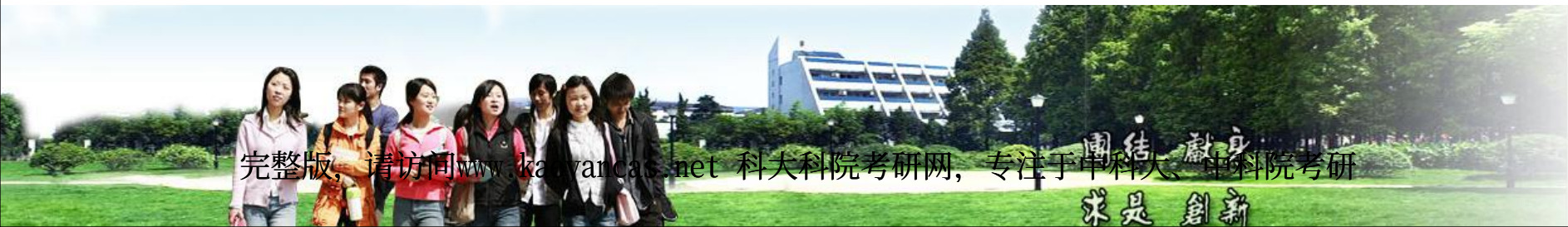




Theory of Machines and Mechanisms (机械原理)





Chapter1 Introduction

Chapter2 Structural Analysis of Planar Mechanisms

Chapter3 Kinematic Analysis of Mechanisms

Chapter4 Planar Linkage Mechanisms

Chapter 5 Cam Mechanisms

Chapter 6 Gear Mechanisms

Chapter 7 Gear Trains

Chapter 8 Other Mechanisms in Common Use

Chapter 9 Balancing of Machinery

Chapter 10 Motion of Mechanical Systems and Its Regulation

Chapter 11 Efficiency of Machine



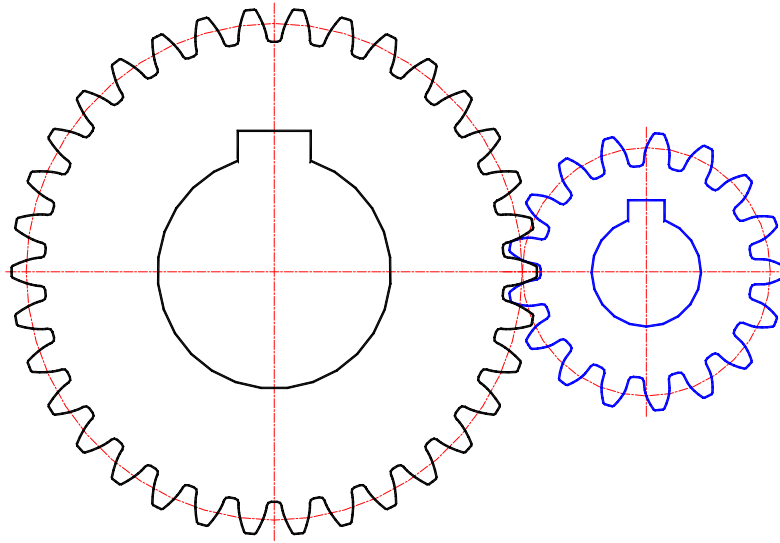
Chapter 6

GEAR MECHANISMS

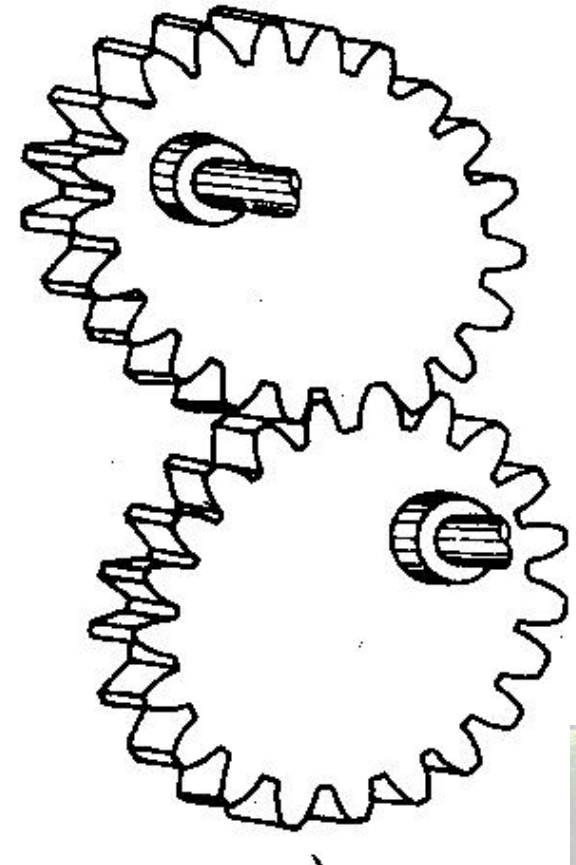


6.1 Types of Gear Mechanisms

- ☀ **circular gears** : constant transmission ratio
- ☀ **non-circular gears** : the ratio varies



In this chapter, only circular gears are considered.

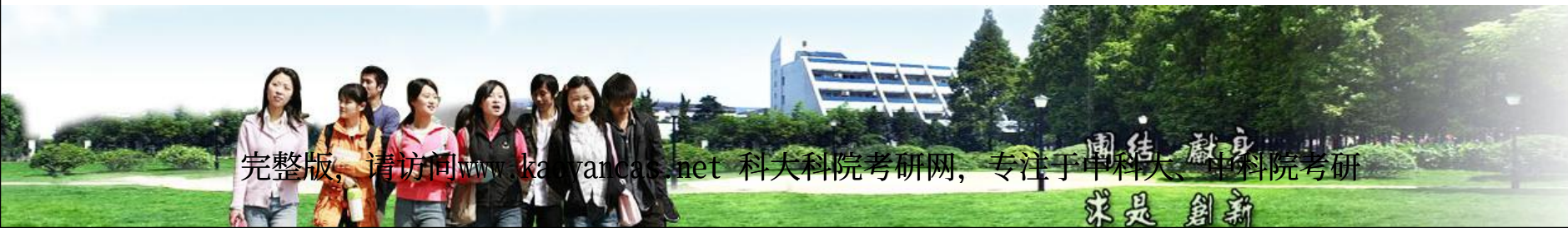




Depending on the relative shafts positions

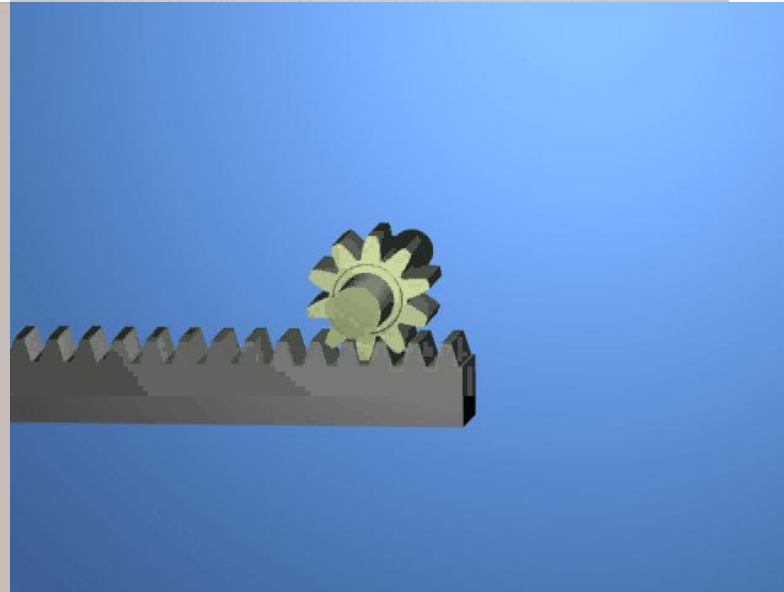
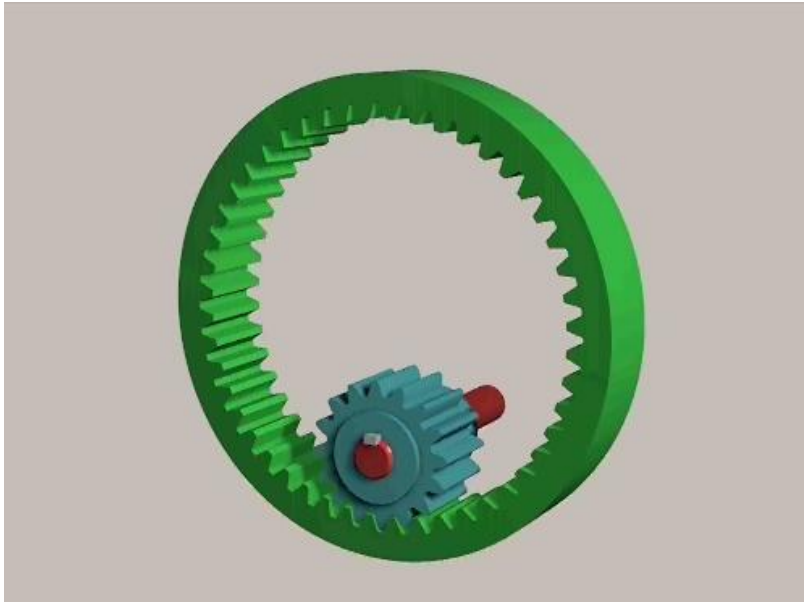
☀ parallel shafts → planar gear mechanism
(平面)

☀ nonparallel shafts → spatial gear mechanism
(空间)



6.1.1 Planar Gear Mechanisms

External gear pair



Internal gear pair

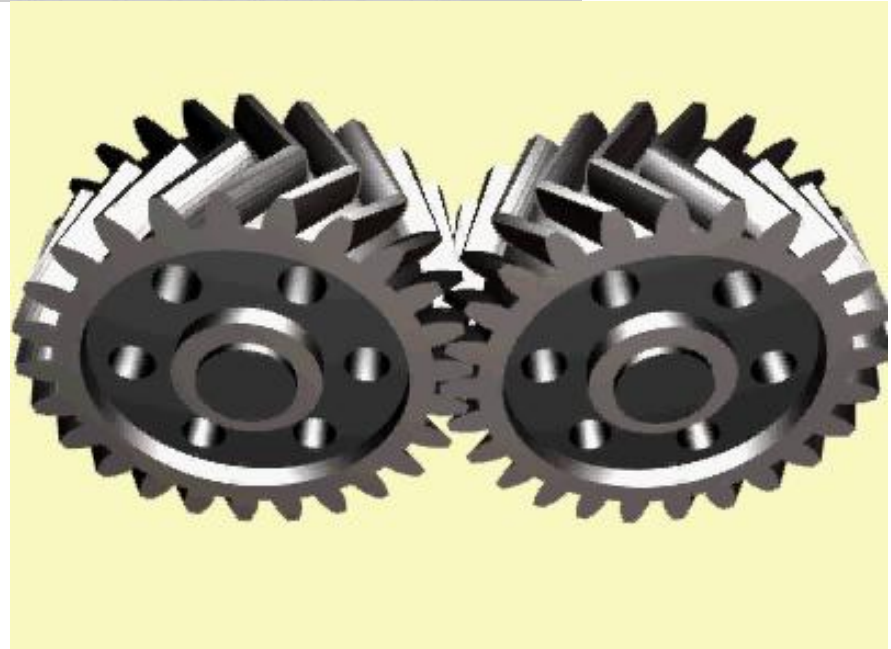
Rack and pinion (齿条和齿轮)

External gear:

Spur gear (直齿轮)



Helical gear (斜齿轮)



Double Helical gear (人字齿轮)



Internal gear pair:

1. Spur internal gear



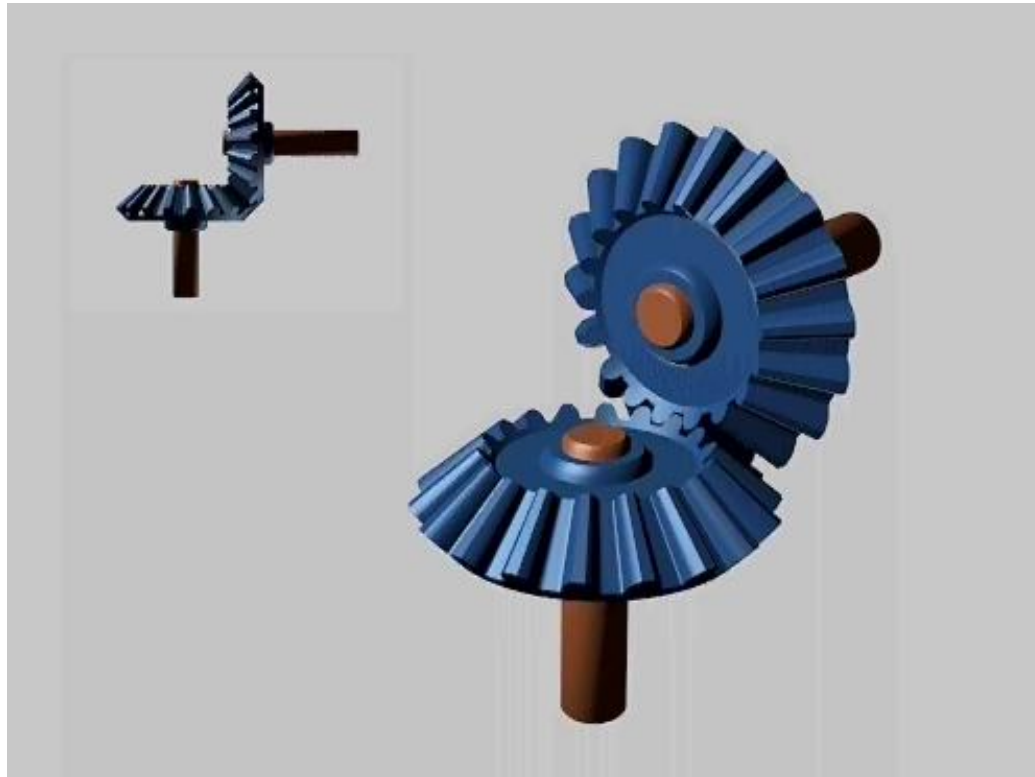
2. Helical internal gear





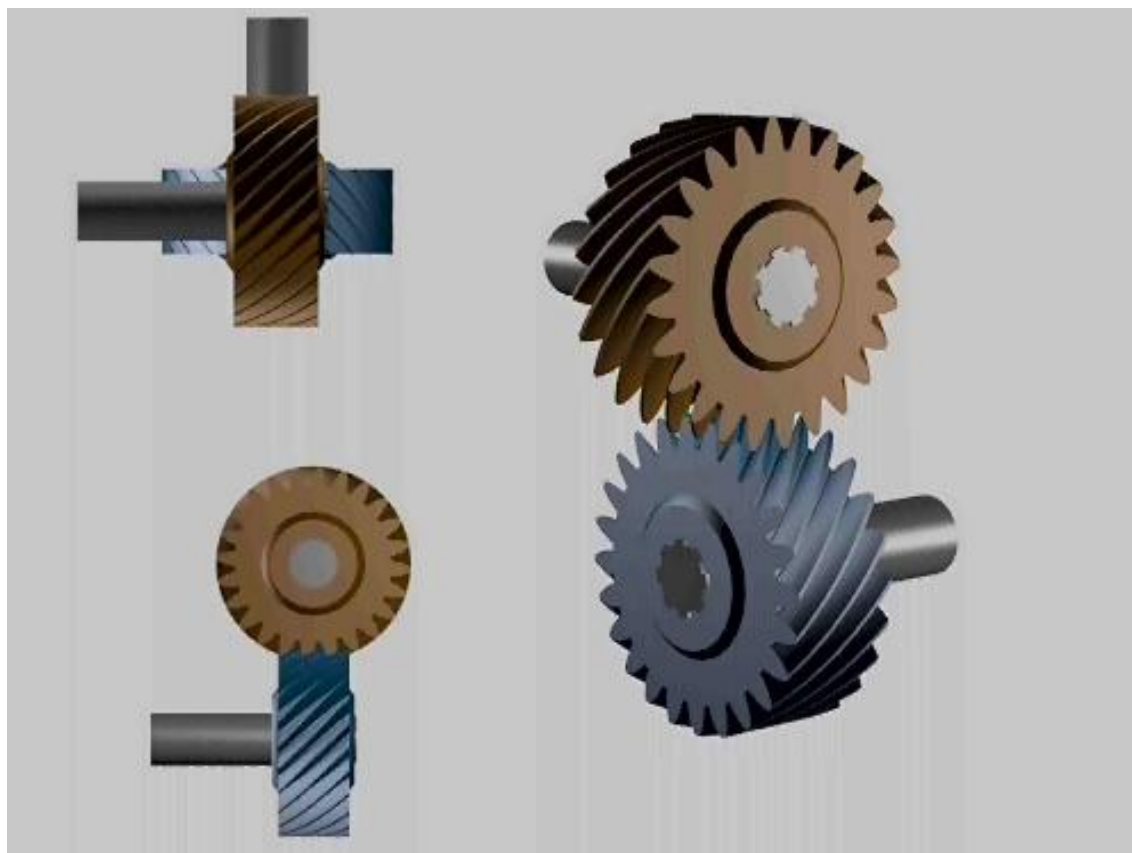
6.1.2 Spatial Gear Mechanisms

1. Bevel gear mechanism(锥齿轮)





2. Crossed helical gear mechanism(螺旋齿轮机构)





3. Worm and worm wheel mechanism (蜗杆蜗轮机构)



6.2 Fundamentals of Engagement of Tooth Profiles (齿廓啮合基本定律)

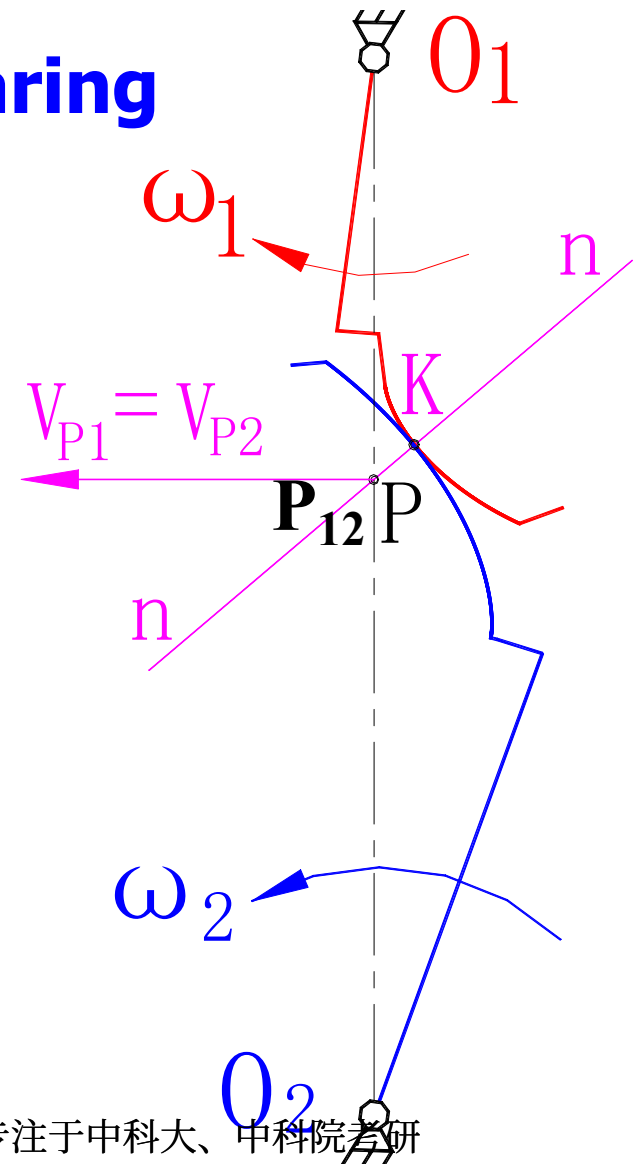
6.2.1 Fundamental Law of Gearing (啮合基本定律)

P_{12} is the instant centre
of the gears 1 and 2.

$$V_{P1} = V_{P2}$$

$$\omega_1 \cdot O_1P = \omega_2 \cdot O_2P$$

$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$



P ----- the pitch point(节点).

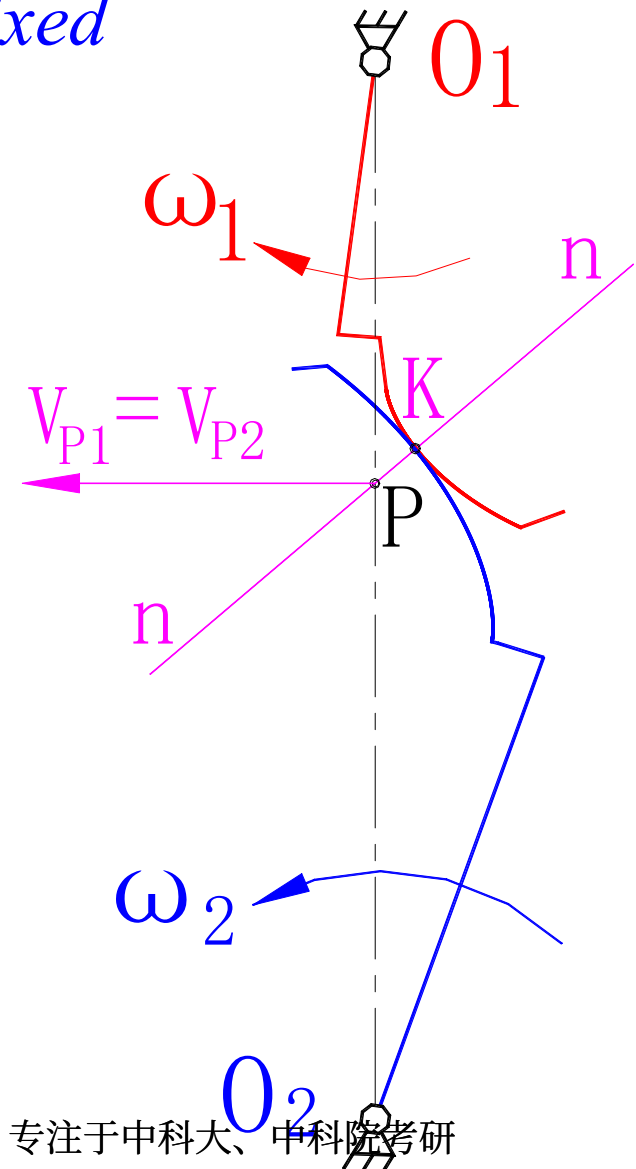
$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{\overline{O_2P}}{\overline{O_1P}} = \text{constant} \Leftrightarrow P \text{ is fixed}$$



If a constant transmission ratio i_{12} is required.



Wherever(无论何处) the teeth contact, the common normal n-n of the tooth profiles through the contact point must intersect the center line at a fixed point.

