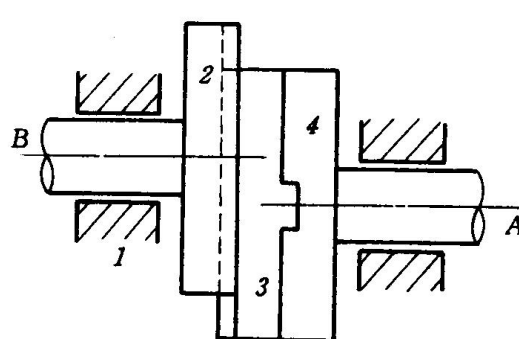
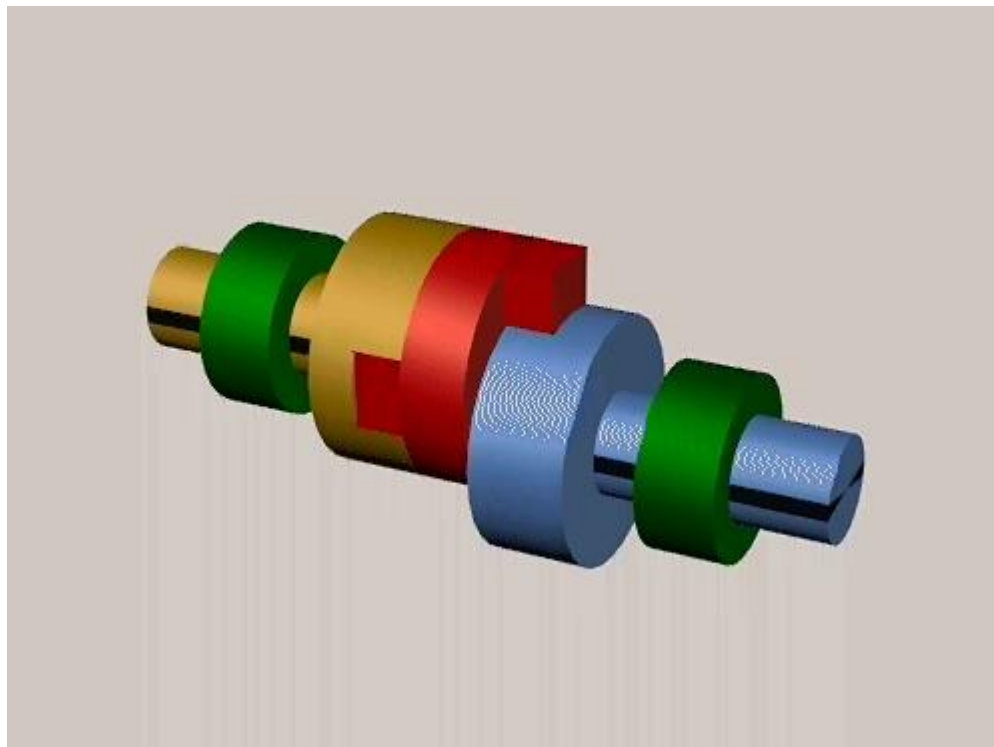
Elliptic trammel 椭圆规



双滑块机构



双转块机构

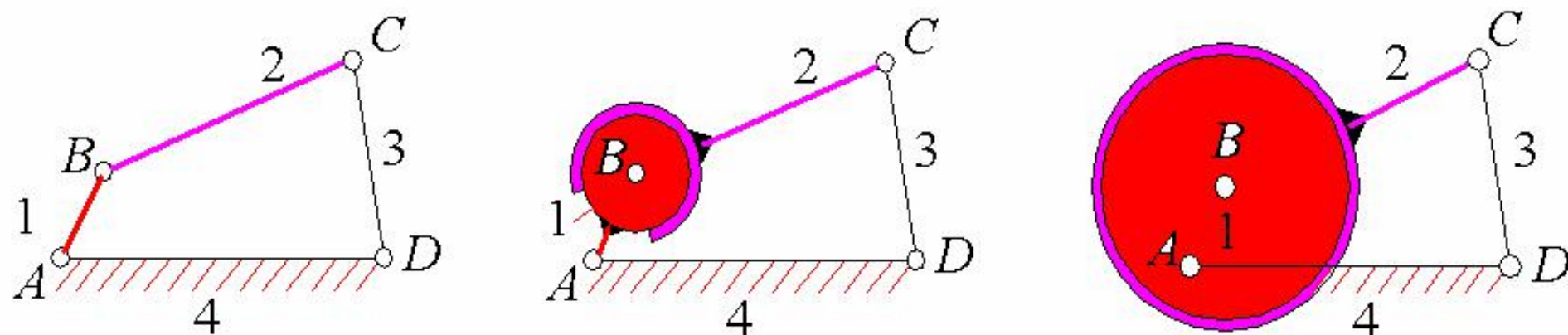


十字滑块机构



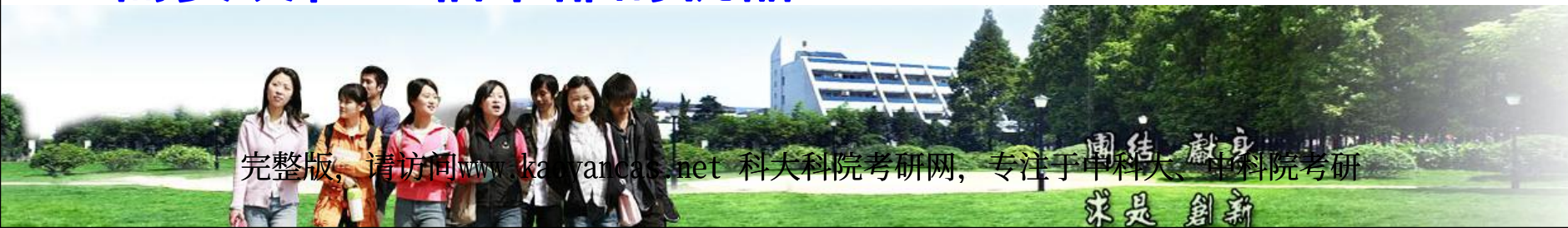
(3) Enlarging a revolute pair

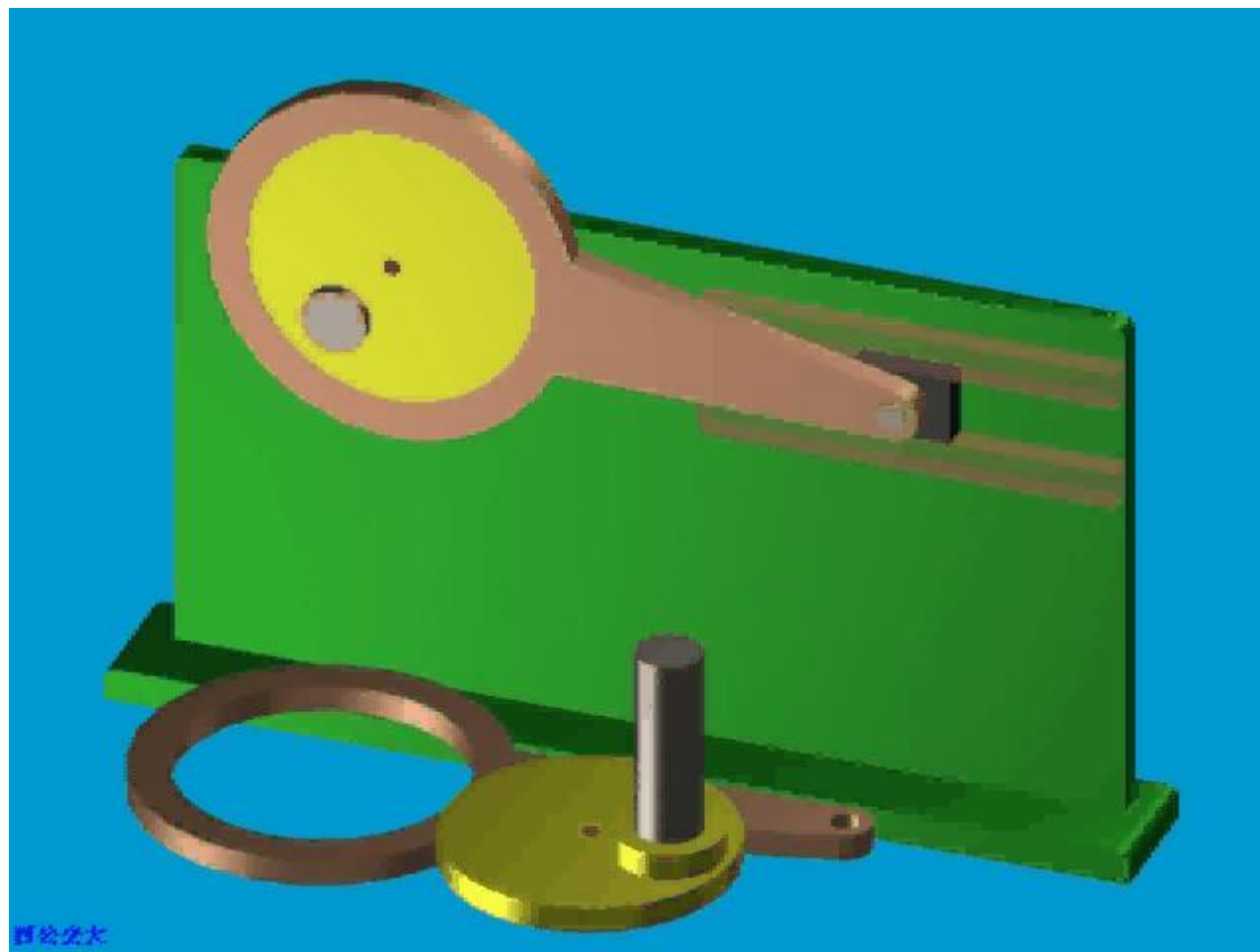
Note: Enlarging a revolute pair in no way changes the motion relationship between any links.



Crank1---eccentric 偏心轮

偏心轮机构多用于曲柄销承受较大冲击，曲柄较短，需要装在直轴中部的机器



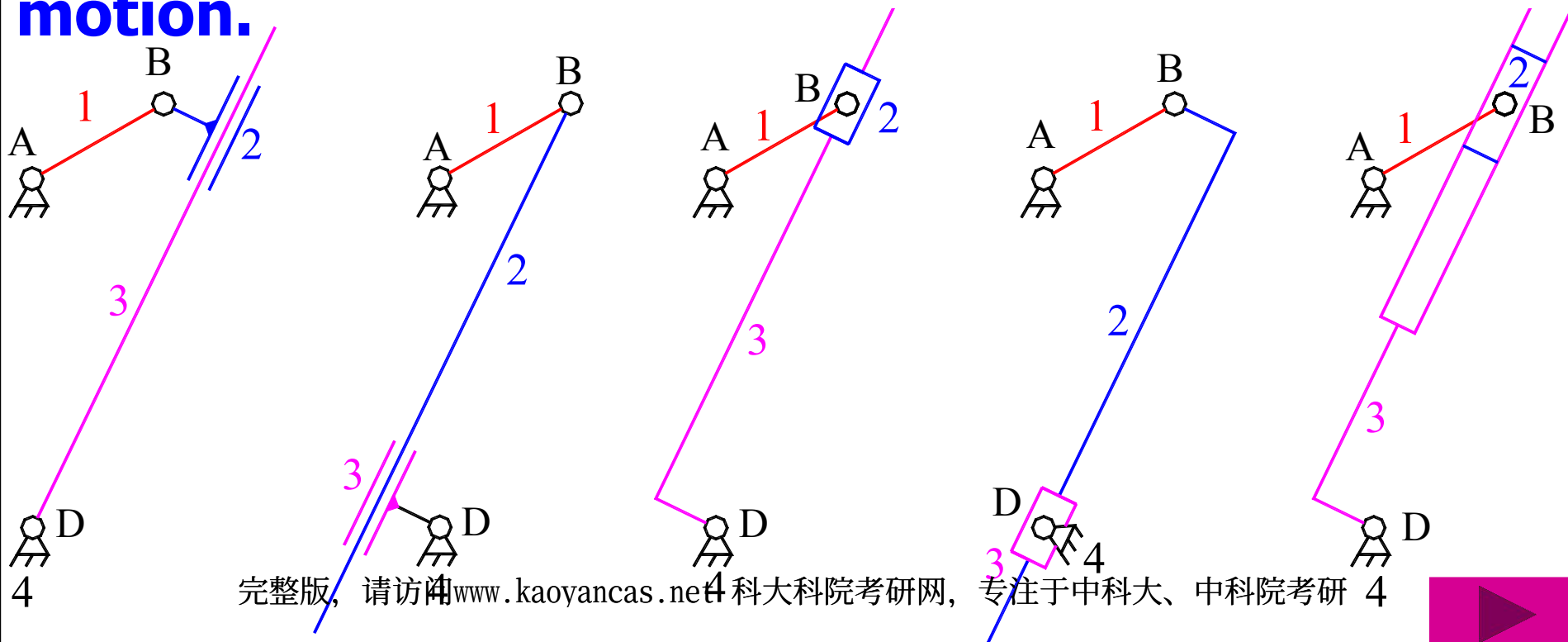


(4) Interchanging guide-bar and sliding block

变换导杆和滑块

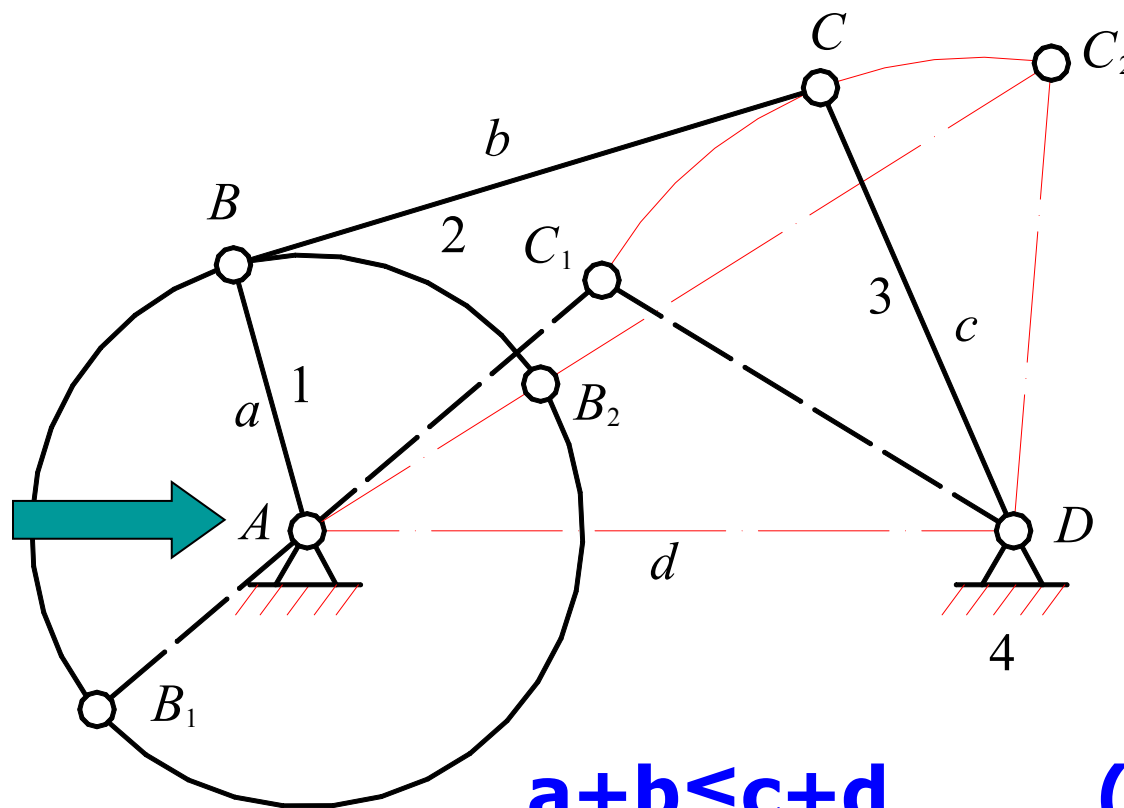
Any link in a sliding pair can be drawn as a guide bar, and the other link as a sliding block.

The centre line of any sliding pair can be translated(平移) without changing any relative motion.