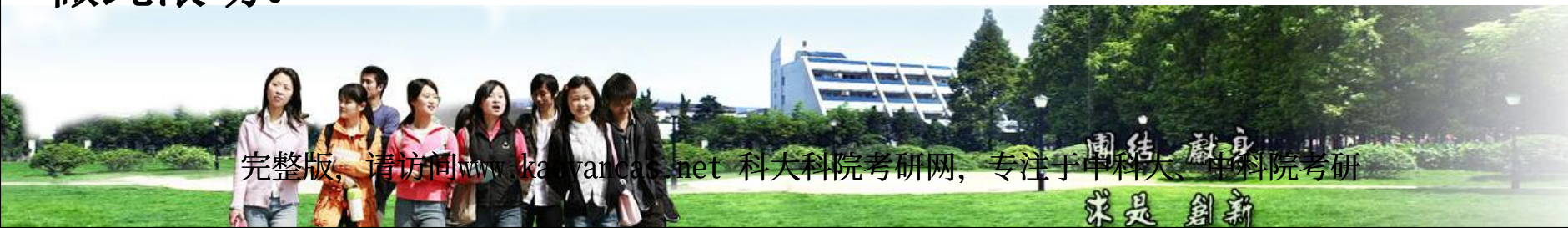
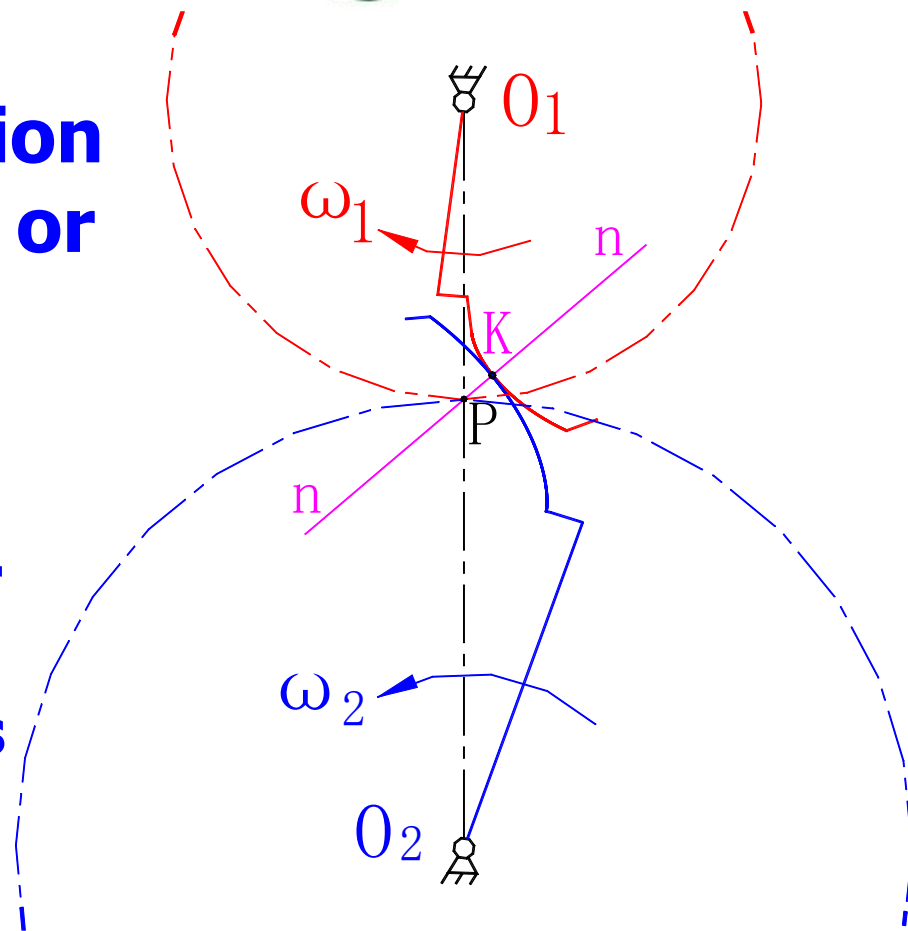




Pitch circle(节圆): the locus of the pitch point P on the motion plane of the pinion or the gear.

Pitch circles are tangent and roll without slipping each other 纯滚动. The gear pair is represented by two chain dotted(点划线) circles tangent to each other.

一对齿轮传动可看成两个节圆做纯滚动。

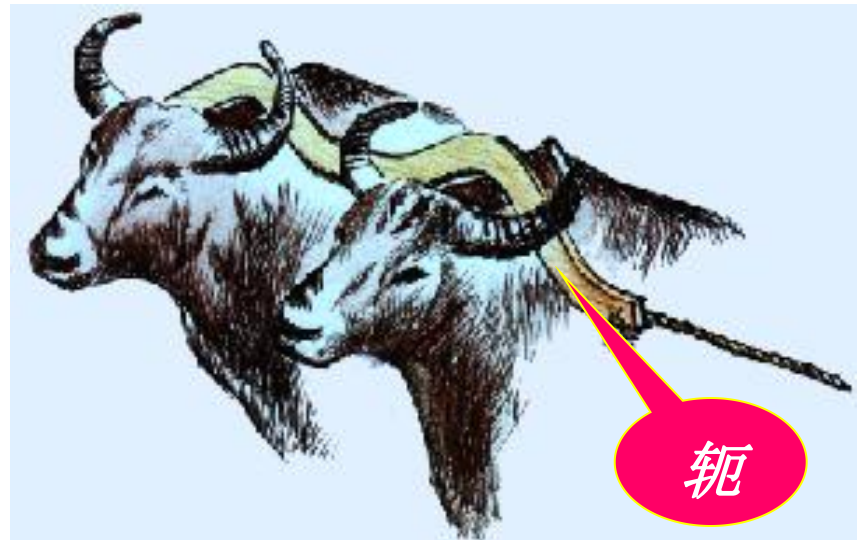


6.2.2 Conjugate Profiles(共轭齿廓)

Meshing(啮合的) profiles of teeth that can yield a desired transmission ratio(传动比) are termed conjugate profiles.

满足啮合基本定律的一对齿廓称为共轭齿廓。

两头牛背上的架子称为**轭**，轭使两头牛同步行走。**共轭**即为按一定规律相配的一对。



For any specific tooth profile, we can find its conjugate profile.

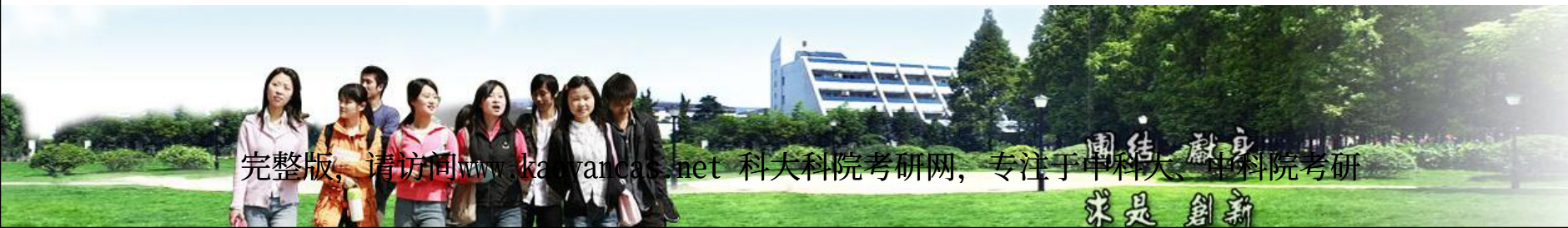


共轭齿廓曲线的选择

1. 满足定传动比的要求；2. 考虑设计、制造等方面。

通常采用渐开线、摆线、变态摆线

Involutes(渐开线) are used most widely since they can be manufactured and assembled easily.

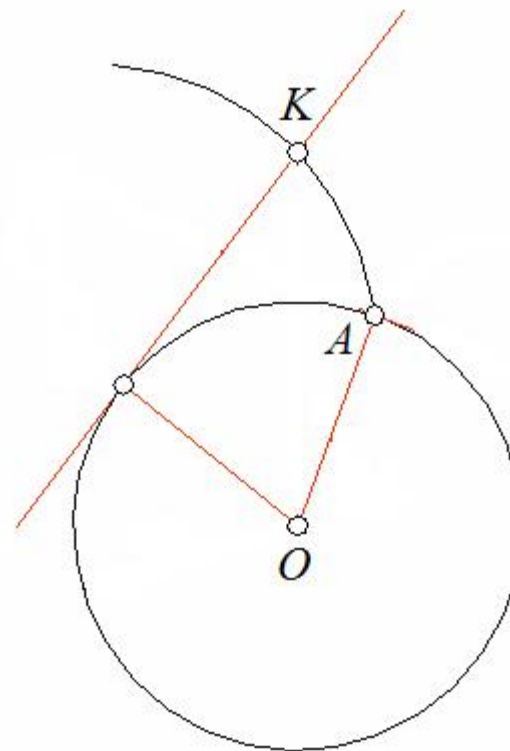


6.3 The Involute(渐开线) and Its Properties

6.3.1 Generation of Involute

An involute is the curve generated by any point on a string which is unwrapped(展开) from a fixed circle.

当一直线 $n-n$ 沿一个圆的圆周作纯滚动时，直线上任一点 K 的轨迹

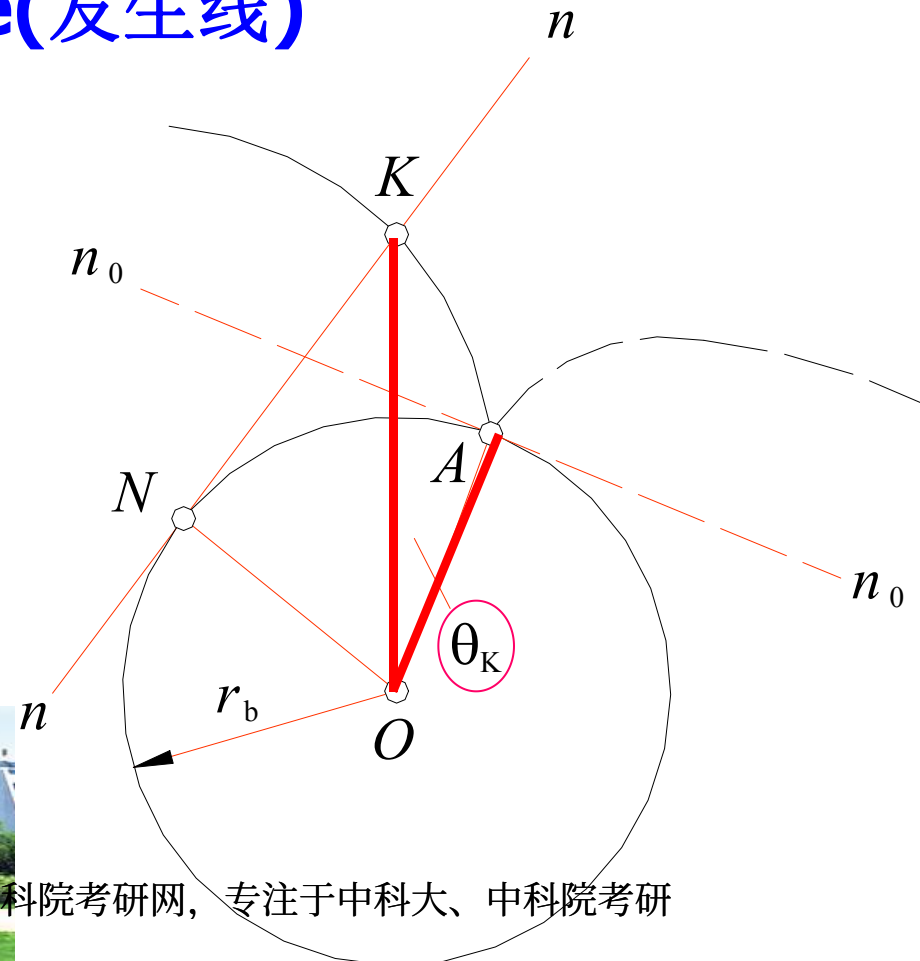




r_b —— The radius of **base circle** (基圆) .

$n-n$ —— Generating line(发生线)

θ_K —— unfolding angle
(展角) of the
involute at
point K.