4.3 Characteristics Analysis of Four-bar Linkages

4.3.1 Conditions for having a crank



$$d < c + b - a \rightarrow a + d \leq c + b \quad (2)$$

$$c < d + b - a \rightarrow a + c \leq b + d \quad (3)$$

即： $a+d \leq b+c$ (1)

$$a+b \leq c+d \quad (2)$$

$$a+c \leq b+d \quad (3)$$

(1)+(2) 得 $2a+b+d \leq 2c+b+d$ 即 $a \leq c$

(1)+(3) 得 $a \leq b$ (2)+(3) 得 $a \leq d$

Grashof Criterion: the Condition for having a crank
两构件作整周相对转动的条件（曲柄存在的条件）

1. The crank must be shortest link.

2. The sum of the shortest and the longest links must be less than the sum of the remaining two links 最短构件与最长构件的长度之和小于等于其它两构件长度之和。（杆长之和的条件）

铰链四杆机构类型判定： (根据是否符合杆长之和条件)

☀ 如不满足杆长之和条件 $l_{\max} + l_{\min} > l_b + l_c$

Double rocker mechanism

☀ 如满足杆长之和条件 $l_{\max} + l_{\min} \leq l_b + l_c$

If the link opposite to the shortest Link is the frame

—— **Double Rocker mechanism**

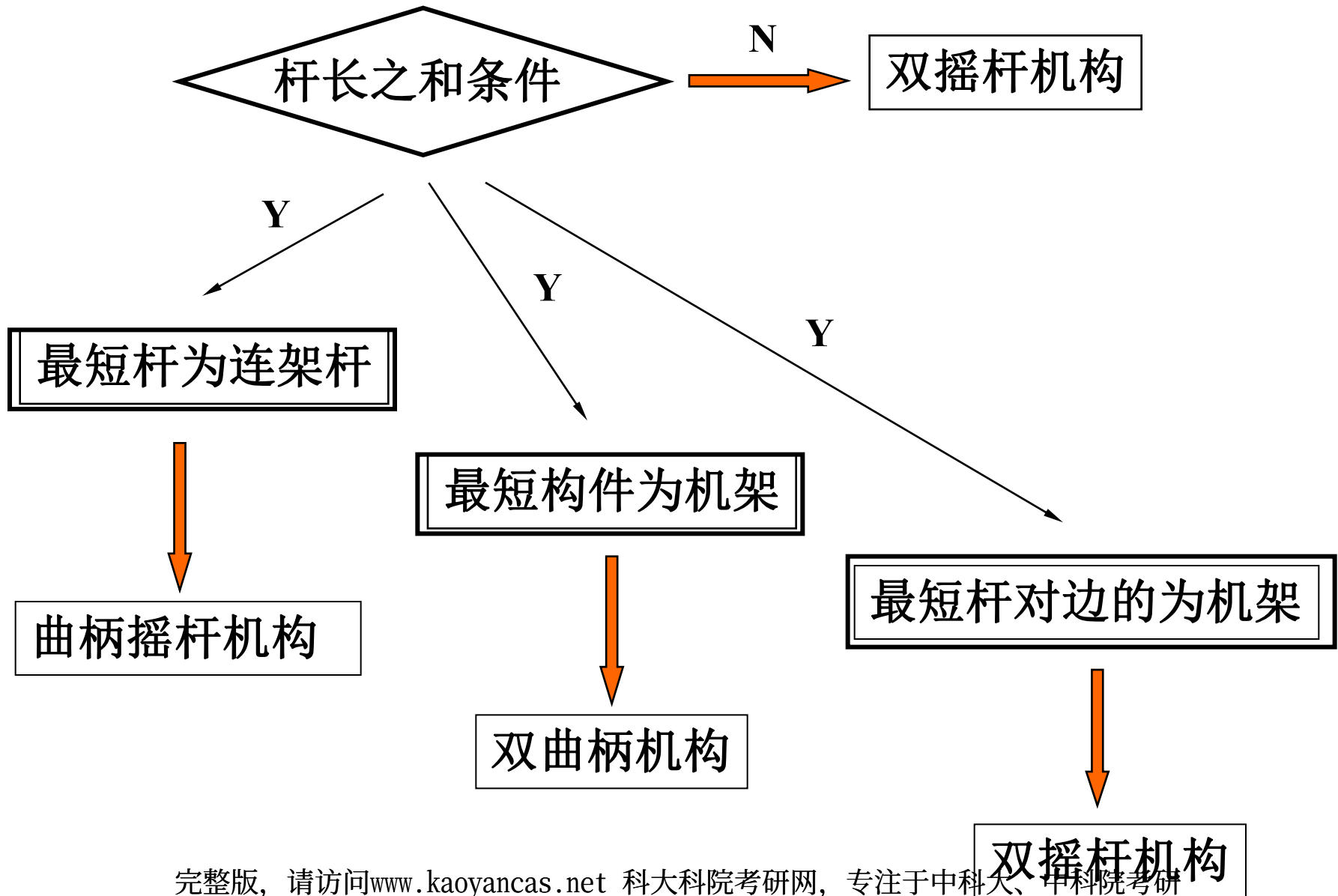
The shortest link is the frame

—— **Double crank mechanism**

If the link adjacent to the shortest Link is the frame

—— **Crank-rocker mechanism**

铰链四杆机构类型判定：





if $l_{\max} + l_{\min} = l_b + l_c$

The centre lines of the four links can become collinear(共线).

At these positions, the output behavior may become indeterminate(不确定的). These positions AB_1C_1D are called **change-points**.

AB_2C_2D

Parallel-crank mechanism

$AB_2C'_2D$

antiParallel-crank mechanism

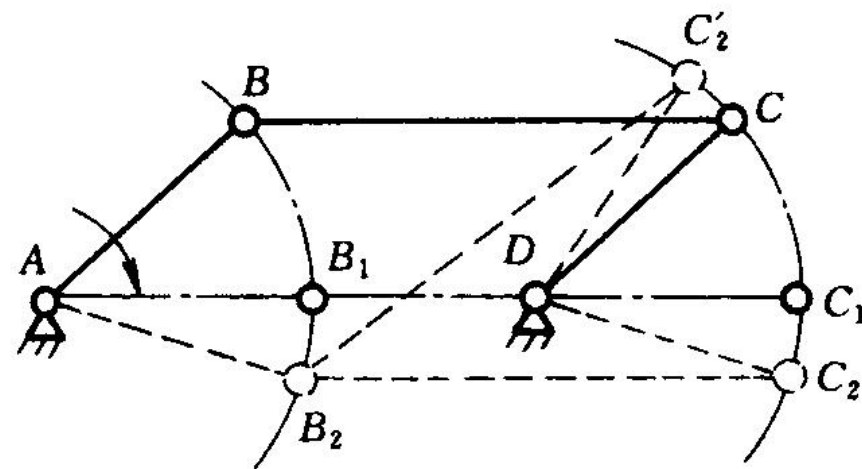
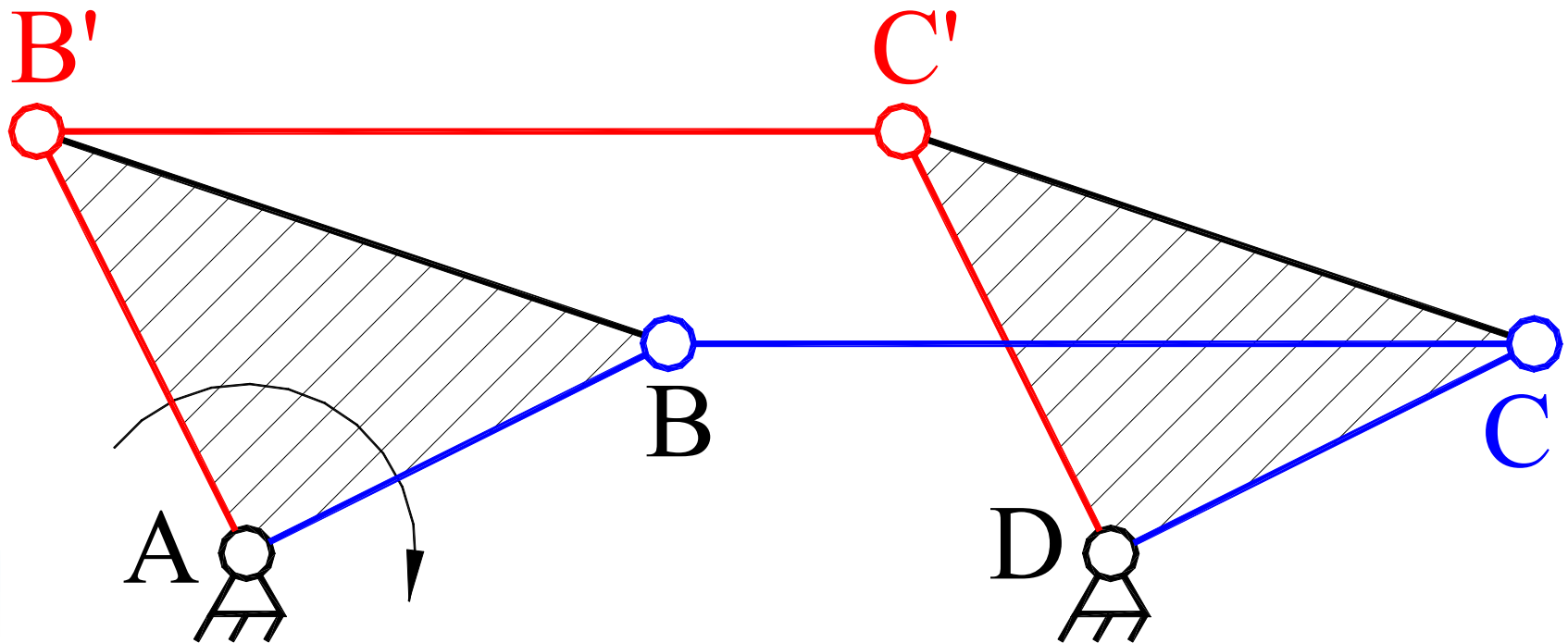


Fig. 4-21



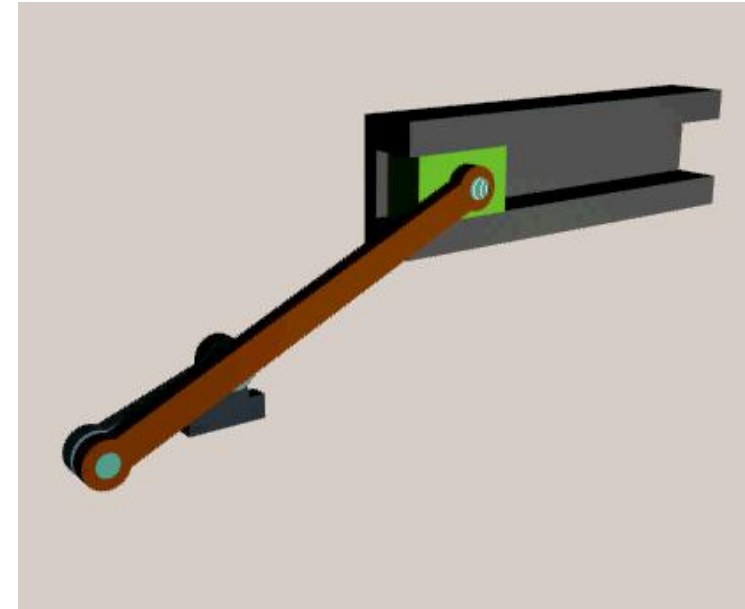
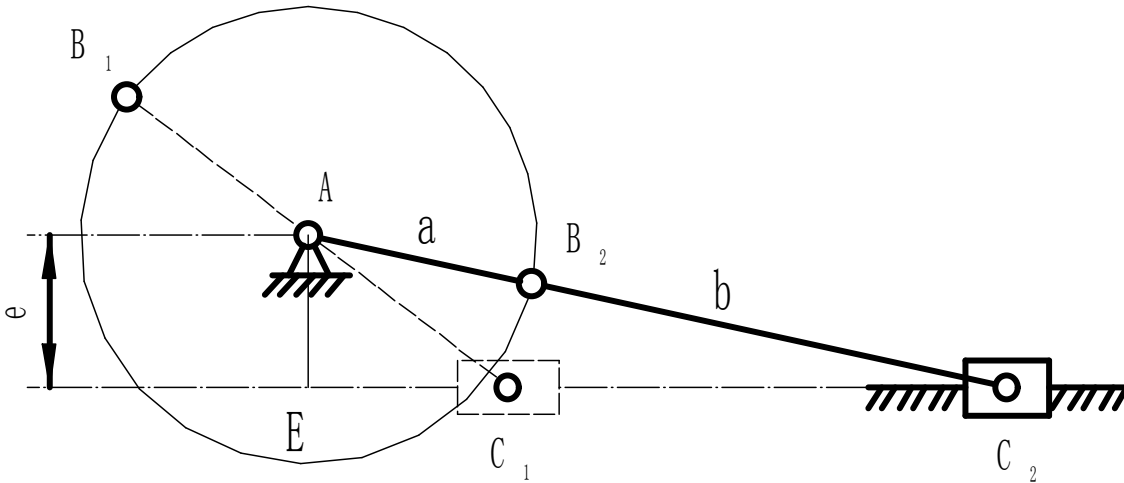


the change-points are handled by inertia of flywheel fixed with the driven crank or providing the duplicate(复制的) linkage 90° out of phase(相位).



Slider-Crank Mechanisms

---conditions for having a crank



直角 $\triangle AC_1E$: $AC_1 > AE$ $b - a > e$

直角 $\triangle AC_2E$: $AC_2 > AE$ $a + b > e$

即曲柄存在的条件: $b > a + e$ 偏置
 $b > a$ 对心

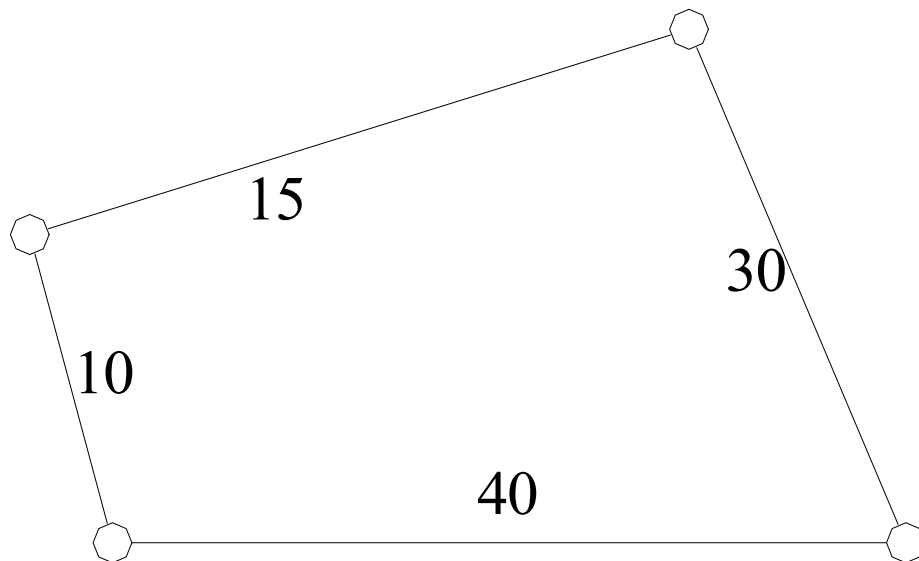
示例：

(1) $10+40 \nless 15+30$

不满足曲柄存在条件



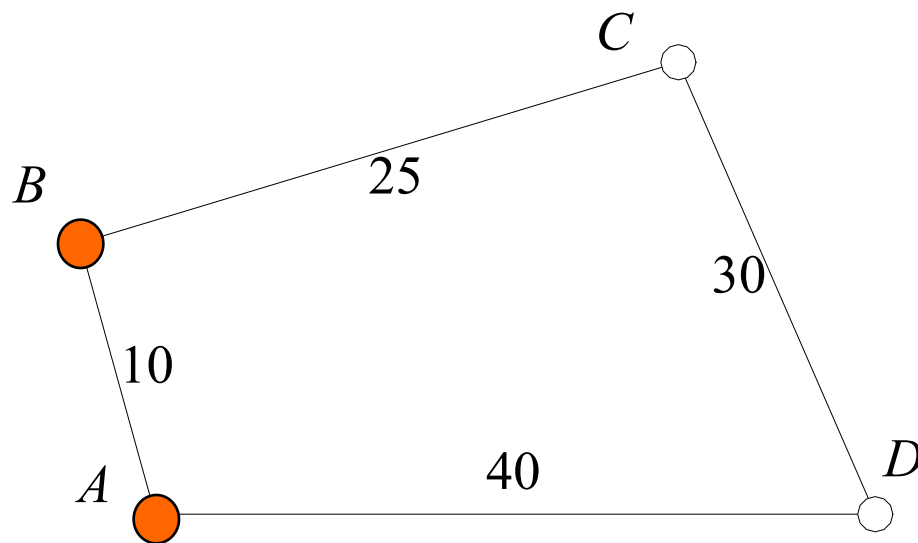
双摇杆机构



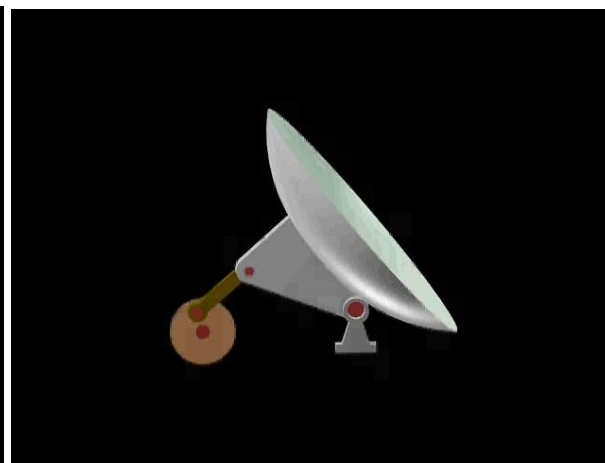
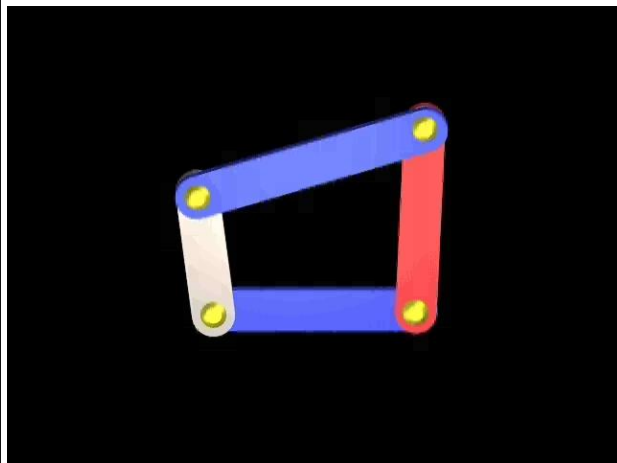
(2) $10+40 < 25+30$

满足曲柄存在条件

根据机架不同来判定

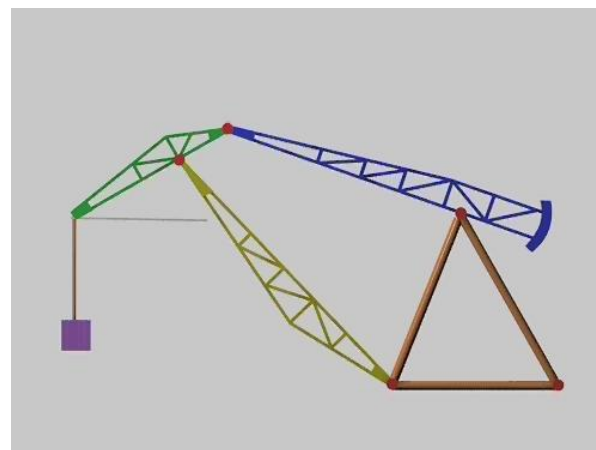
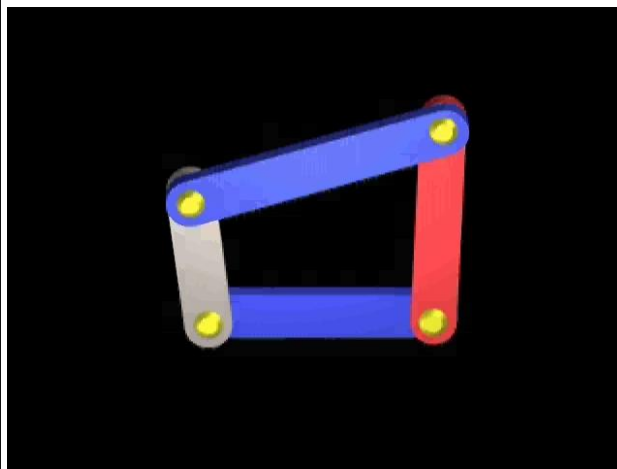


(1) 曲柄摇杆机构



雷达俯仰机构

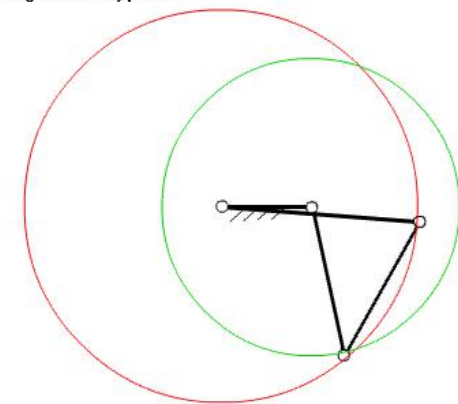
(2) 双摇杆机构



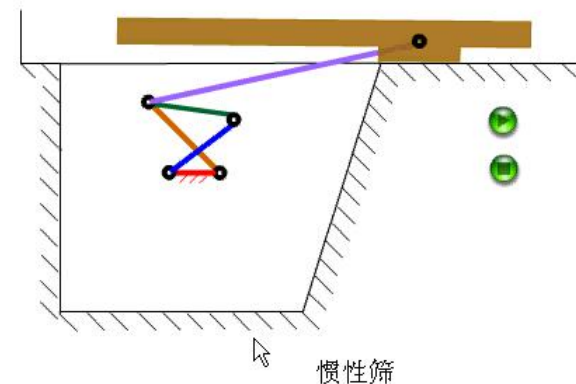
鹤式起重机

(3) 双曲柄机构

Unregistered HyperCam



Unregistered HyperCam



惯性筛

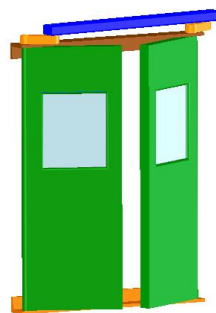
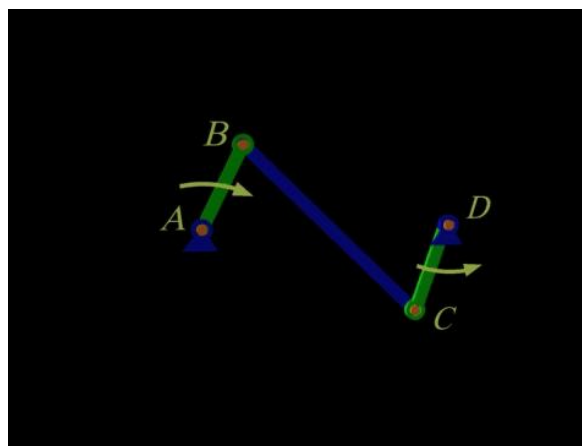
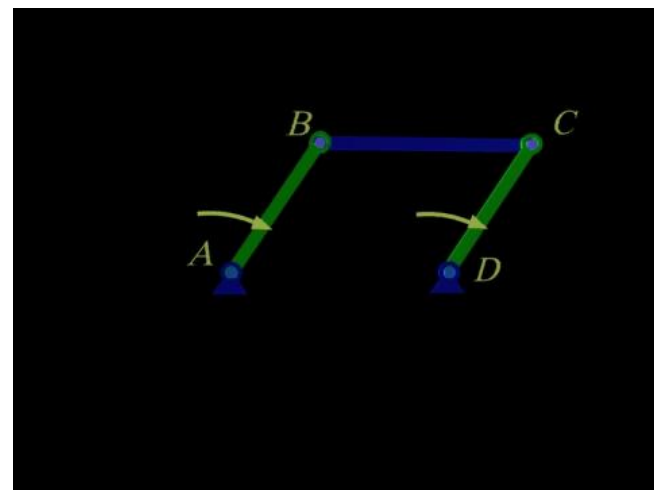
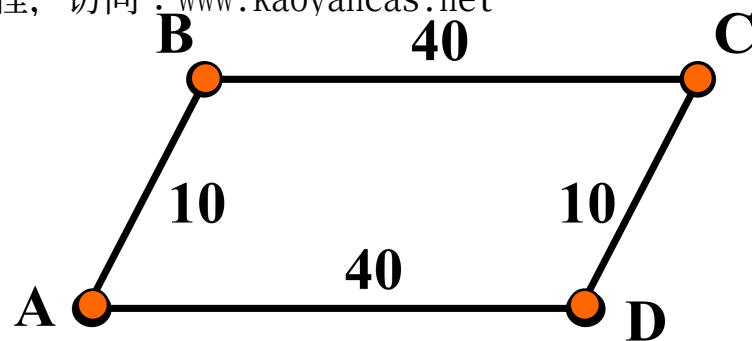
(3) $10+40=10+40$

满足周转副存在条件

↓
双曲柄机构

↓
当两曲柄
长度相等

平行四边形机构



4.3.2 Pressrue Angle α 压力角 and Transmission Angle γ 传动角

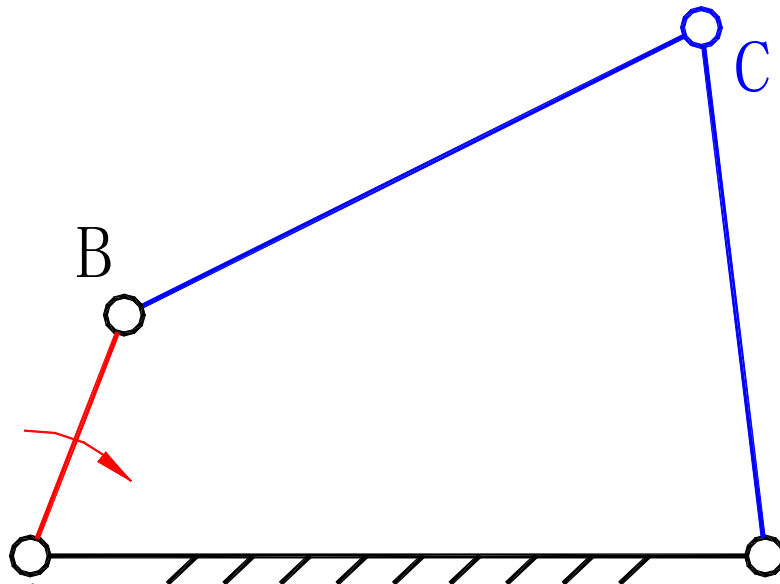
crank-rocker mechanism:

input link-AB

output link-CD

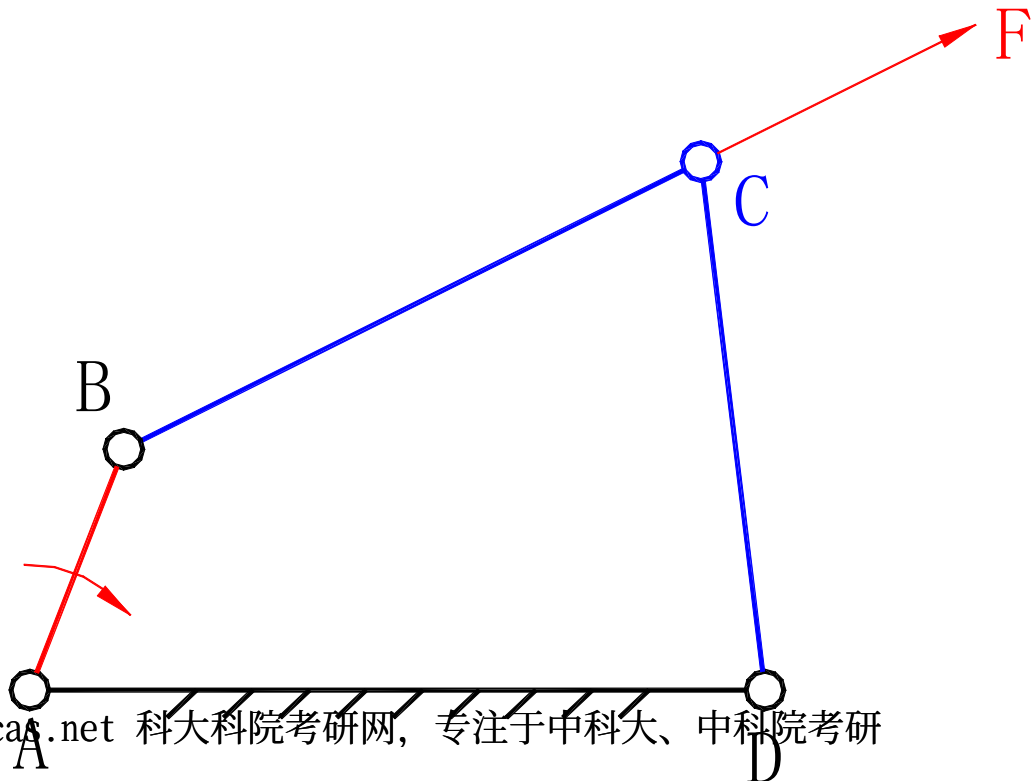
neglect the friction force & gravity force

不考虑摩擦力和重力



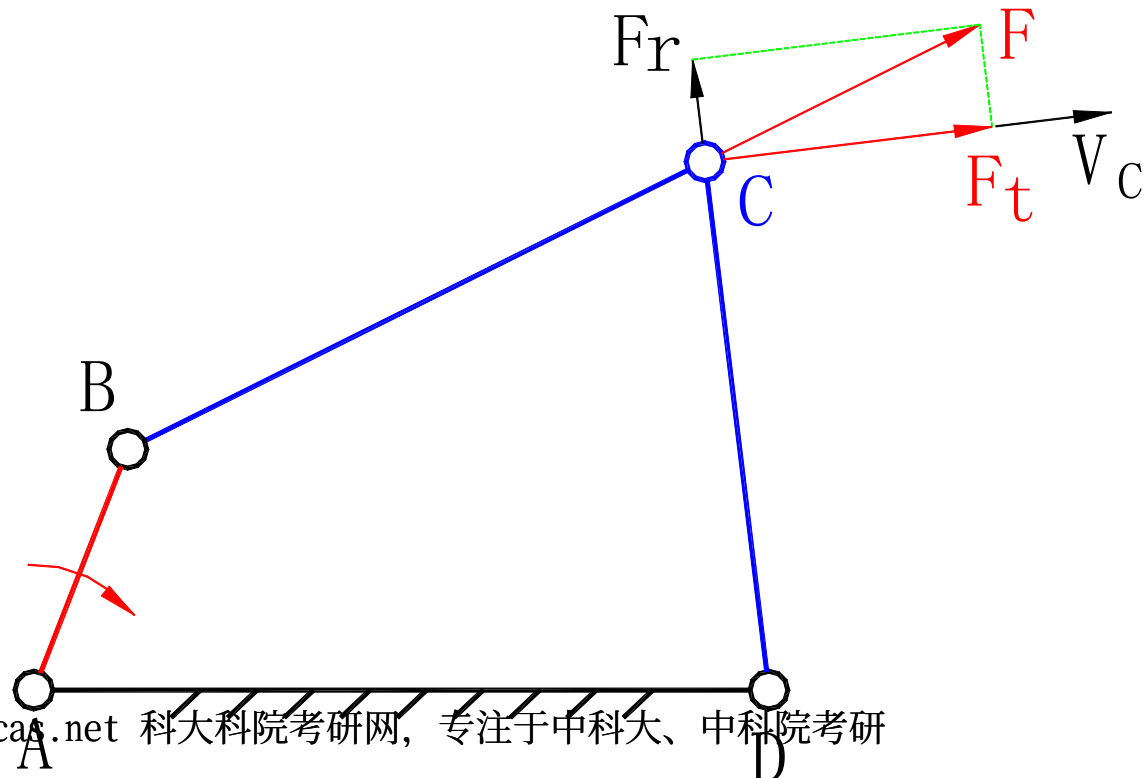


**the force applied to the CD $\rightarrow F$,
direction: along to BC**



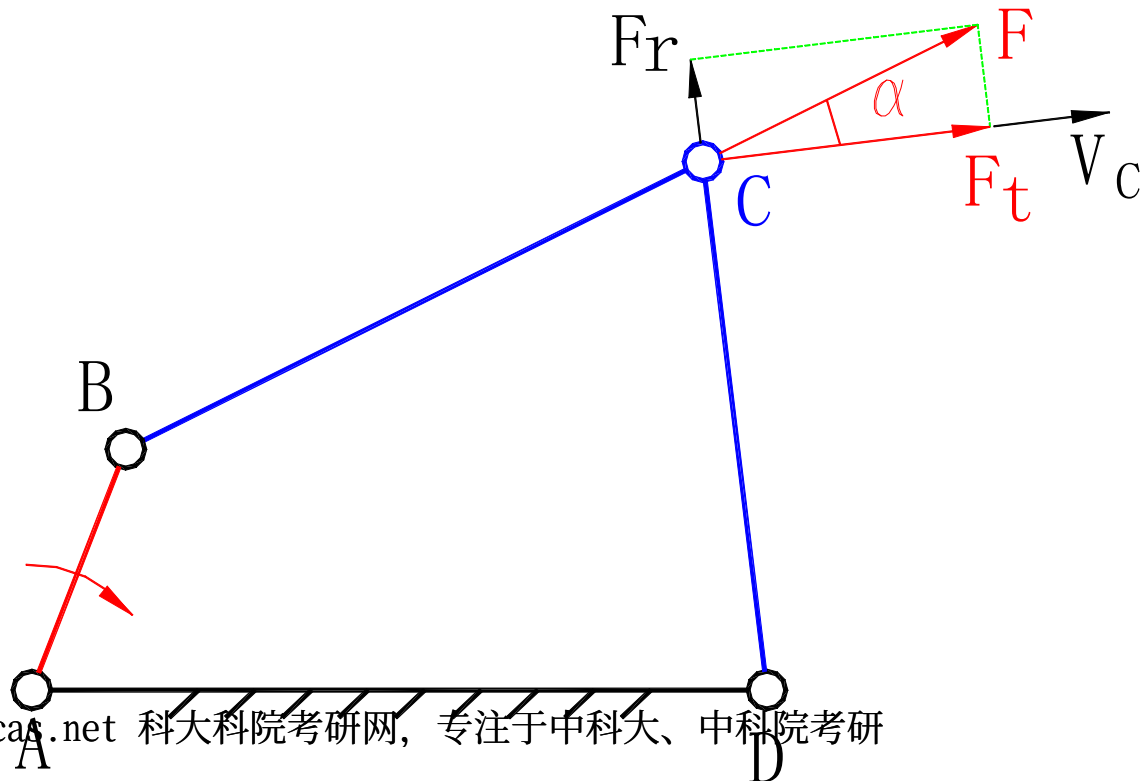


the velocity of point C $\rightarrow V_c$,
direction: vertical to CD





The acute angle(锐角) between the directions of the force F and the velocity of the point receiving(受到) the force on the follower is defined as the pressure angle(压力角) α of the mechanism at that position.