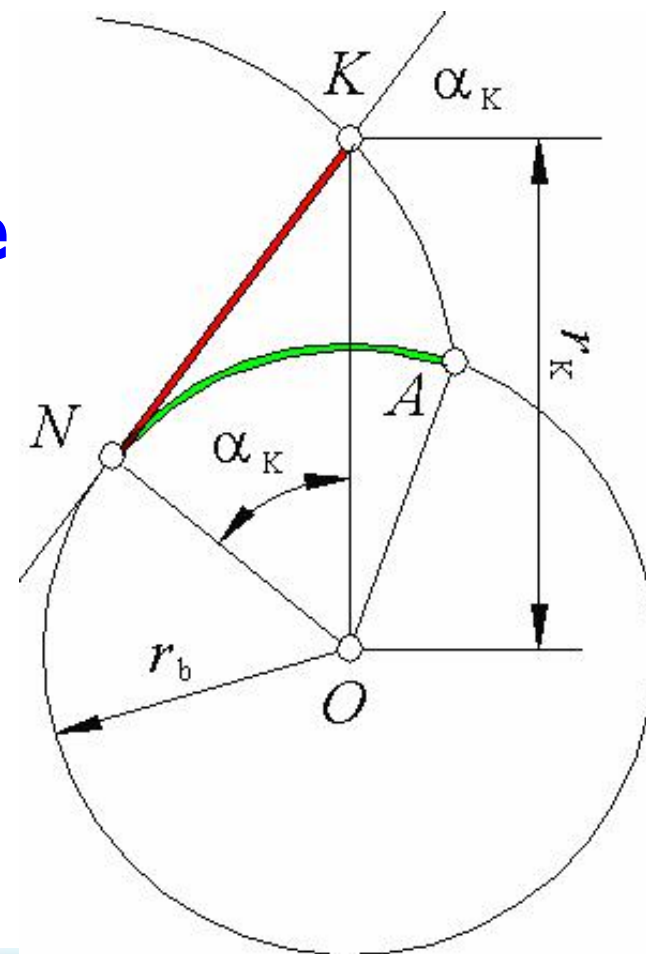




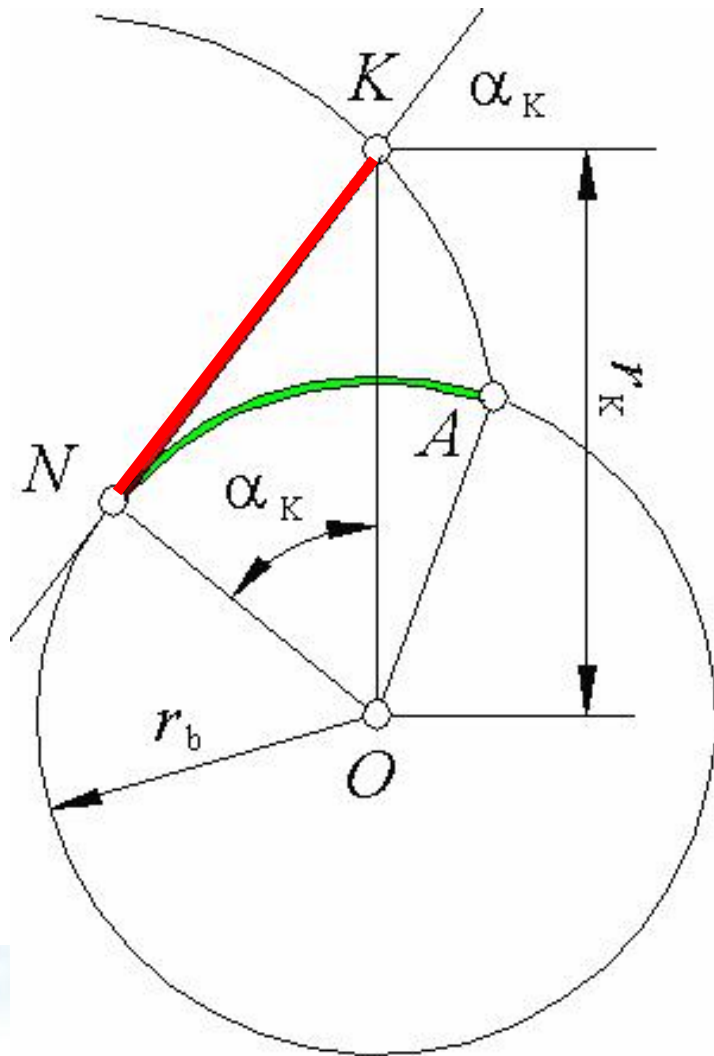
6.3.2 Properties of the Involute

1. The length of the generating line(发生线) segment unrolled from the base circle is equal to the arc length of the base circle rolled.

$$KN = \widehat{AN}$$



2. The length of segment NK is the radius of curvature of the involute at the point K.



NK为渐开线在K点的法线，NK为曲率半径，渐开线上任一点的法线与基圆相切。

$$\rho_K = \overline{NK} = \sqrt{r_K^2 - r_b^2}$$

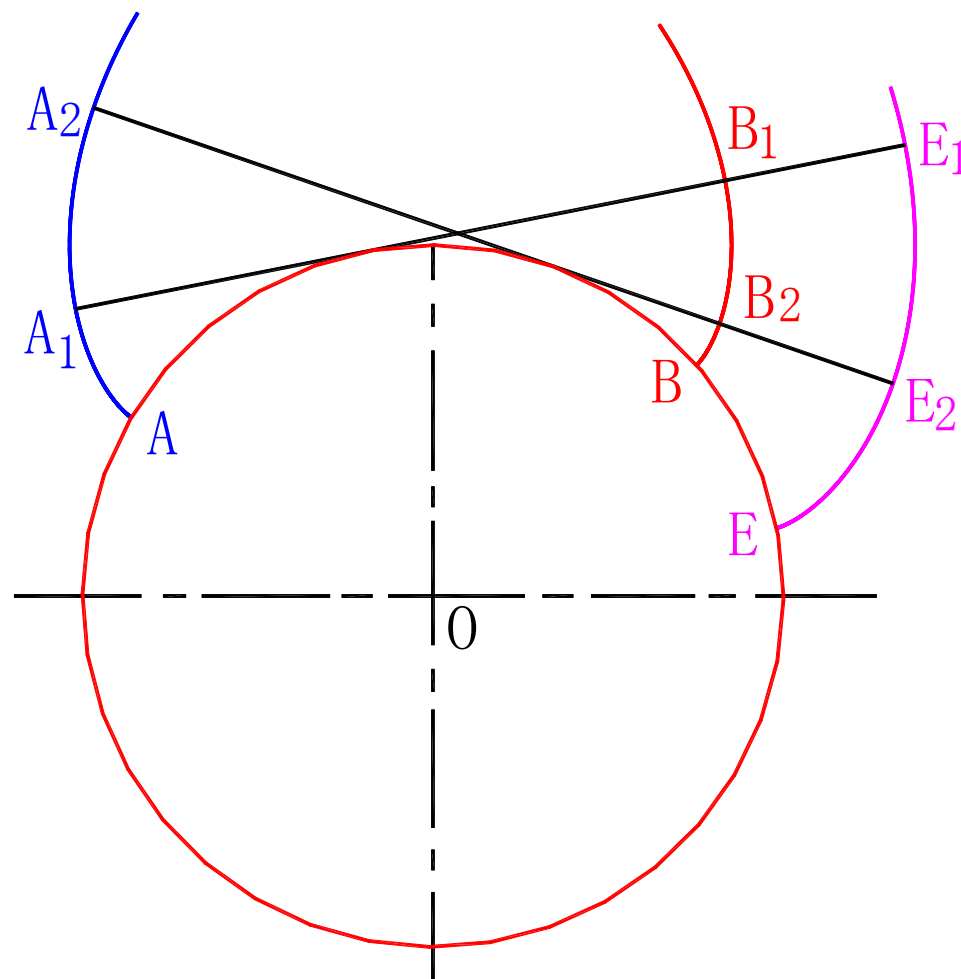
$$\rho_A = 0$$



3. The normal distance between two involutes of the same base circle remains the same.

$$\overline{A_1B_1} = \overline{A_2B_2} = AB$$

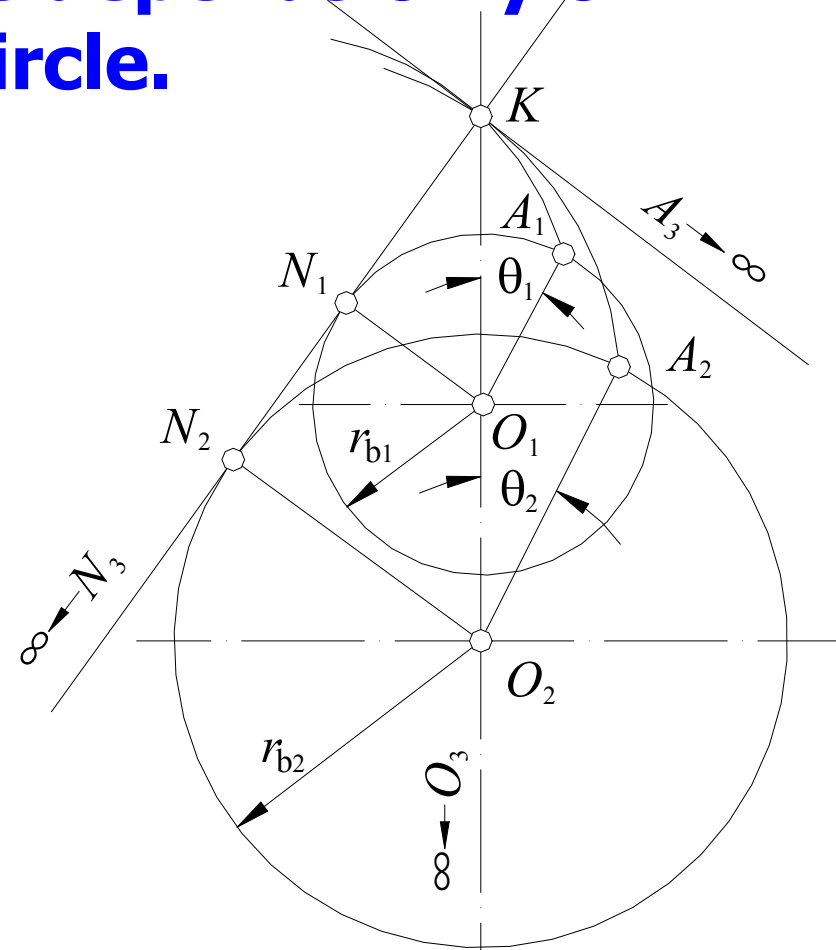
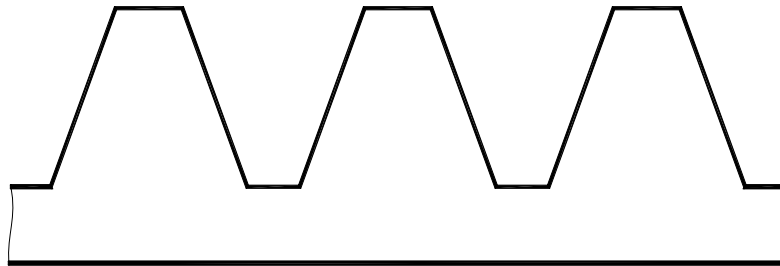
$$\overline{B_1E_1} = \overline{B_2E_2} = BE$$





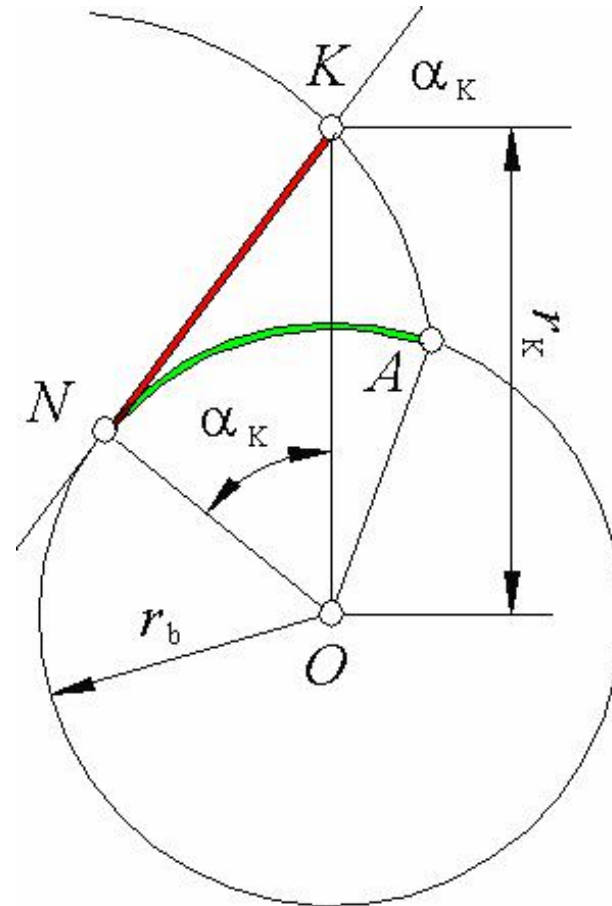
4. The shape of an involute depends only on the radius of its base circle.