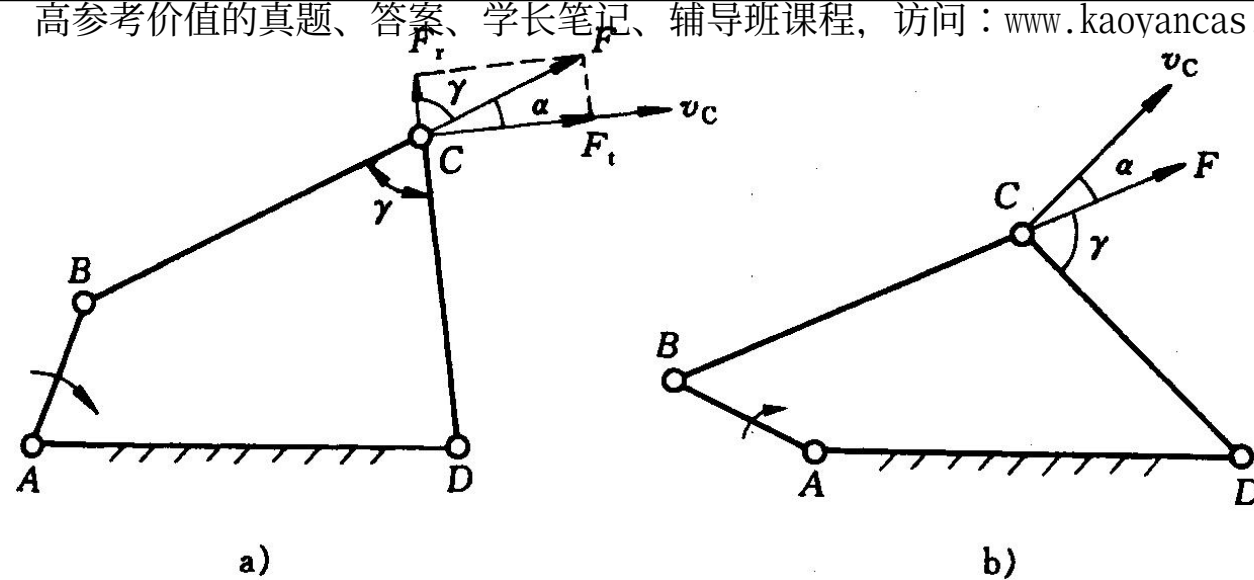
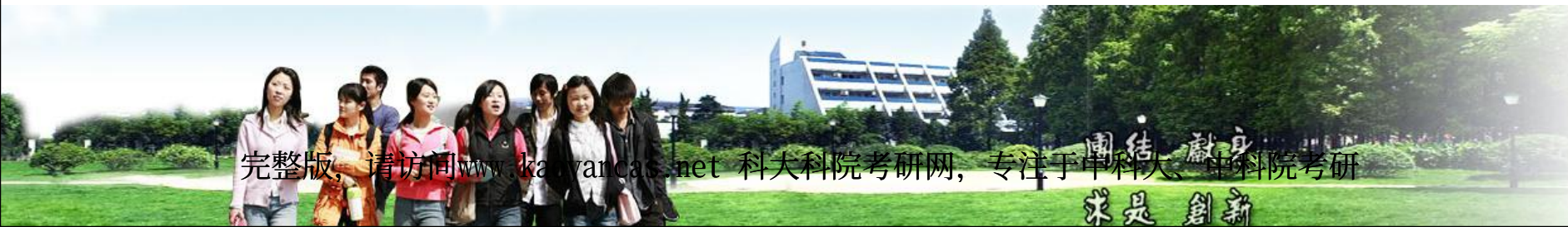


Fig. 4-23

$$F_t = F \cos \alpha = F \sin \gamma \quad \text{——有效分力}$$

$$F_n = F \sin \alpha = F \cos \gamma \quad \text{——无效分力}$$

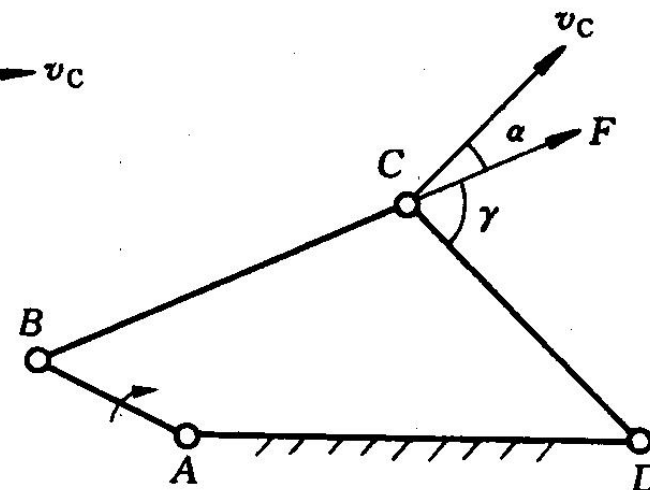
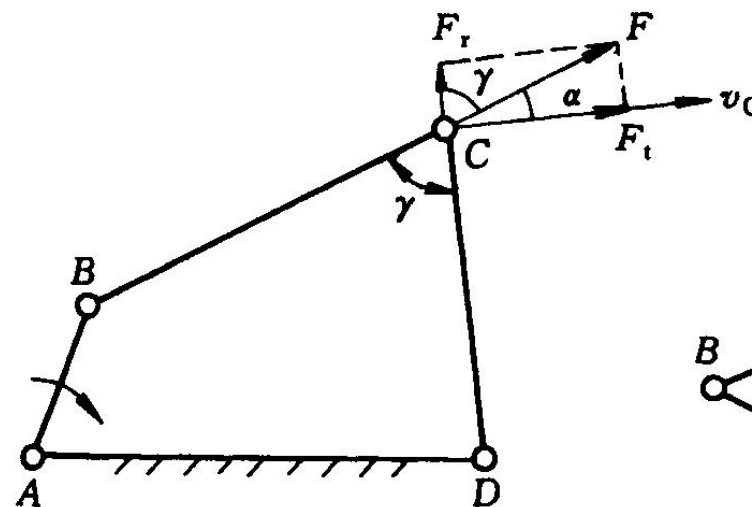
α 越小， F_t 越大，对机构传递越有利（效率高，不易自锁）





传动角 γ : The complement of the pressure angle α is transmission angle γ 压力角余角

$$\alpha + \gamma = 90^\circ$$



$\angle BCD < 90^\circ$ 时, $\gamma = \angle BCD$ $\angle BCD > 90^\circ$ 时, $\gamma = 180^\circ - \angle BCD$

平面连杆机构经常用 γ 衡量机构的传动质量 (易于测量)

γ 越大, 对机构传递越有利



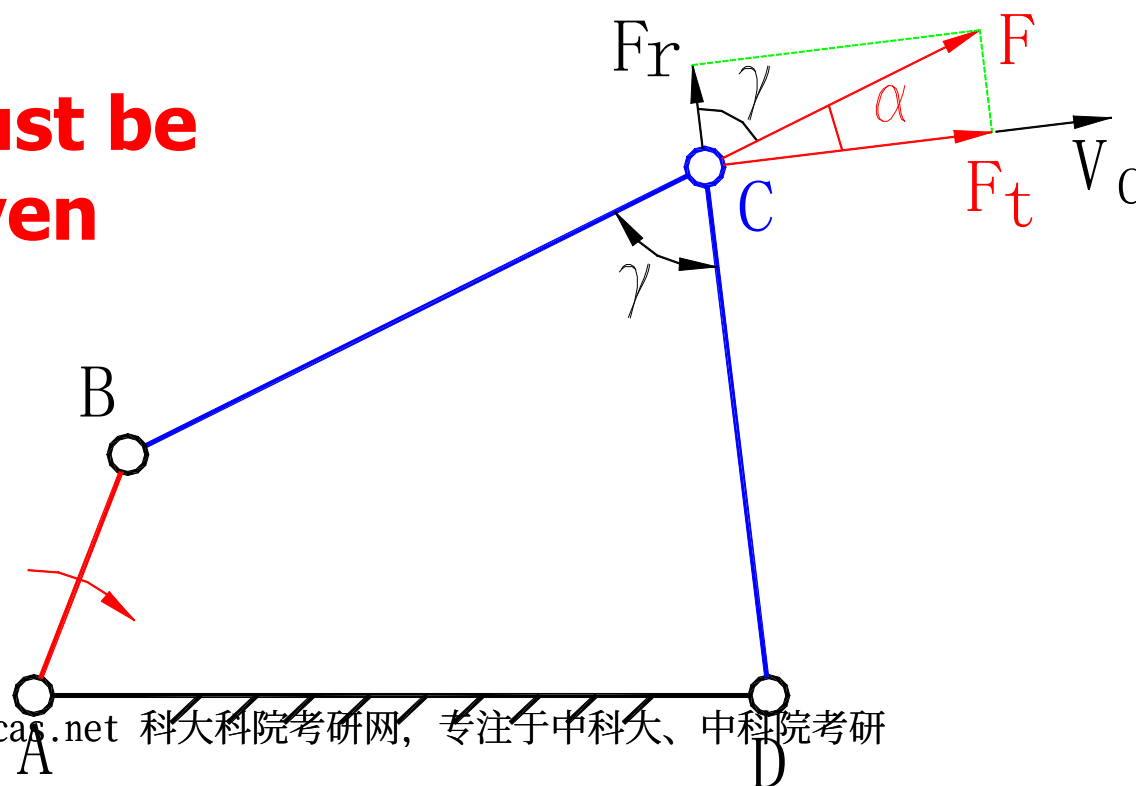
α 越小， γ 越大， F_t 越大，对机构传递越有利（效率高，不易自锁）

机构不停地运动， α 和 γ 不停在变化，为使机构不出现冲击

$$\alpha \leq [\alpha] = 40^\circ$$

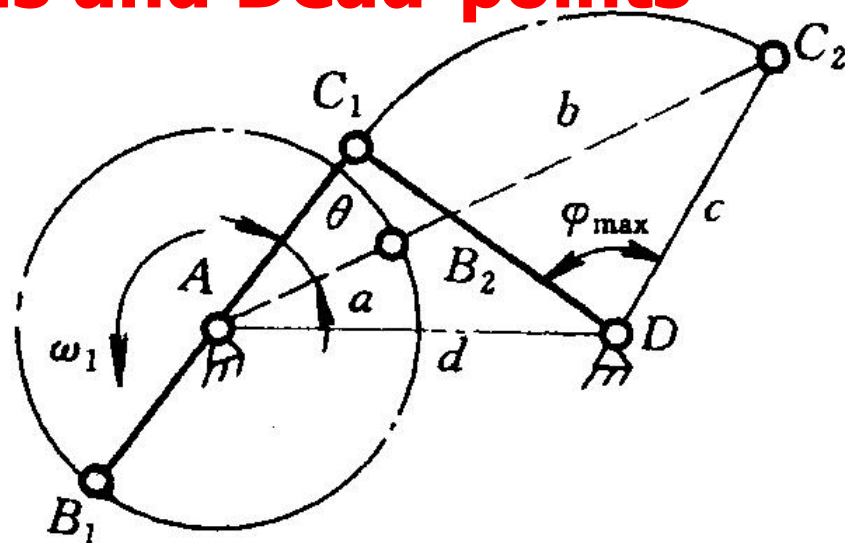
$$\gamma \geq [\gamma] = 50^\circ$$

Note: α and γ must be drawn on the driven link!!



4.3.3 Toggle (肘节) Positions and Dead-points

1.the limiting position C_1D, C_2D



如果曲柄 **AB** 为主动件, 转到极限位置时, 只用很小的力, 可克服作用在 **CD** 上很大的力。

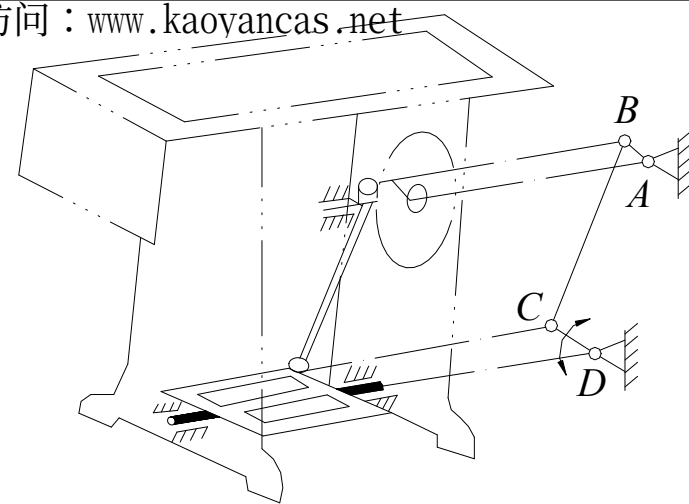
→ **toggle position.** (杠杆增力位置)

如果摇杆 **CD** 为主动件, 曲柄的压力角 $\alpha = 90^\circ$ ($\gamma = 0^\circ$)

加在 **AB** 上的力通过铰链中心 **A**, 使从动件 **AB** 运动的有效分力等于 0
曲柄 **AB** 不能运动 → **dead-point**

2. 死点位置的不利影响及克服

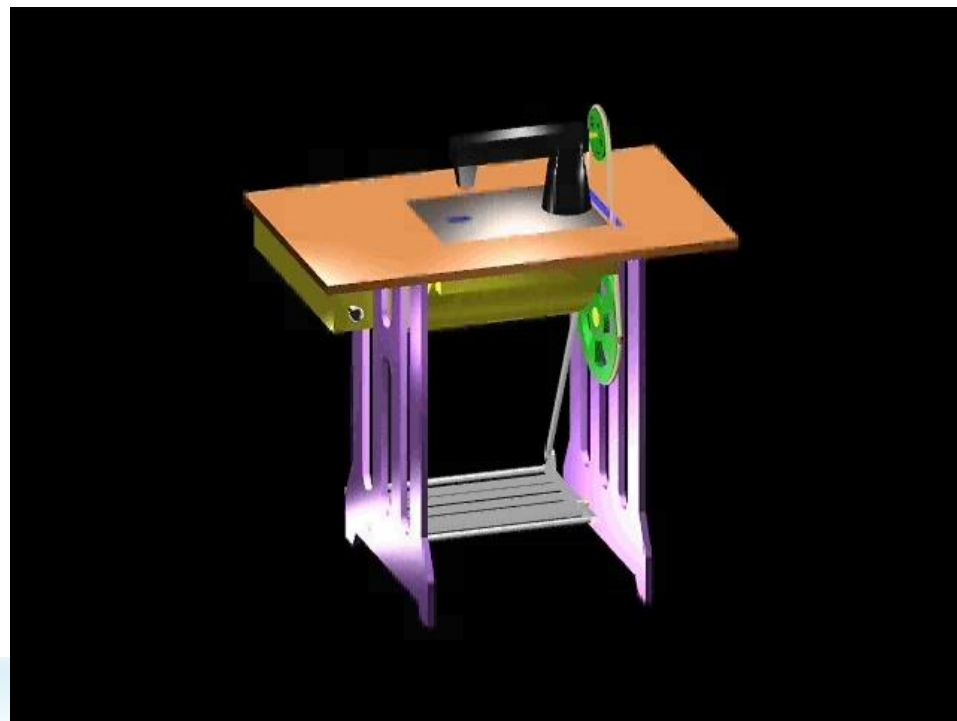
- ①运动不确定性
- ②运动突然停止, 卡死



Example: sewing machine

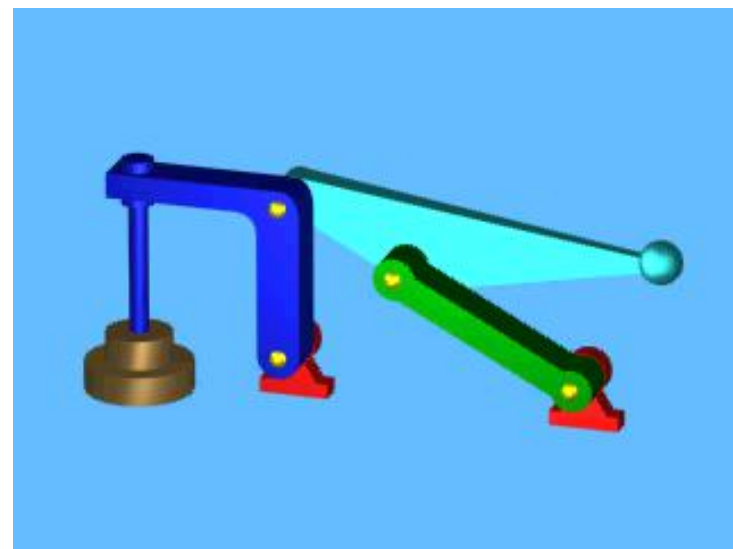
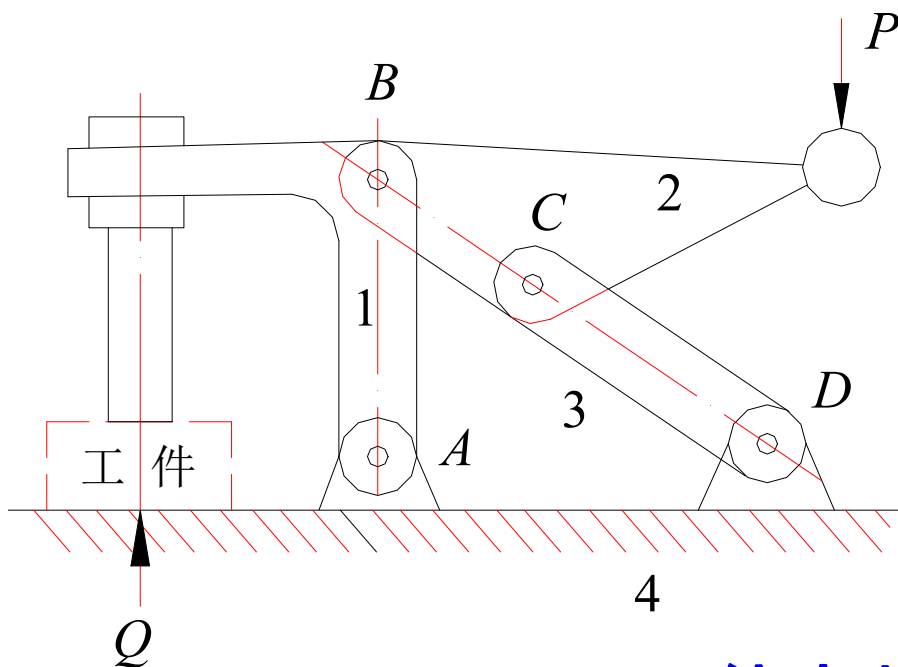
☀ 避免由死点处启动

☀ 运转时利用惯性通过死点





3.死点位置的应用--- In some circumstances(情况), the dead point is very useful. It can provide a self-locking(自锁) feature(性能).



工件夹紧机构

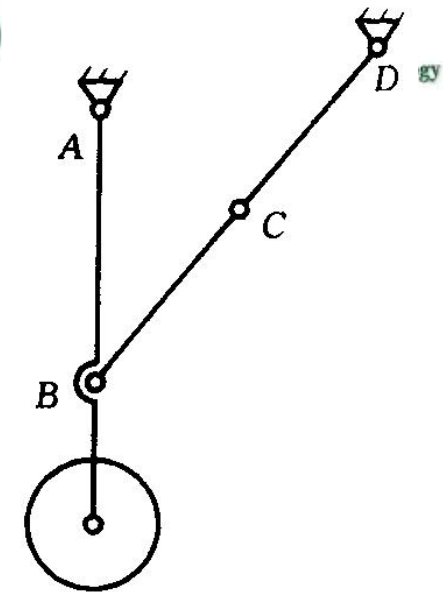
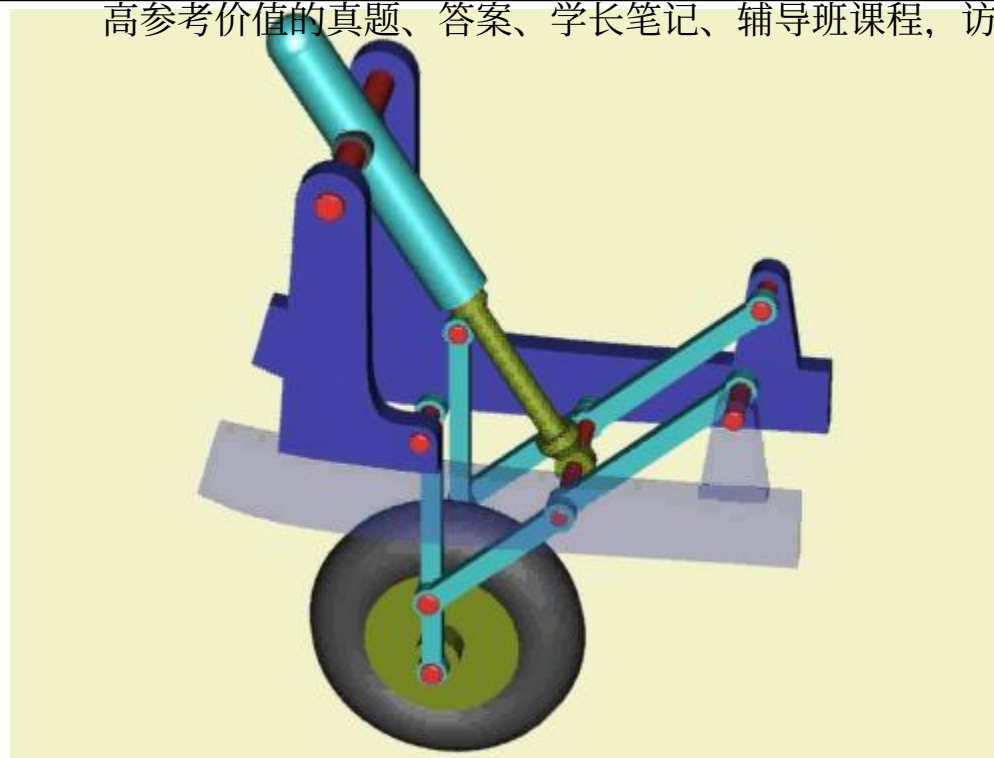


Fig. 4-30

Landing mechanism in airplane 飞机起落架
轮子上的力将不能引起机构运动，力通过转动中心D——**dead position;**
需要收起落架时，转动CD即可。



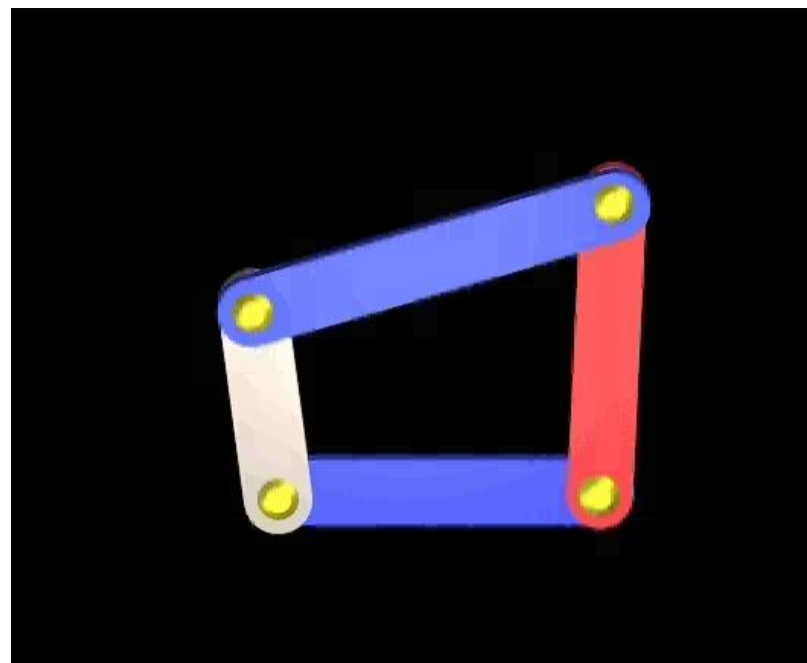
4.3.4 Quick Return Characteristics

---急回运动特性

crank→rotate in a constant angle velocity
driven links→ the working stroke→slower speed
(rocker,slider) the return stroke→faster speed



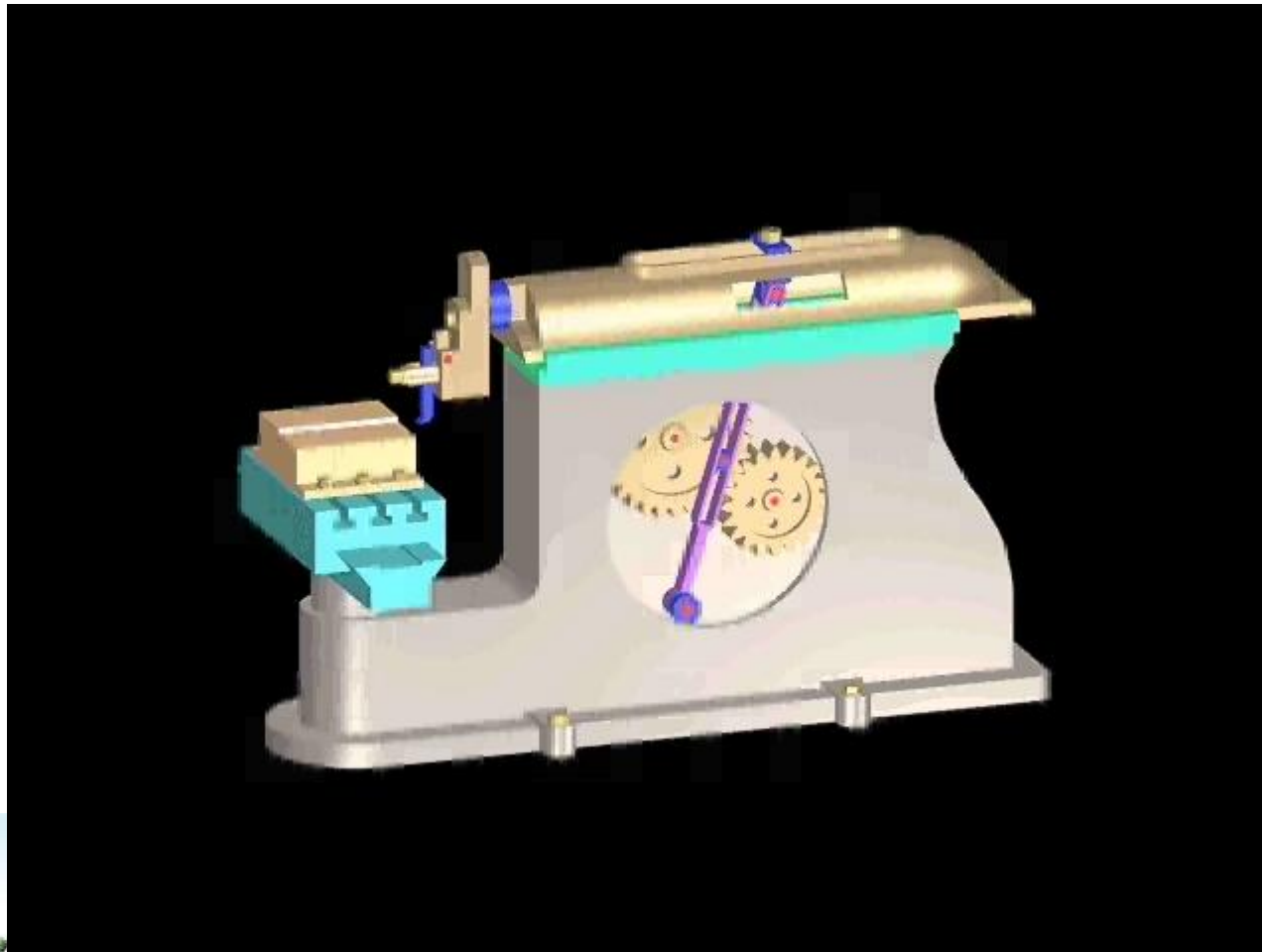
**Quick-return
mechanism**

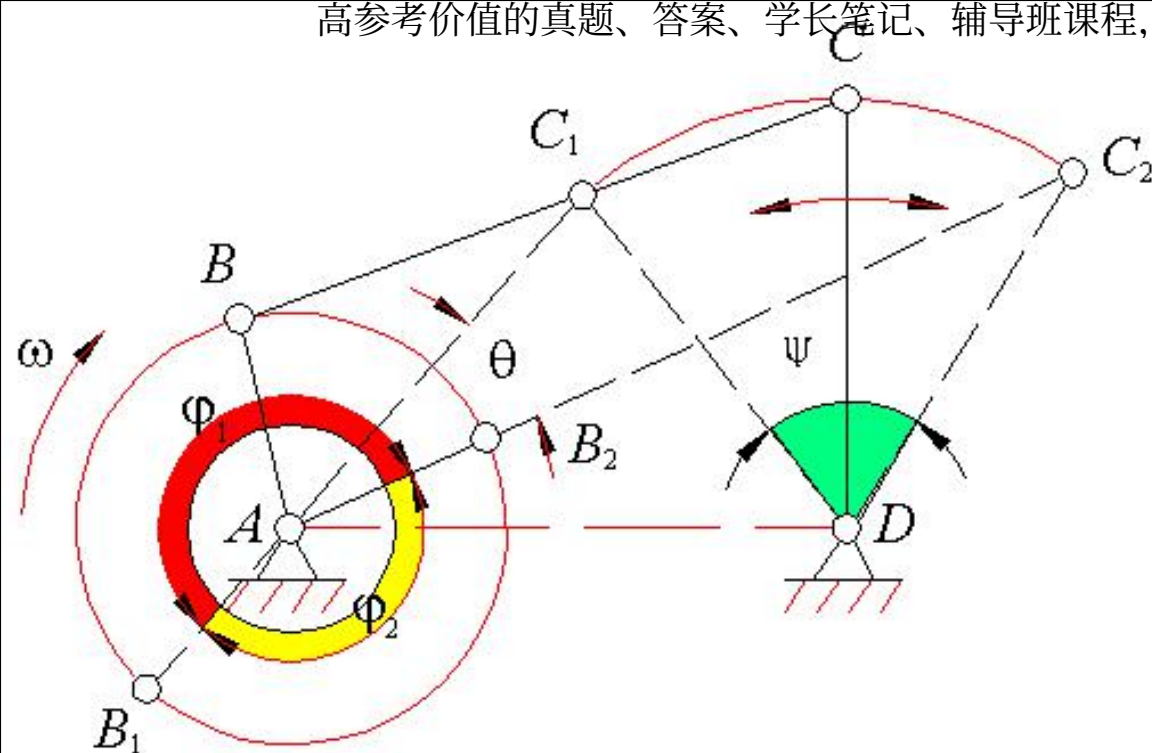


原动件作匀速转动，从动件相对机架作往复运动的机构，从动件正行程和反行程的平均速度不相等。这种特性称急回运动特性。



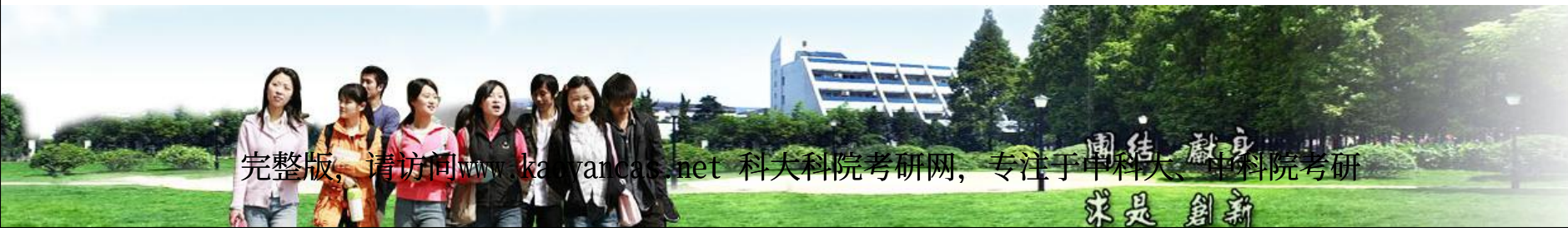
$V_{\text{工作}} < V_{\text{回程}}$ ，所以急回运动特性可缩短非生产时间，
提高劳动生产率。例：牛头刨床机构

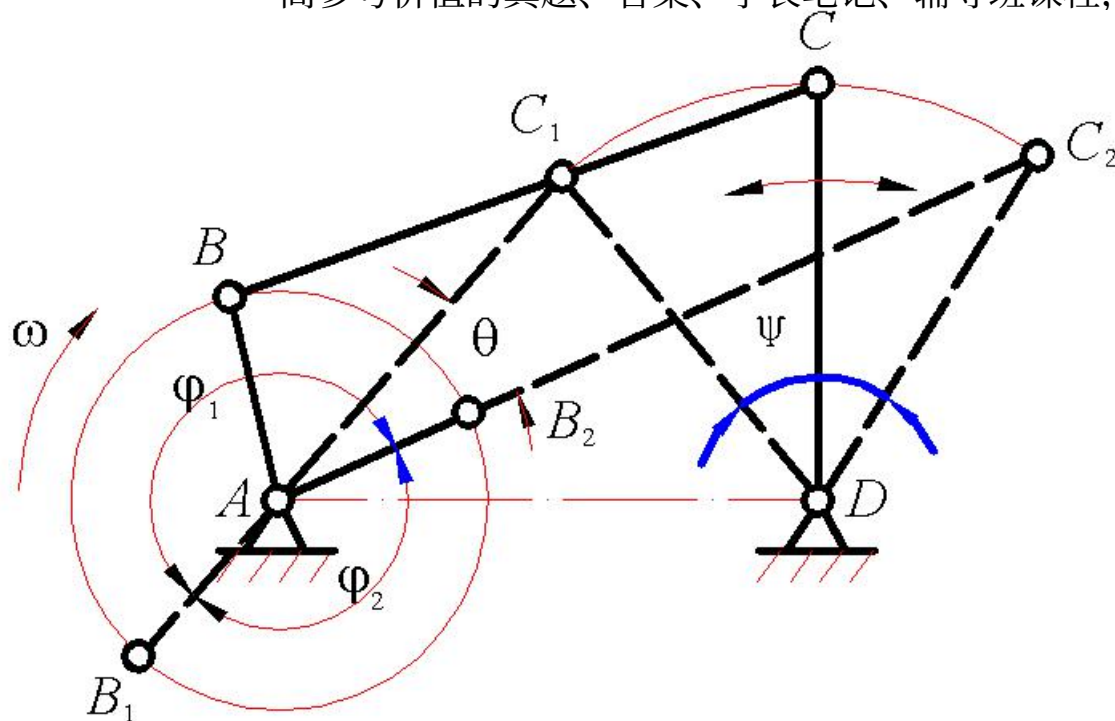




正行程：曲柄 **$AB_1—AB_2$** ， φ_1 ， $t_1 = \varphi_1 / \omega_1$
摇杆 **$C_1D—C_2D$** ， ψ ， $\omega'_3 = \psi / t_1$ ——慢