As the radius r_b of the base circle approaches infinity, the involute becomes a straight line.





5. No involute exists inside its base circle.

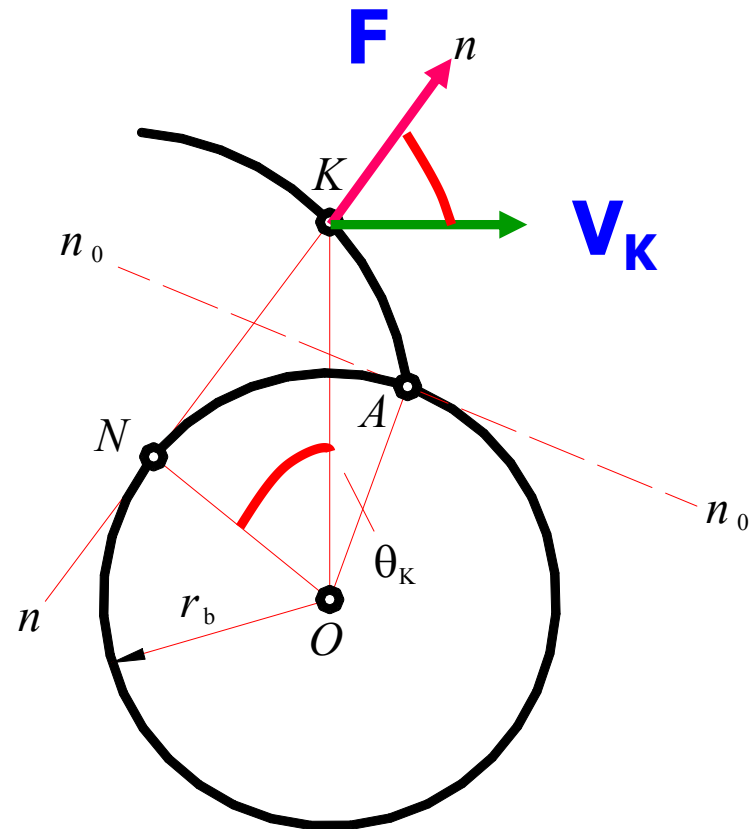


6.3.3 Equation of the Involute 渐开线方程



Pressure angle $\alpha_K = \angle NOK$

$$\begin{aligned}\tan \alpha_K &= \frac{\overline{NK}}{\overline{AN}} \\ &= \frac{r_b}{r_b(\alpha_K + \theta_K)} \\ &= \alpha_K + \theta_K\end{aligned}$$

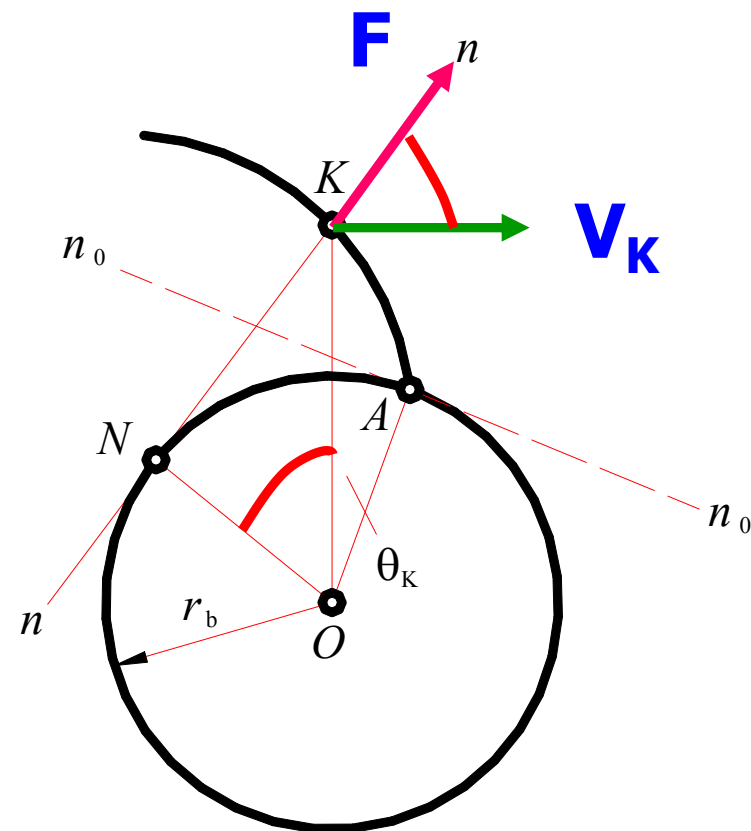


$$r_b = r_K \cos \alpha_K$$



The polar parametric equation of the involute 渐开线方程 with α_K as a parameter is

$$\begin{cases} r_K = \frac{r_b}{\cos \alpha_K} \\ \theta_K = \text{inv} \alpha_K = \tan \alpha_K - \alpha_K \end{cases}$$



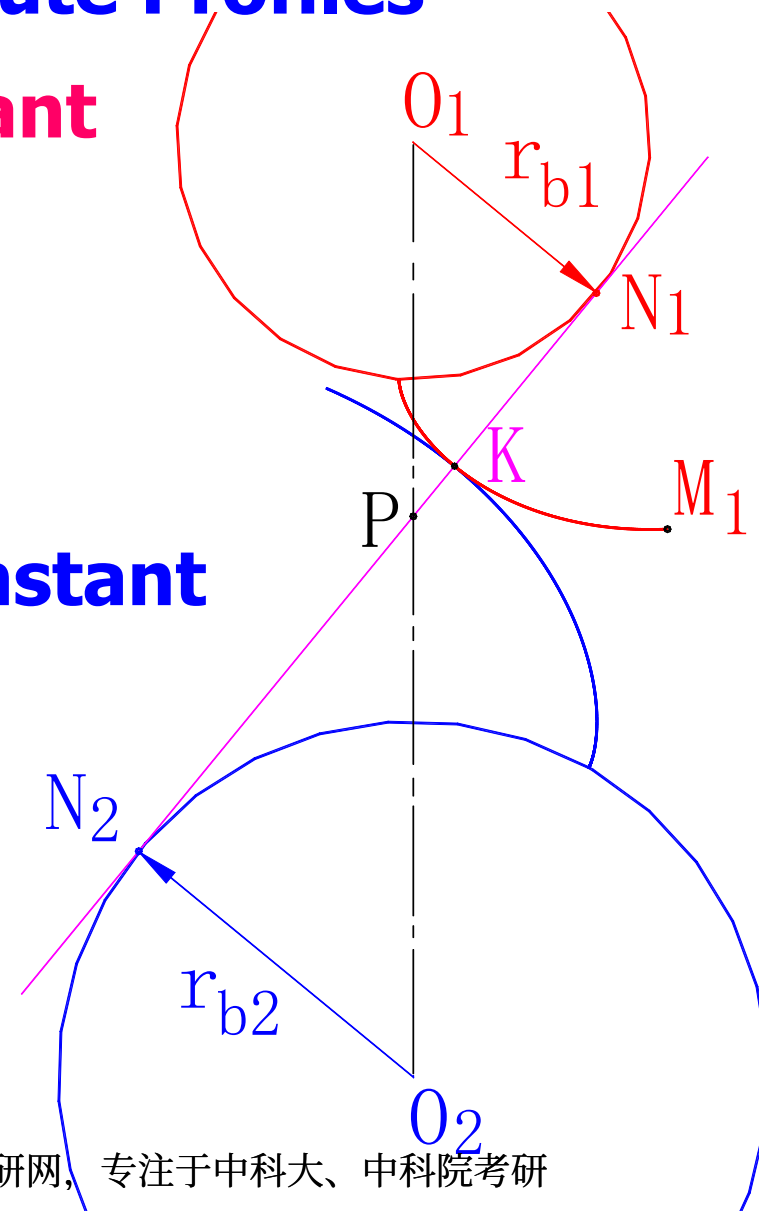


6.3.4 Gearing(啮合) of Involute Profiles

Involute gears have constant transmission ratio.

$\triangle O_1N_1P$ 和 $\triangle O_2N_2P$

$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{r_2'}{r_1'} = \frac{r_{b2}}{r_{b1}} = \text{constant}$$