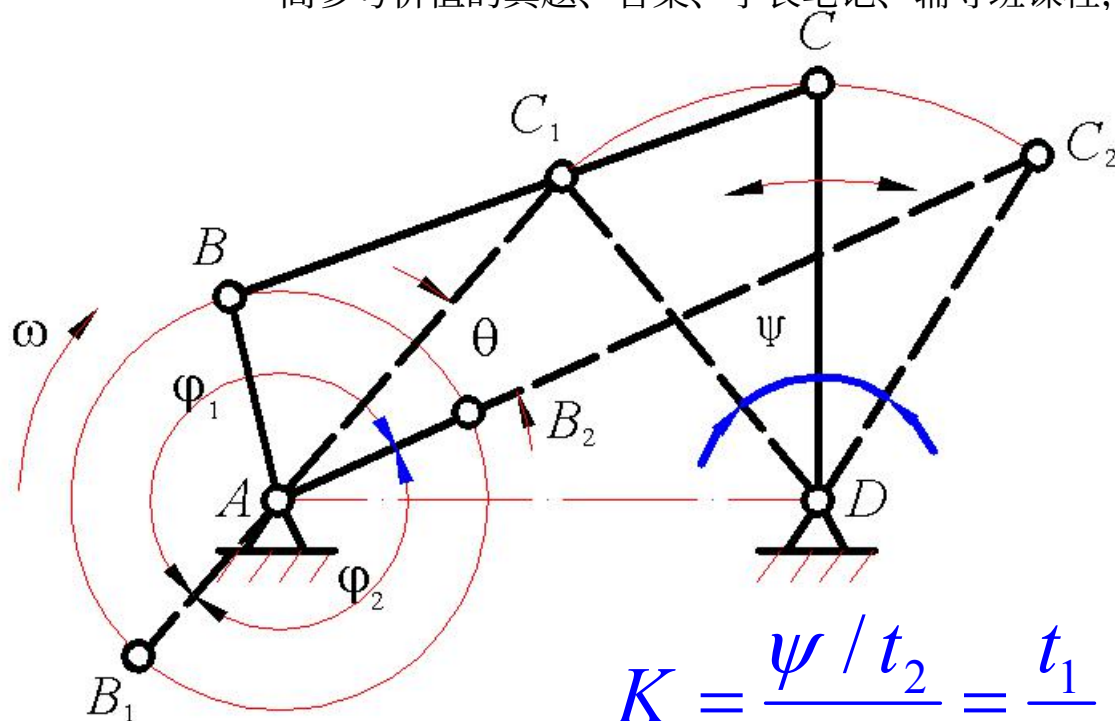
反行程：曲柄 **$AB_2—AB_1$** ， $\varphi_2 (\varphi_2 < \varphi_1)$ ， $t_2 = \varphi_2 / \omega_1 (< t_1)$
摇杆 **$C_2D—C_1D$** ， ψ ， $\omega''_3 = \psi / t_2$ ——快





K—coefficient of travel speed variation. $K \geq 1$
从动件行程速度变化系数。

$$K = \frac{\text{the average angular velocity of faster stroke}}{\text{the average angular velocity of slower stroke}} \geq 1$$



$$K = \frac{\psi / t_2}{\psi / t_1} = \frac{t_1}{t_2} = \frac{\varphi_1 / \omega}{\varphi_2 / \omega} = \frac{\pi + \theta}{\pi - \theta}$$

$\theta \rightarrow$ crank acute angle between the two limiting positions $[0, 180^\circ]$

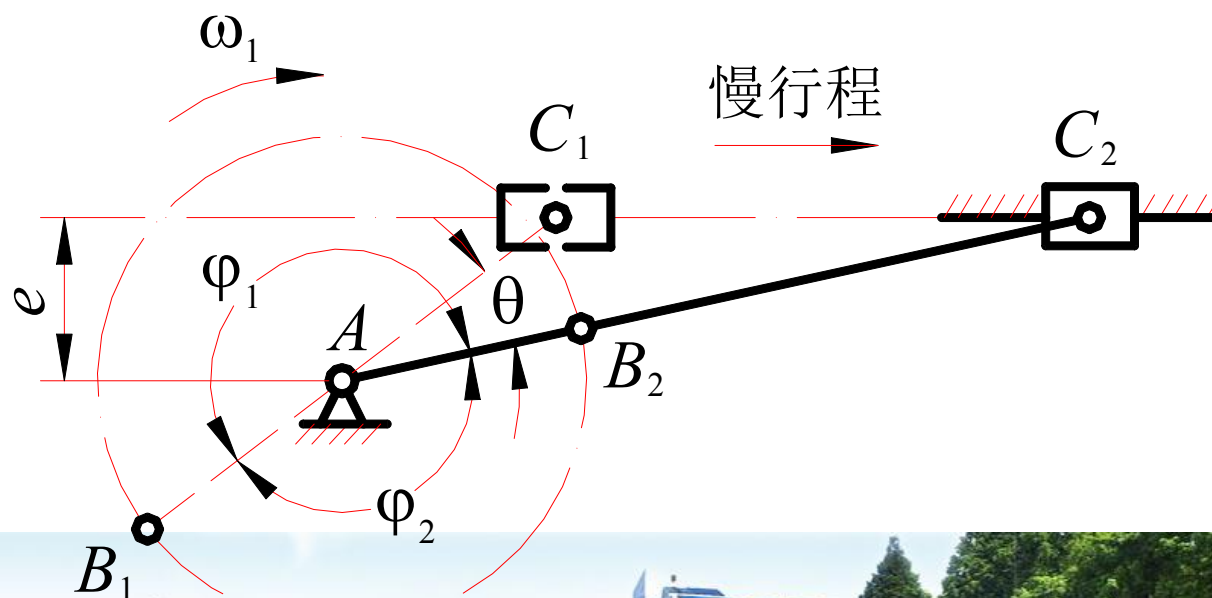
θ 为极位夹角: $\theta = 180^\circ \frac{K-1}{K+1}$ θ 越大，急回特性越显著



3. 其他具有急回特性的机构

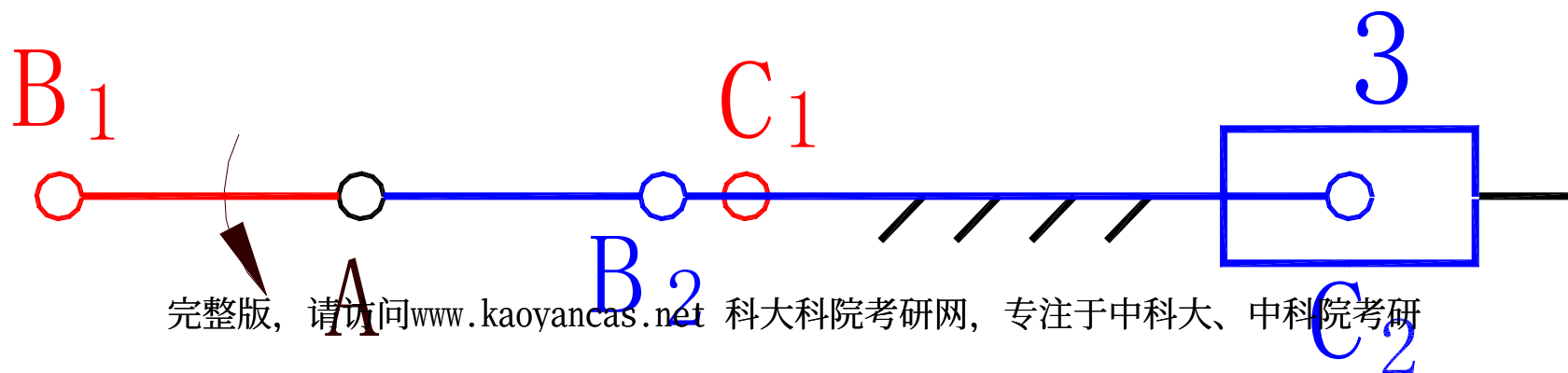
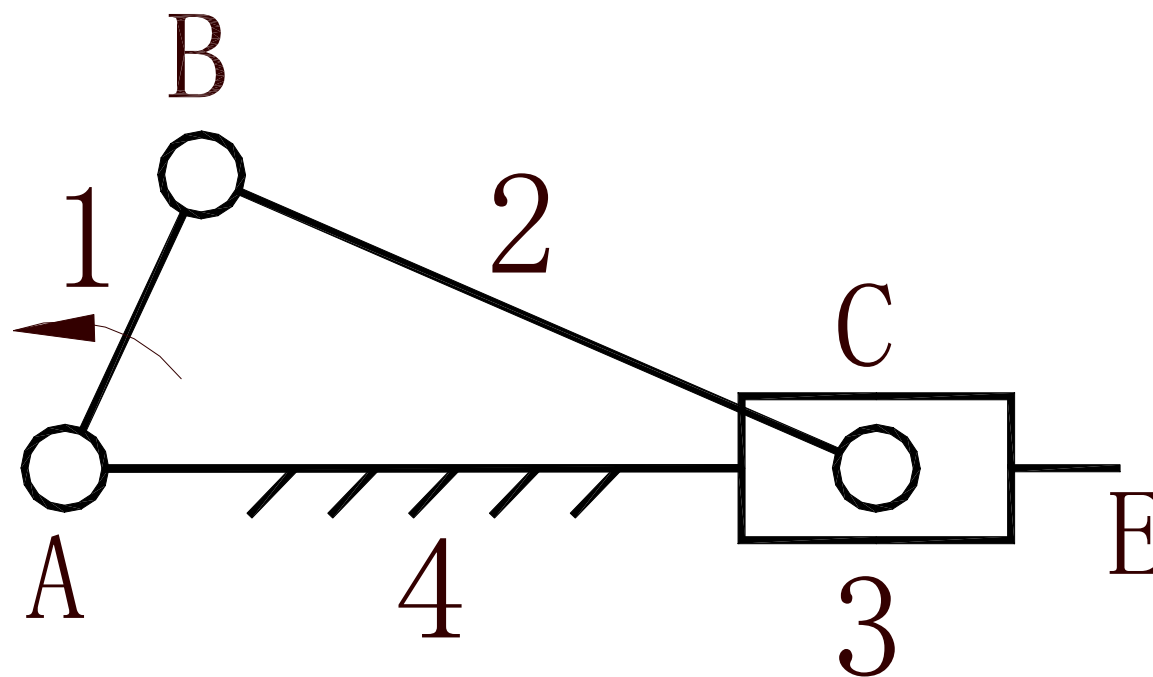
offset slider-crank mechanism 偏置曲柄滑块机构:

If the driving crank AB rotates clockwise with constant angular velocity, the slider will take a longer time in its rightward(向右) stroke than in its leftward(向左) stroke.





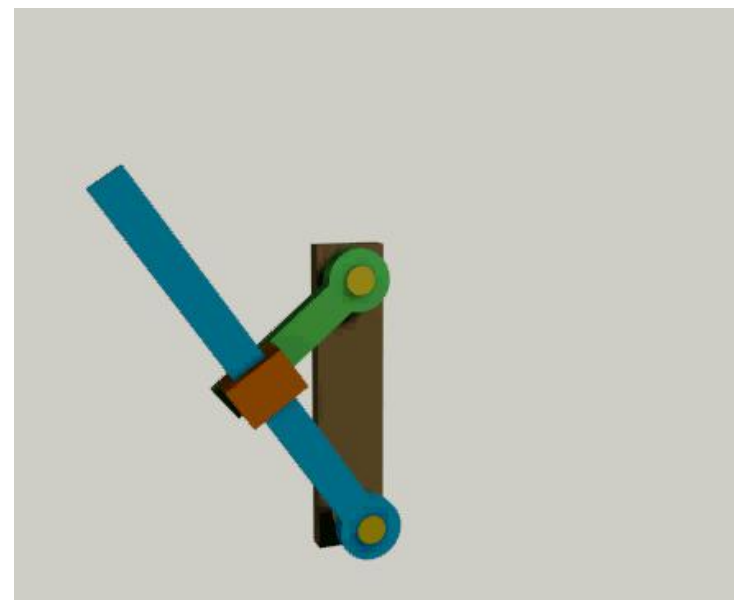
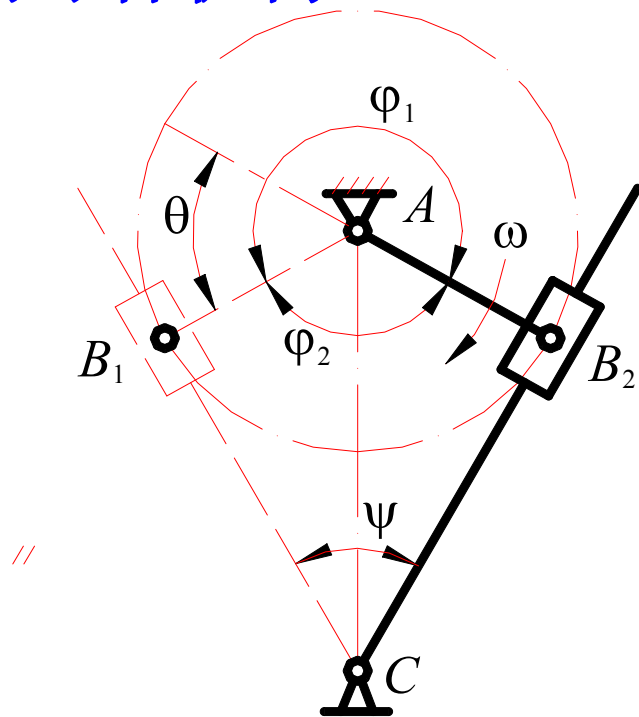
Since $\theta=0^\circ$, an in-line(对心) slider-crank mechanism has no quick-return characteristics.



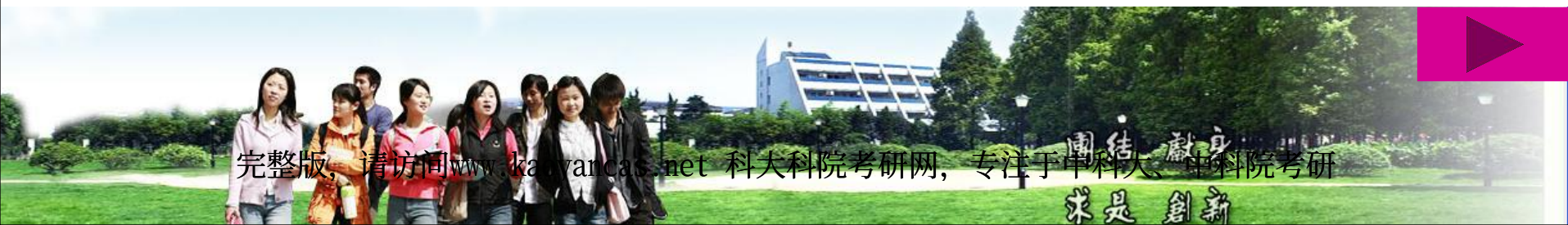


oscillating guide-bar mechanism

摆动导杆机构：



$$\theta = \psi$$



4.4 Dimensional Synthesis of Four-bar Linkages

Dimensional synthesis(综合) of a mechanism is the determination of the kinematic dimensions necessary to achieve(获得) the required motion.

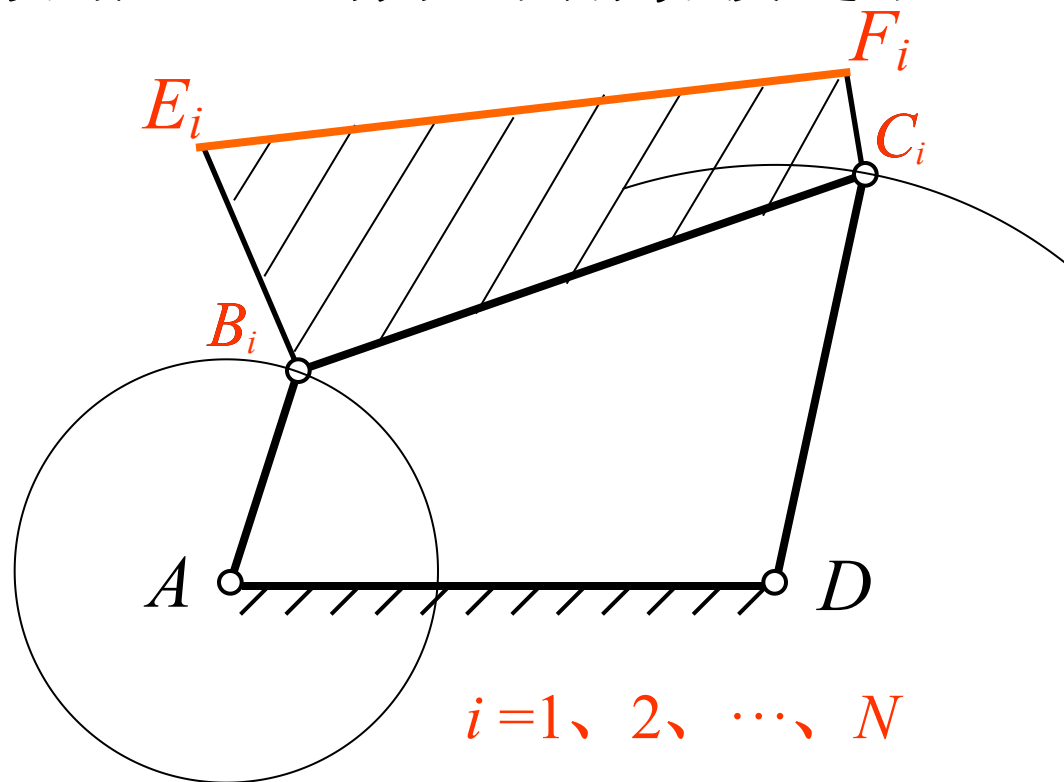
连杆机构的设计有二个任务：**机构选型**（称为**型综合**）和**确定机构的尺寸**（称为**尺度综合**）。

Usually, different problems will use different methods.

- **Graphical method**图解法
- **Analytical method**解析法
- **Experimental method**实验法

图解设计的基本原理

➤ 图解设计问题——作图求解各铰链中心的位置问题。

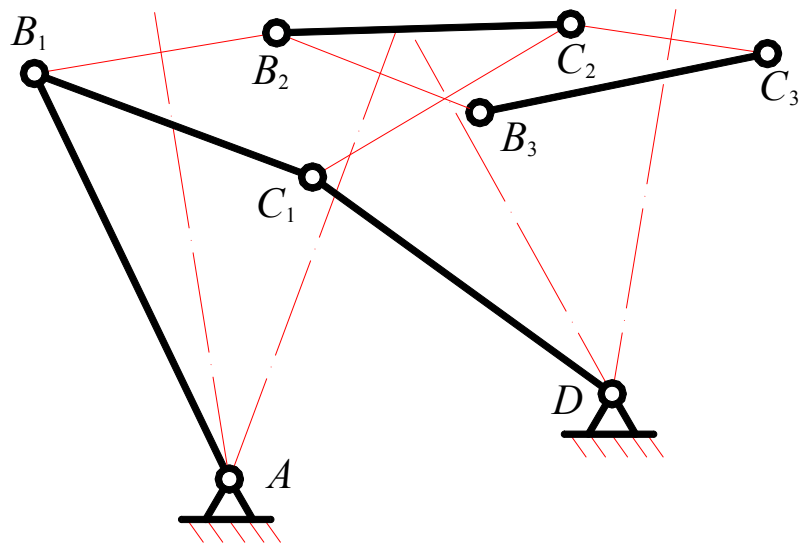


➤ 各铰链间的运动关系：

固定铰链 A 、 D ：圆心

活动铰链 B 、 C ：圆或圆弧

1、已知B，C及连杆的三个位置，设计该铰链四杆机构。



∵ B点绕A点、C点绕D点定轴转动

- ① 连接B₁B₂、B₂B₃;
- ② 作B₁B₂、B₂B₃垂直平分线交于A点;
- ③ 连接C₁C₂、C₂C₃;
- ④ 作C₁C₂、C₂C₃垂直平分线交于D点。

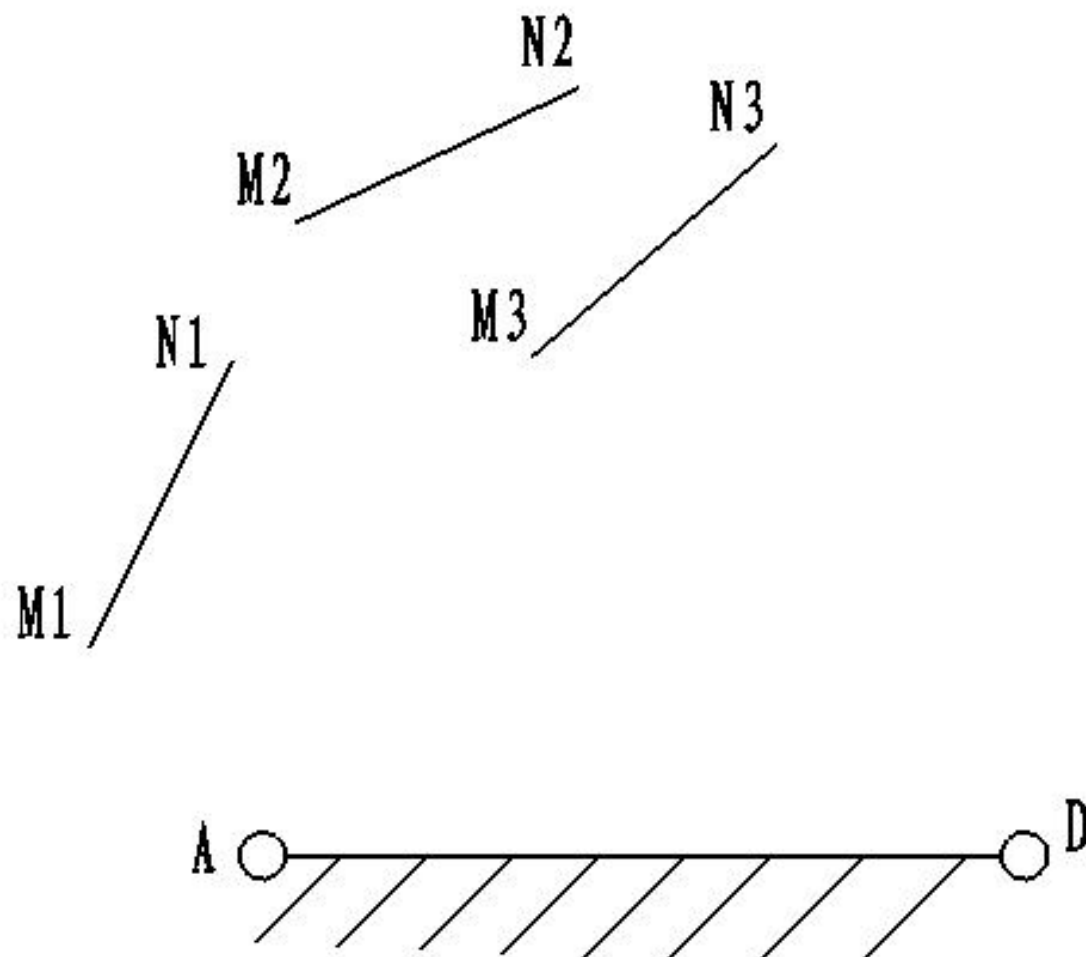
给出B、C三个位置，A、D位置唯一

给出B、C二个位置，A、D位置无穷

(需加其他条件方可获得唯一解)



2、已知A，D及连杆MN的三个位置，设计该铰链四杆机构。



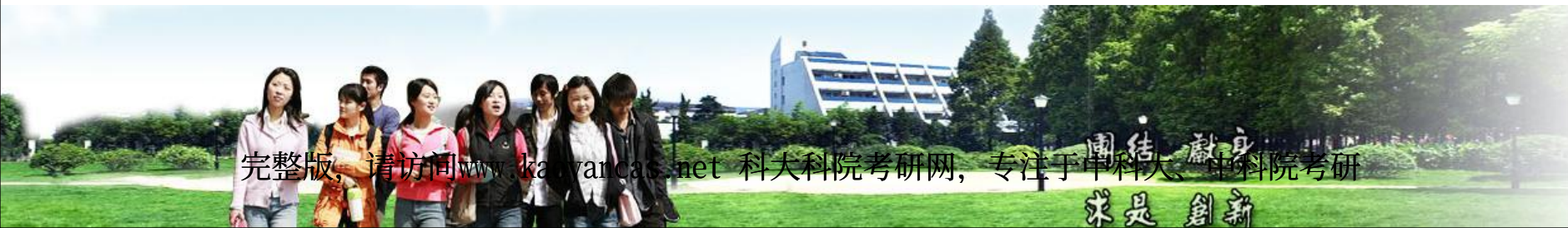


➤ 机构的倒置原理

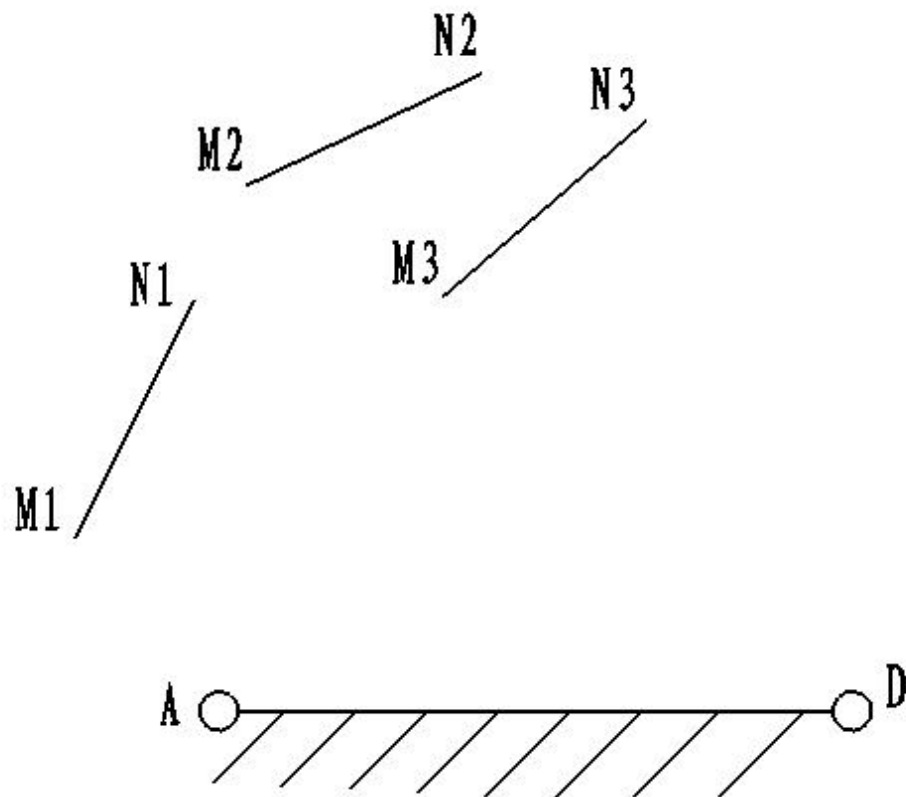
为了求活动铰链的位置，可将待求活动铰链所在的杆视作**新机架**，而将其相对的杆视为**新连杆**。

接下来，将原机构的各位置的构型均视为刚体，并向某一选定位置相对移动，使新机架的各杆位置重合，便可得新连杆相对于新机架的各个位置，即实现了**机构的倒置**。

这样，就将求活动铰链的位置问题转化为求固定铰链的位置问题了。这种方法又称为反转法。



求解步骤动画



以 M_1N_1 为机架， M_1N_1 为原位

- 1、刚化 AM_2N_2D ，使得 M_2N_2 与 M_1N_1 重合，求得 A_2' 、 D_2'
- 2、刚化 AM_3N_3D ，使得 M_3N_3 与 M_1N_1 重合，求得 A_3' 、 D_3'
- 3、作 $A_1'A_2'$ 、 $A_2'A_3'$ 的公垂线交于 B_1
- 4、作 $D_1'D_2'$ 、 $D_2'D_3'$ 的公垂线交于 C_1

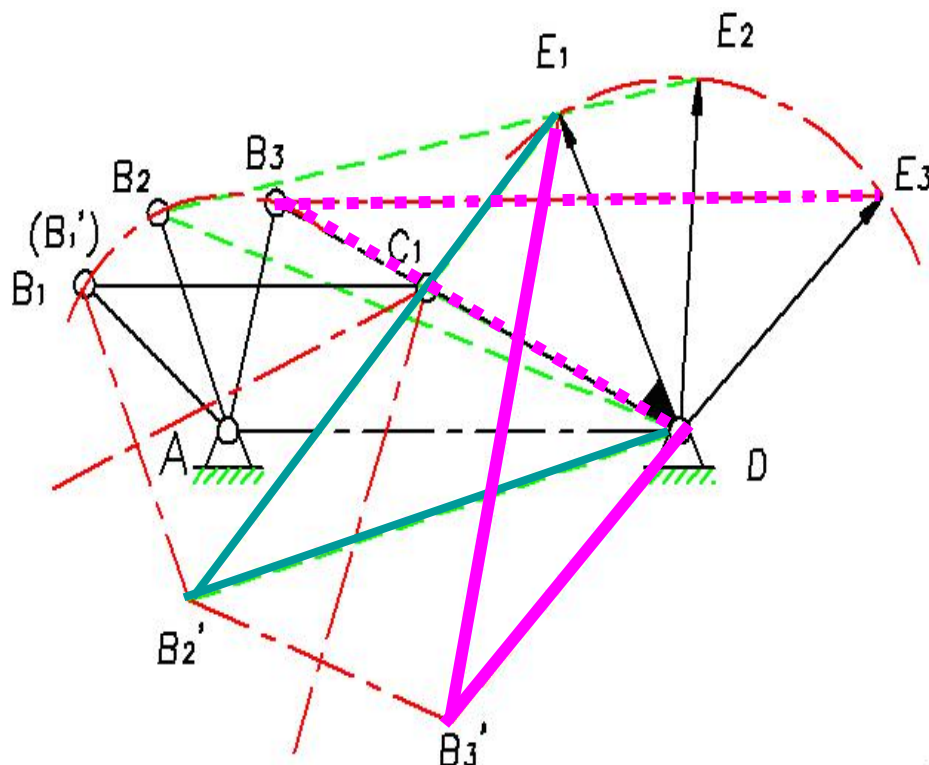
AB_1C_1D 便是所求



3、按给定两连架杆的位置设计四杆机构

目的：求解出铰链 C 。

引入转换机架的思想，采用反转法求得。



① 作 $\triangle DE_1B_2' \cong \triangle DE_2B_2$ ，得 B_2' ；

② 作 $\triangle DE_1B_3' \cong \triangle DE_3B_3$ ，得 B_3' ；

③ 作 $B_1'B_2'$ 、 $B_2'B_3'$ 的垂直平分线，交点就是所求 C_1 。

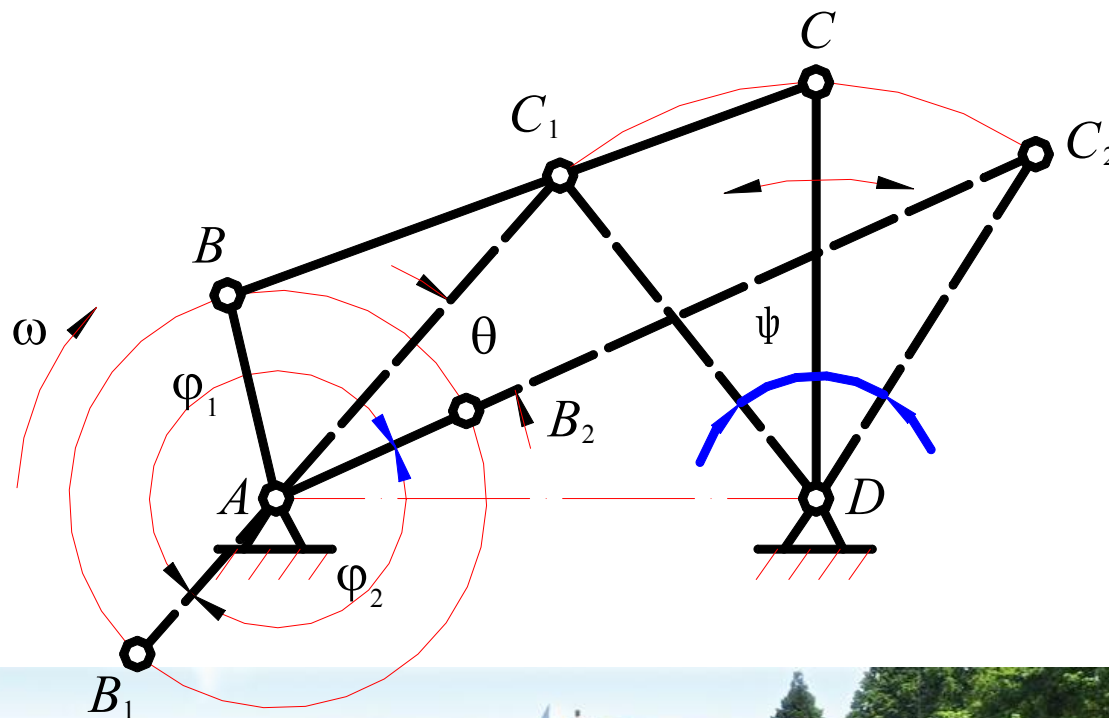
连杆长度： $b = u_l \times B_1C_1$

摇杆长度： $c = u_l \times D_1C_1$



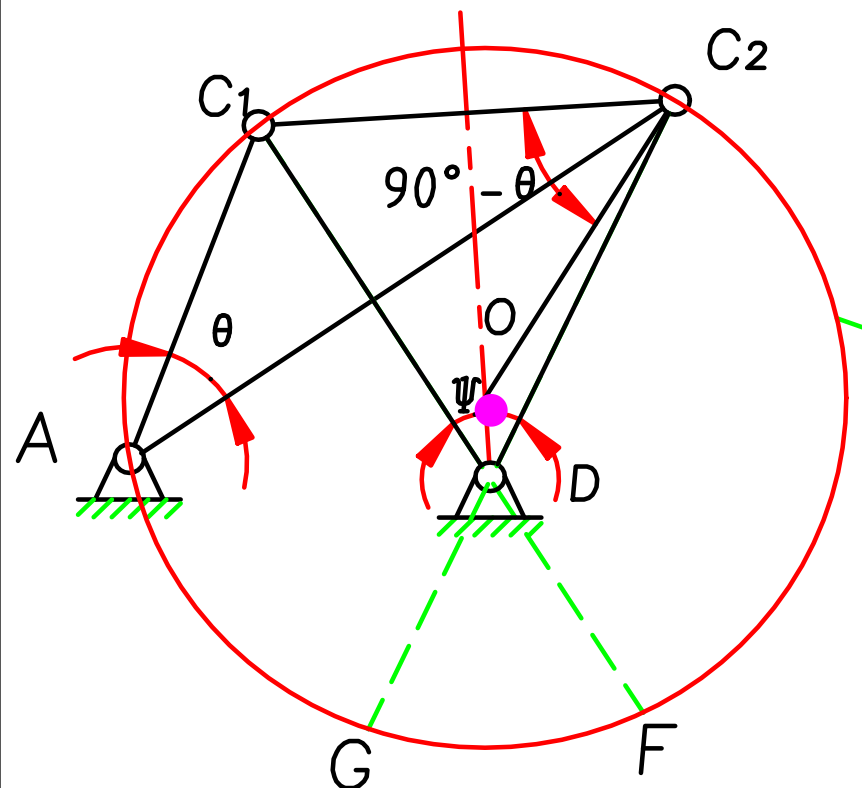
4、按K设计四杆机构

已知：曲柄摇杆机构，摇杆**CD**长度，摆角， **K**
设计此机构（确定**A**, 曲柄和连杆长）



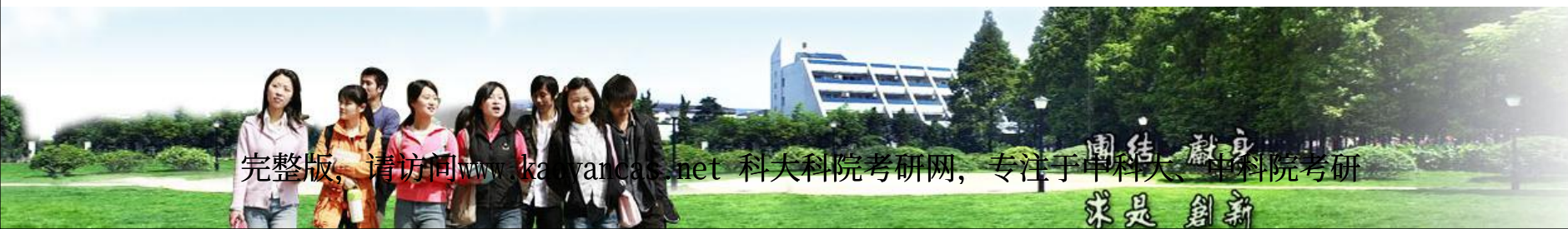


$$\theta = 180^\circ \frac{K-1}{K+1}$$



- ① 由(1)式算出极位夹角 θ ;
- ② 任选 D 的位置，按已知条件作出摇杆极限位置 DC_1 和 DC_2 ;
- ③ 作 $\angle C_2C_1O = \angle C_1C_2O = 90^\circ - \theta$ 得 C_1O 与 C_2O 交点 O ;
- ④ 以 O 为圆心， OC_1 为半径作辅助圆， A 点在圆弧上任选。