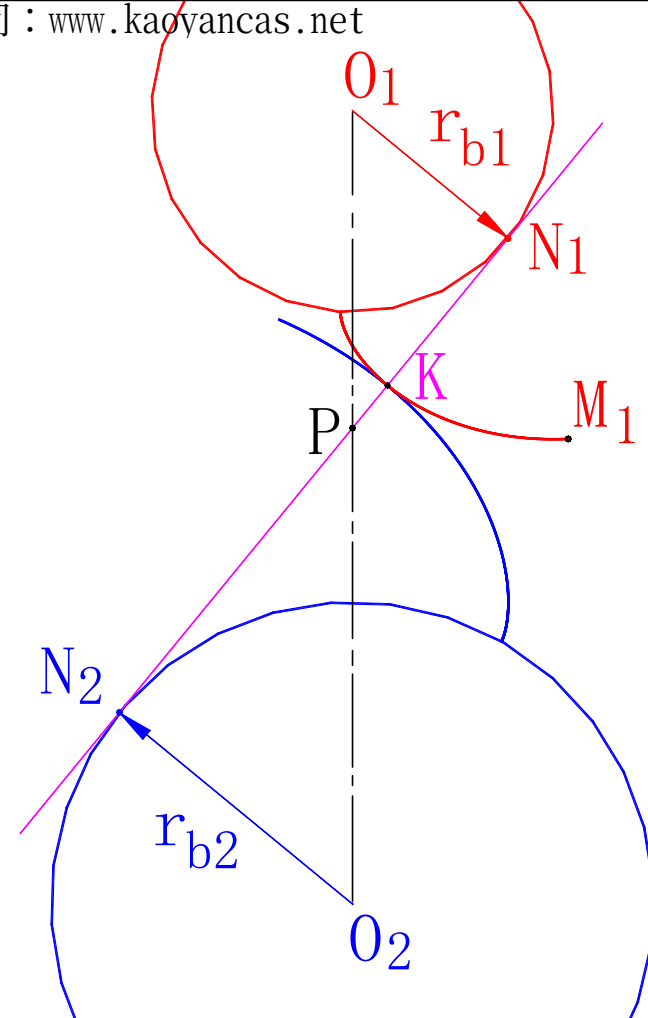


渐开线齿廓啮合的特点

1. Locus of contact point, the line of action(啮合线)

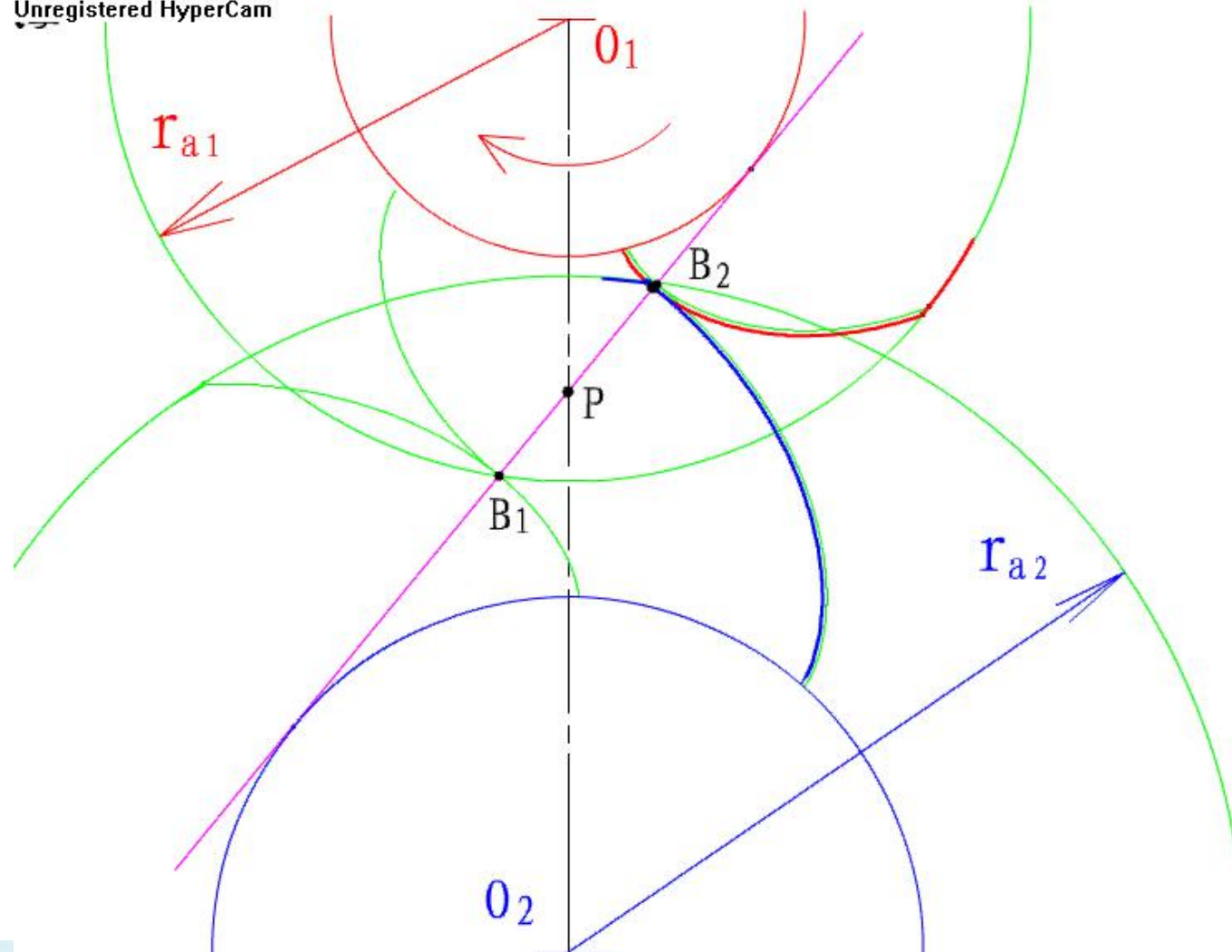
= common normal to the involute profiles

= the common tangent to the base circles





Unregistered HyperCam

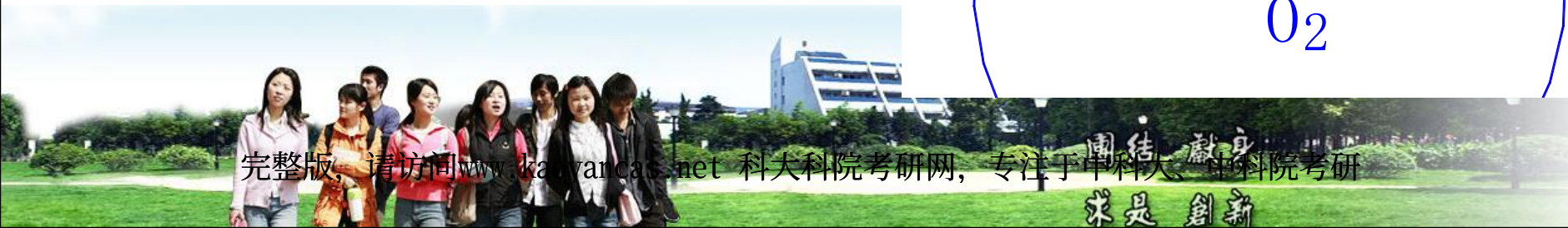
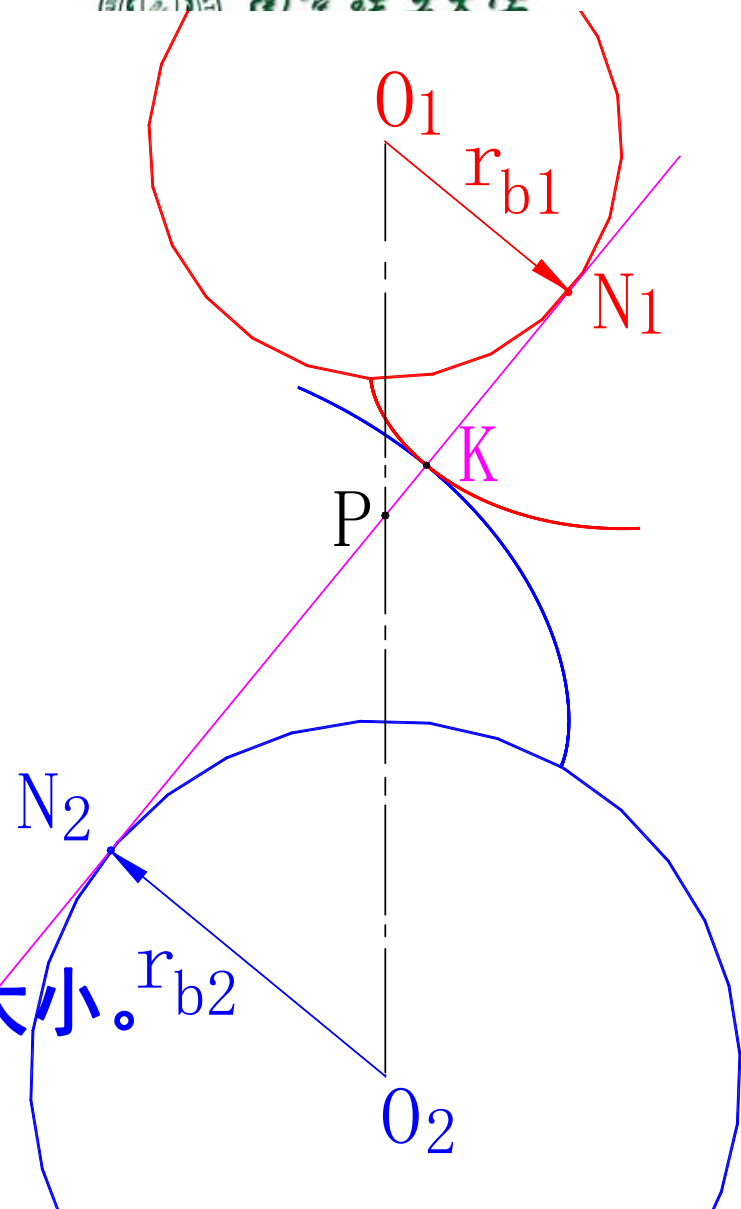


2、Involute gears have separability(可分离性) of the centre distance.

Since $\triangle O_1PN_1 \approx \triangle O_2PN_2$,

$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{\overline{O_2P}}{\overline{O_1P}} = \frac{r_{b2}}{r_{b1}}$$

中心距略有变动时不影响传动的大小。

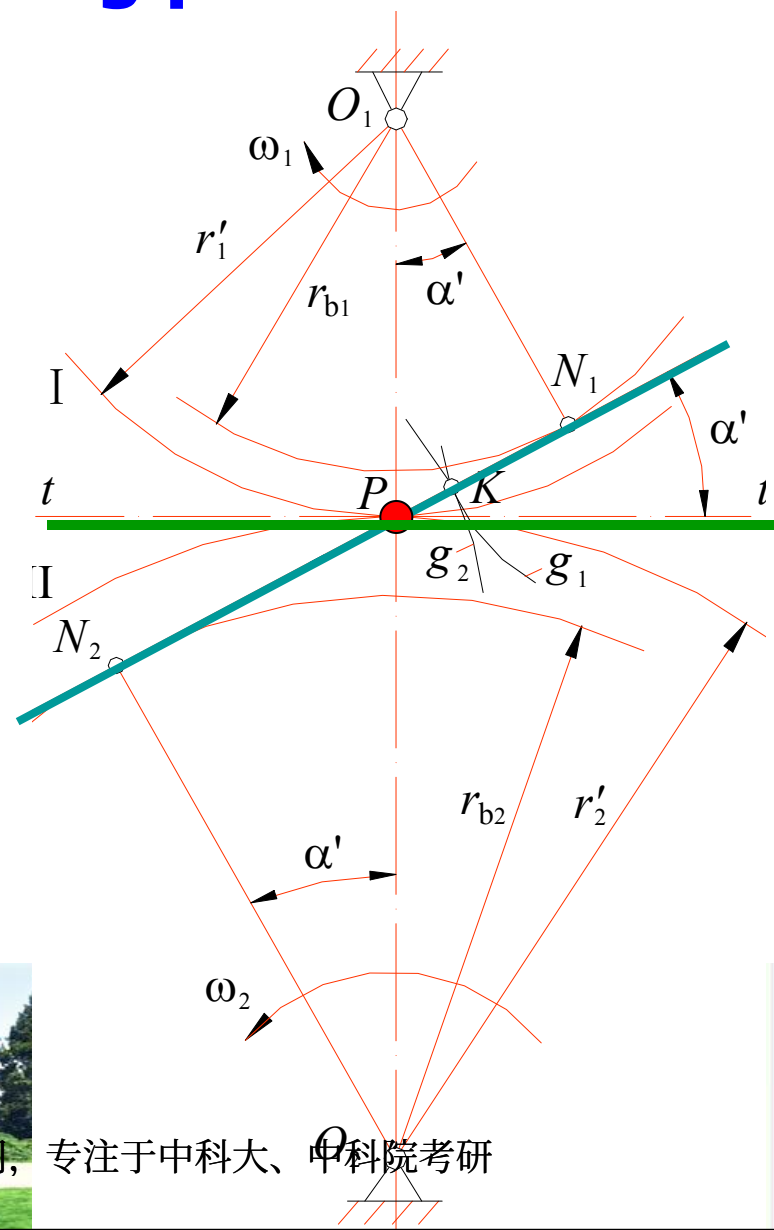




3、渐开线齿廓啮合的啮合角 Working pressure angle α' 不变

α' is defined as the angle between the velocity of pitch point P (on both gears) and the line of action N_1N_2 .
 N_1N_2 与节圆公切线之间的夹角