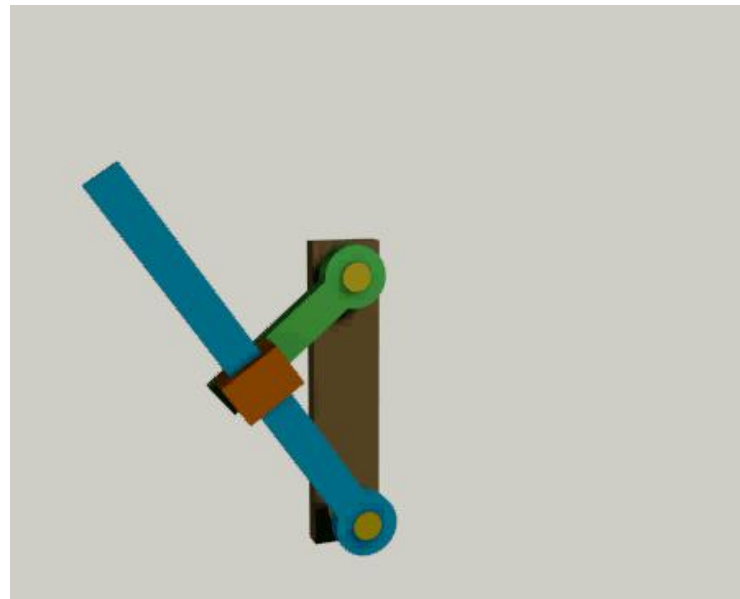
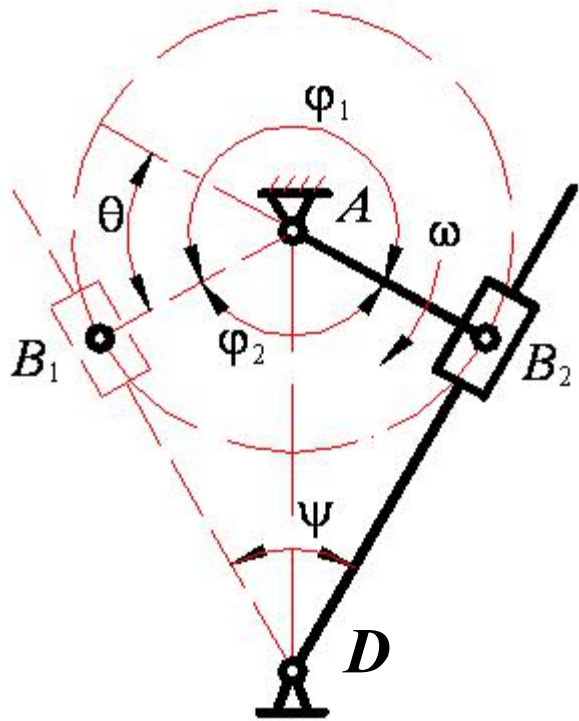
$$\left. \begin{aligned} l_{AC_1} &= b - a \\ l_{AC_2} &= b + a \end{aligned} \right\} \Rightarrow a, b$$



for example: Oscillating guide-bar mechanism

Known: K (time ratio)
 I_{AD} (the frame)

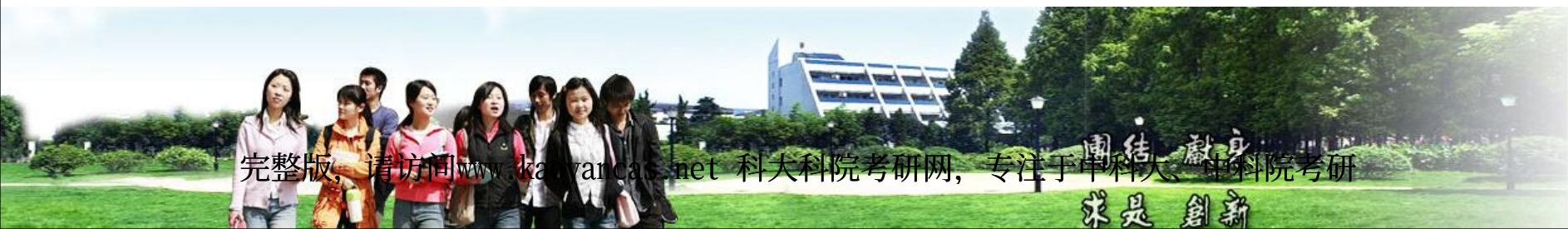
Require: I_{AB}





Attentions (this chapter):

- ☀ **The types of four-bar linkages**
- ☀ **Conditions for having a crank**
- ☀ **Pressure angle and transmission angle**
- ☀ **Dead point and quick return characteristic**
- ☀ **Dimensional synthesis of fourbar linkages**



Question ?



南京理工大学

Nanjing University of Science and Technology



杆件数目？

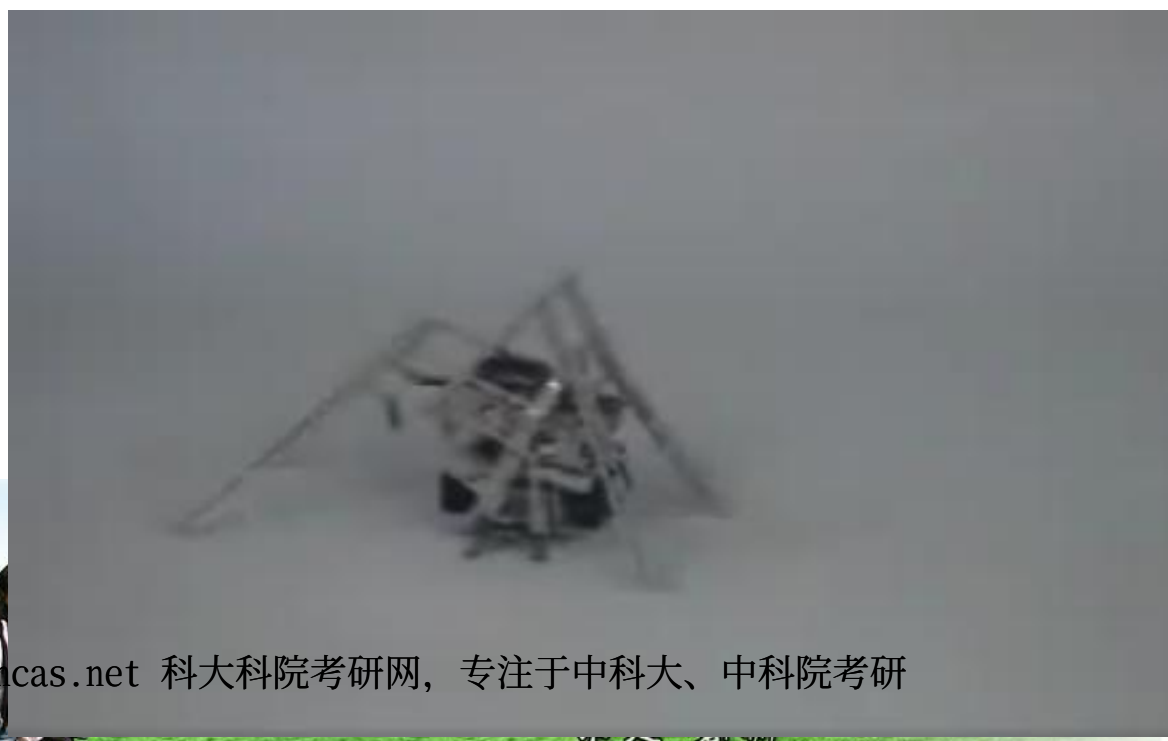
运动副类型？

连杆机构类型？



课外实践作品

——仿生尺蠖机构





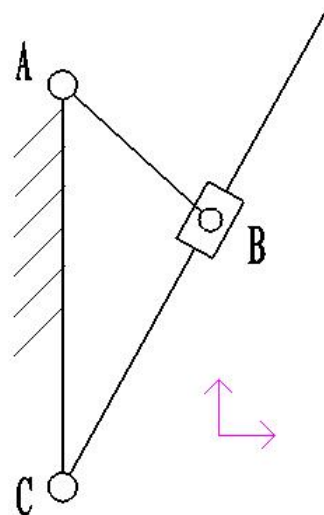
Homework

1. Read book p43-57.
2. Exercise p66:4-3,4-4(1),(2):a,b,c,d
4-5,4-6



复习思考题

- 1、如何依照各杆长度判别铰链四杆机构的型式？
- 2、平面四连杆机构最基本形态是什么？由它演化为其它平面四杆机构，有哪些具体途径？
- 3、图示摆动导杆机构中，**AB**杆匀角速转动。该机构存在的几何条件是什么？给定其摆杆的行程速度变化系数 K 后，怎么用 K 值表达**AB**和**AC**杆的长度关系？





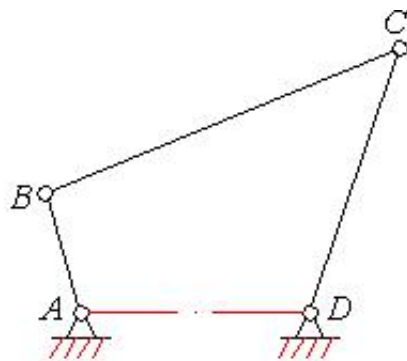
习题

1、图示铰链四杆机构，已知： $l_{BC}=50\text{mm}$ ， $l_{CD}=35\text{mm}$ ， $l_{AB}=30\text{mm}$ ，AD为机架，

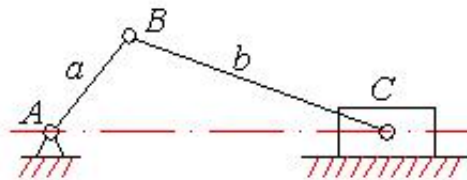
(1) 若此机构为曲柄摇杆机械，且AB为曲柄，求 l_{AB} 的最大值；

(2) 若此机构为双曲柄机构，求 l_{AB} 的范围；

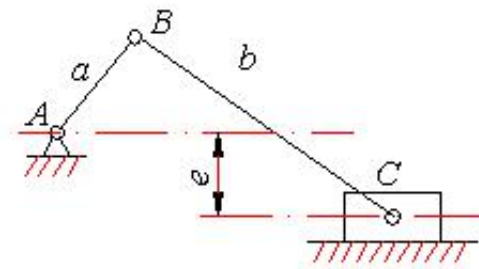
(3) 若此机构为双摇杆机构，求 l_{AB} 的范围。



题1图



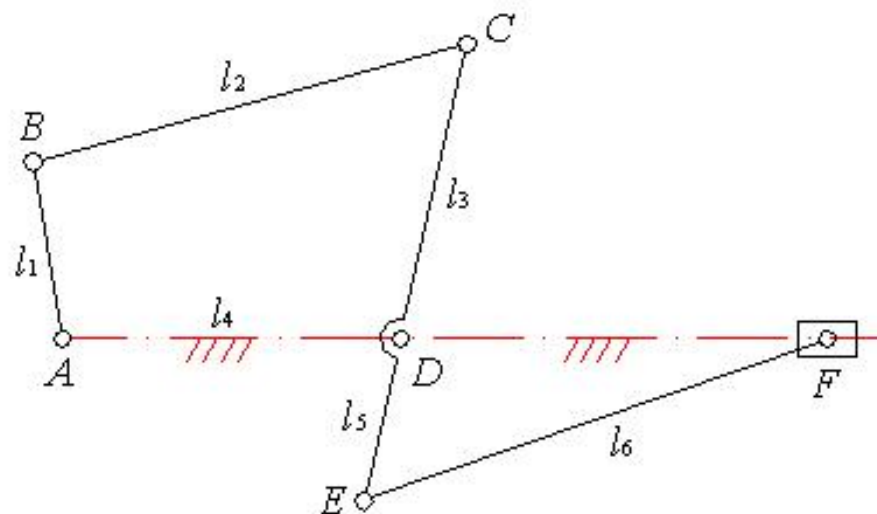
题2图



2、图示两种曲柄滑块机构，若已知 $a=120\text{mm}$ ， $b=600\text{mm}$ ，对心时 $e=0$ 及偏置时 $e=120\text{mm}$ ，求此两机械的极位夹角 θ 及行程速比系数 K 。又在对心曲柄滑块机构中，若连杆BC为二力杆件，则滑块的压力角将在什么范围内变化？

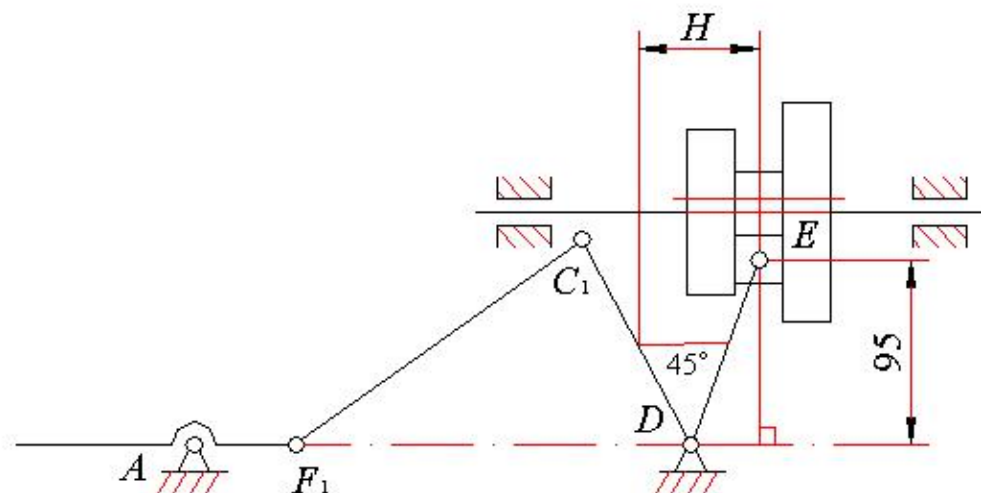


3、图示六杆机构，已知（单位mm）： $l_1=20$ ， $l_2=53$ ， $l_3=35$ ， $l_4=40$ ， $l_5=20$ ， $l_6=60$ ，试确定：1）构件AB能否整周回转？2）滑块行程 h ；3）滑块的行程速度变化系数 K ；4）机构DEF中的最大压力角 $\alpha_{\max}$ 。





4、图示机床变速箱中操纵滑动齿轮的操纵机构，已知滑动齿轮行程 $H=60\text{mm}$ ， $l_{DE}=100\text{mm}$ ， $l_{CD}=120\text{mm}$ ， $l_{AD}=250\text{mm}$ ，其相互位置如图所示。当滑动齿轮在行程的另一端时，操纵手柄朝垂直方向，试设计此机构。





5、设计一偏置曲柄滑块机构，已知滑块的行程速度变化系数 $K=1.5$ ，滑块的冲程 $l_{C_1C_2}=50\text{mm}$ ，导路的偏距 $e=20\text{mm}$ ，求曲柄长度 l_{AB} 和连杆长度 l_{BC} 。