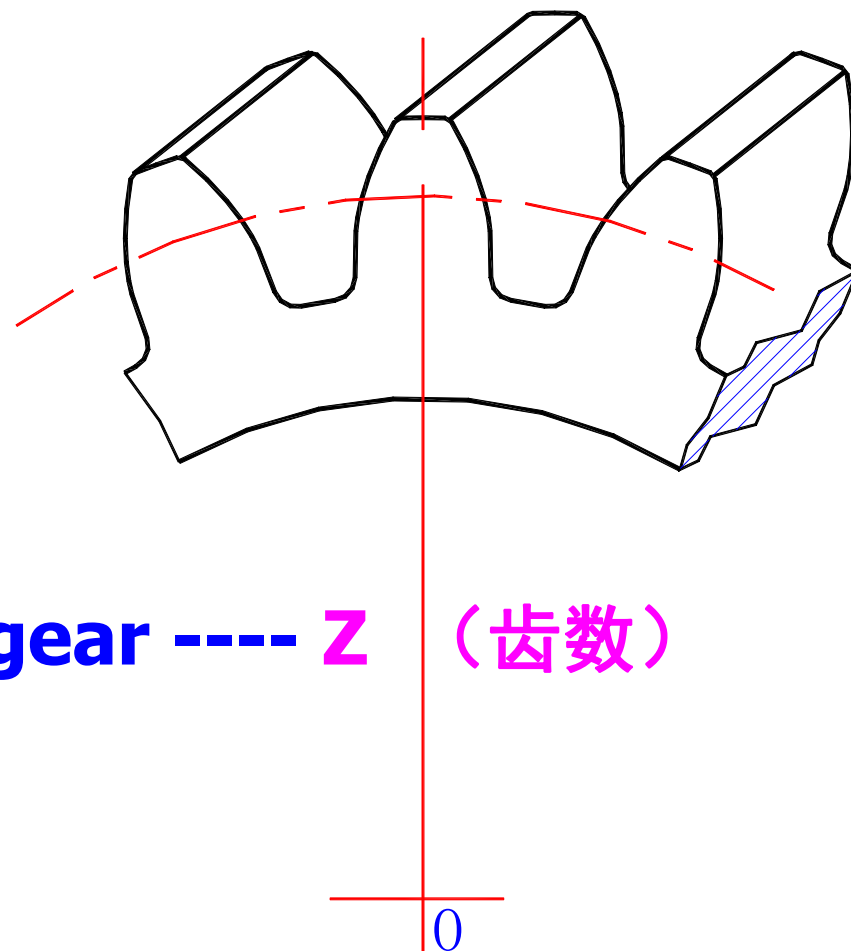
α' = 渐开线在节点处啮合的压力角





6.4. Standard Involute Spur Gears

6.4.1. External Gears

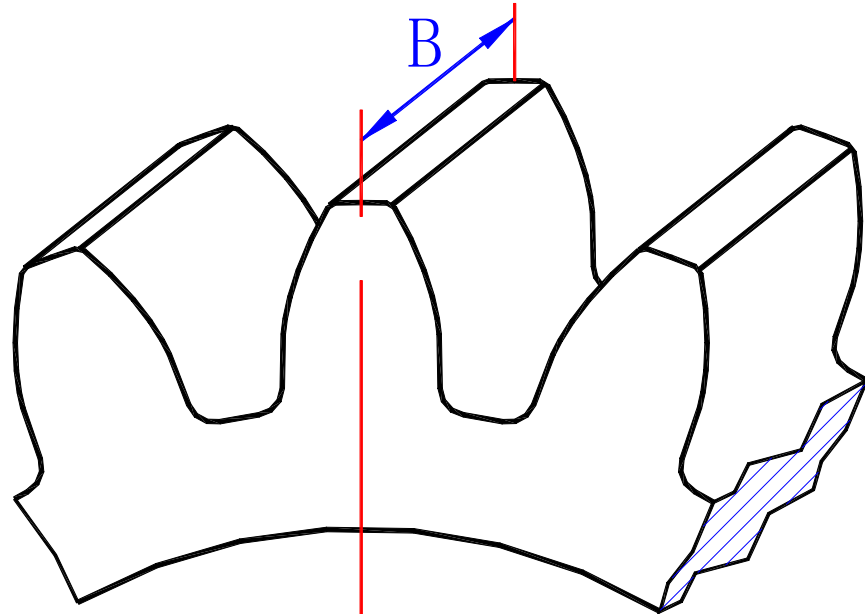


Parameters of gear:

The tooth number of a gear ---- **Z** (齿数)

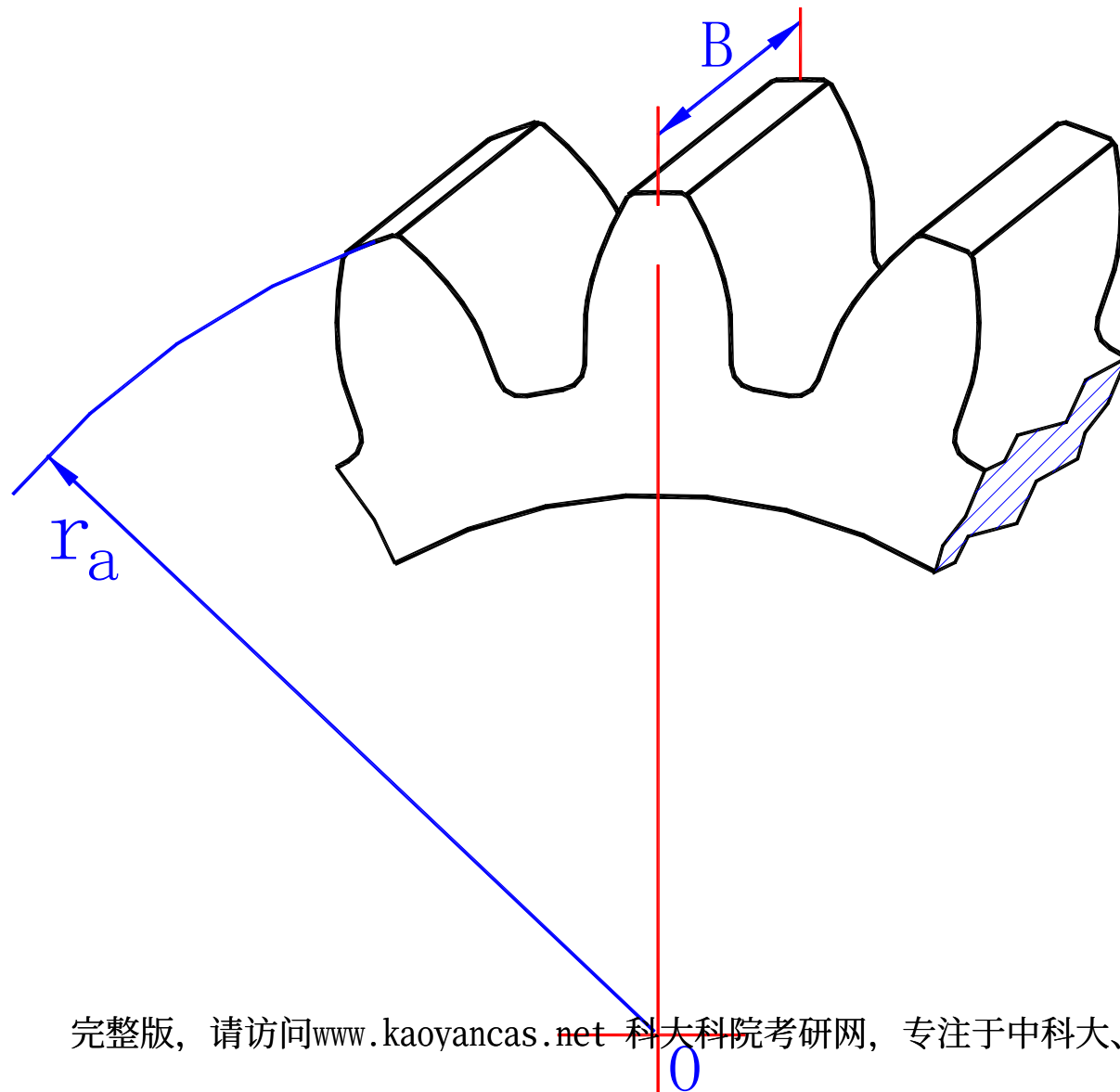


the facewidth(齿宽) ---- B



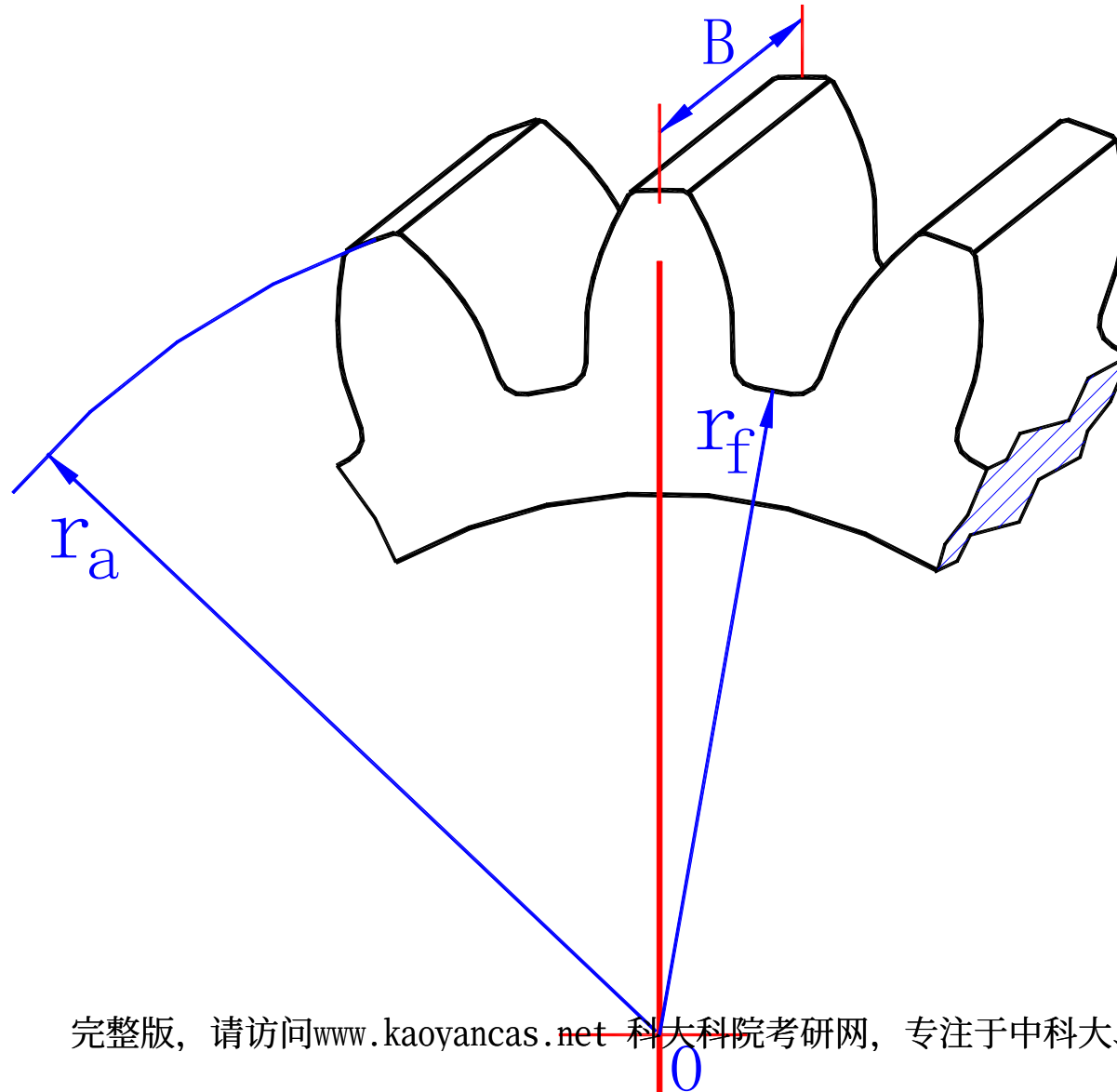


addendum circle (or tip circle) (齿顶圆) ---- d_a , r_a



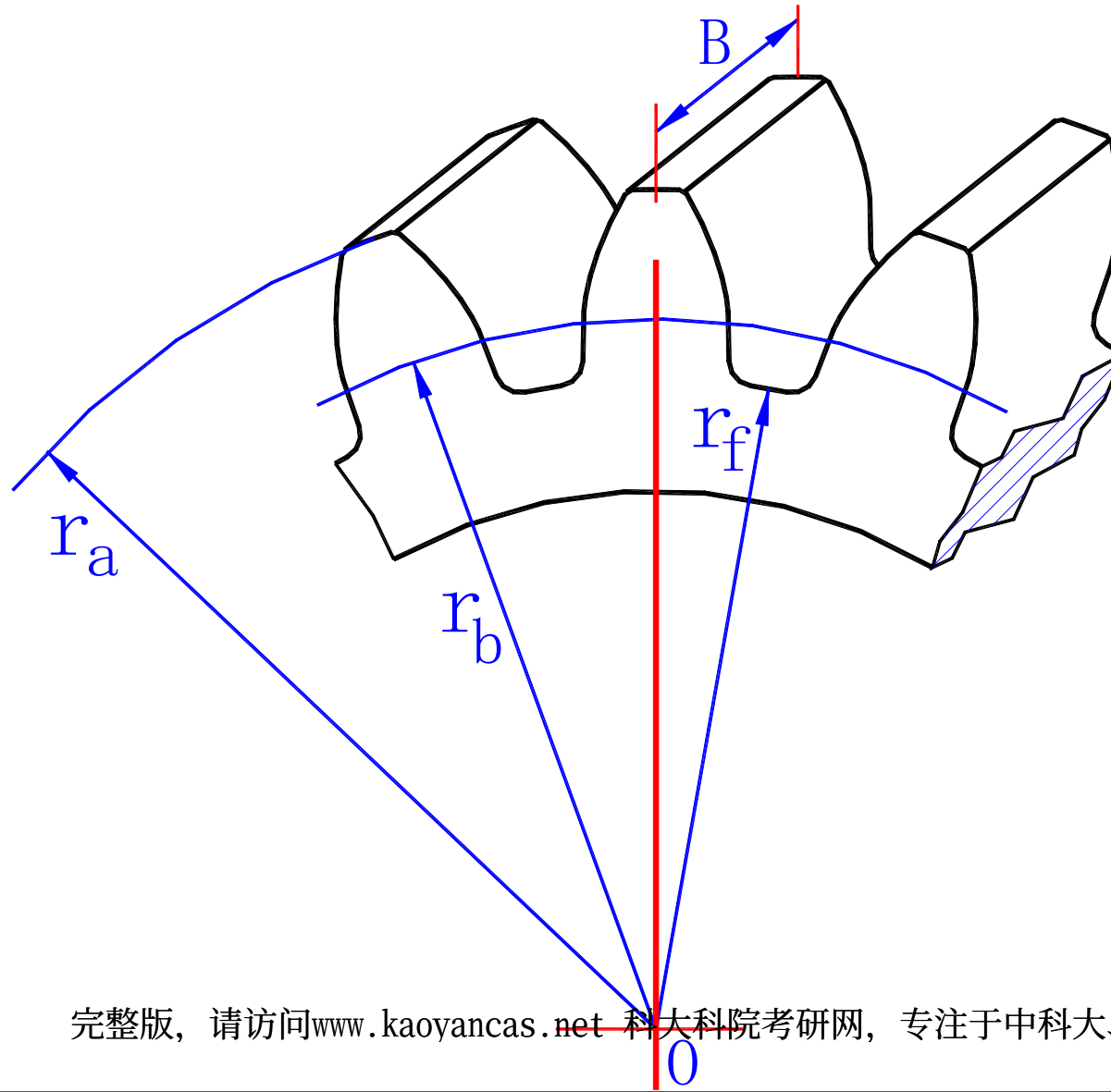


dedendum circle (or root circle) (齿根圆) ---- d_f , r_f



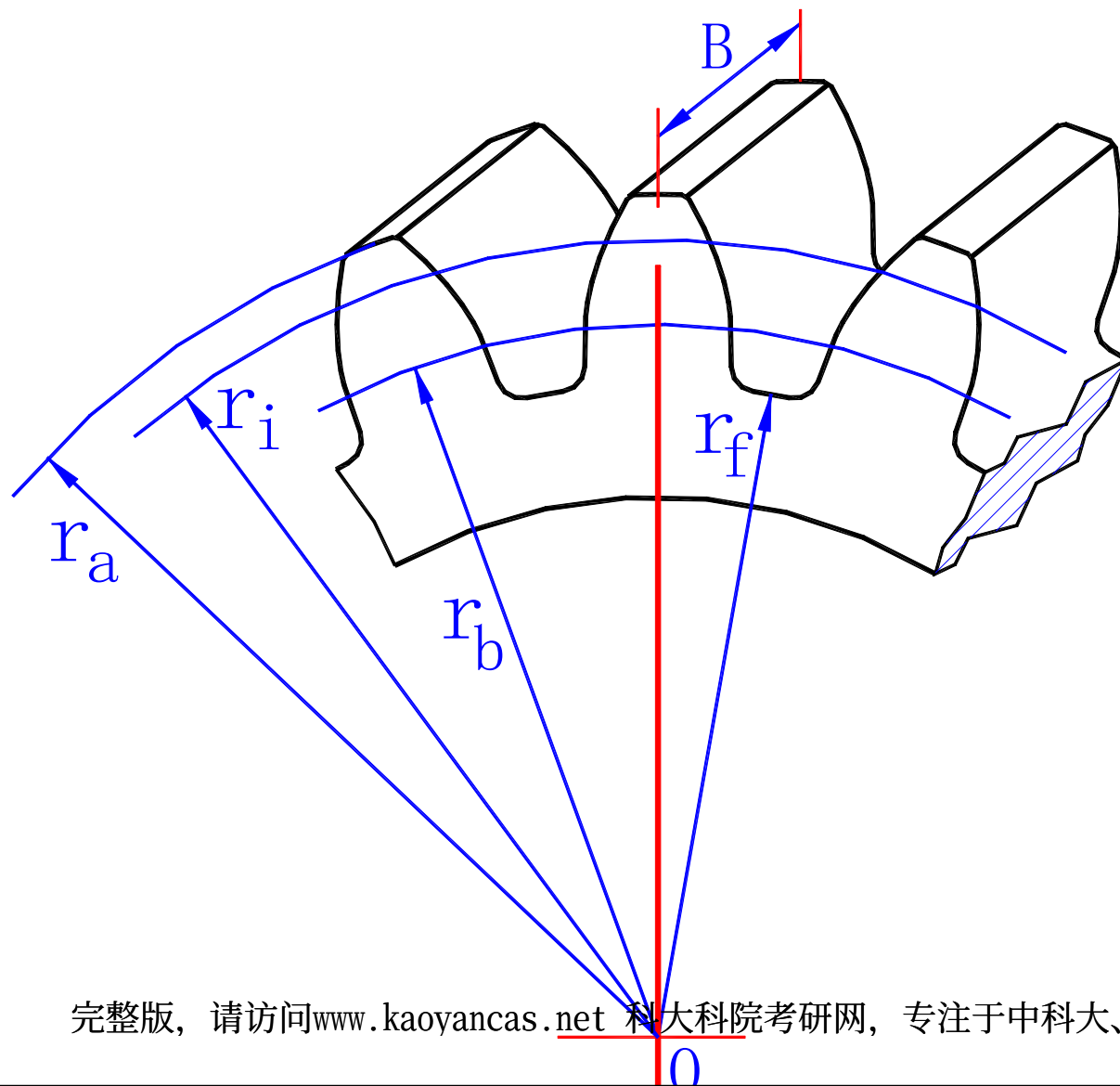


base circle ---- d_b , r_b



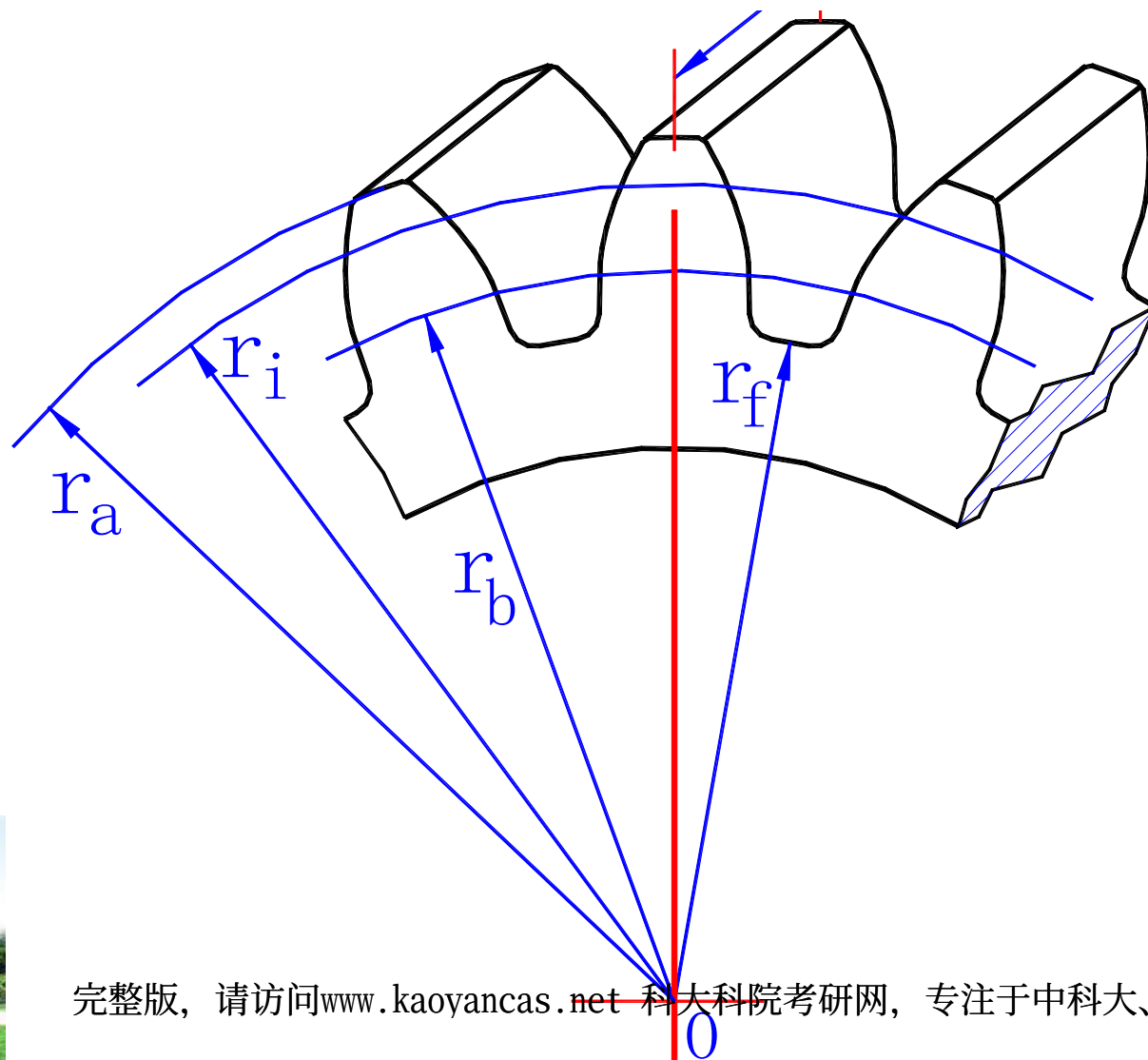


an arbitrary circle (任意圆) ---- d_i, r_i



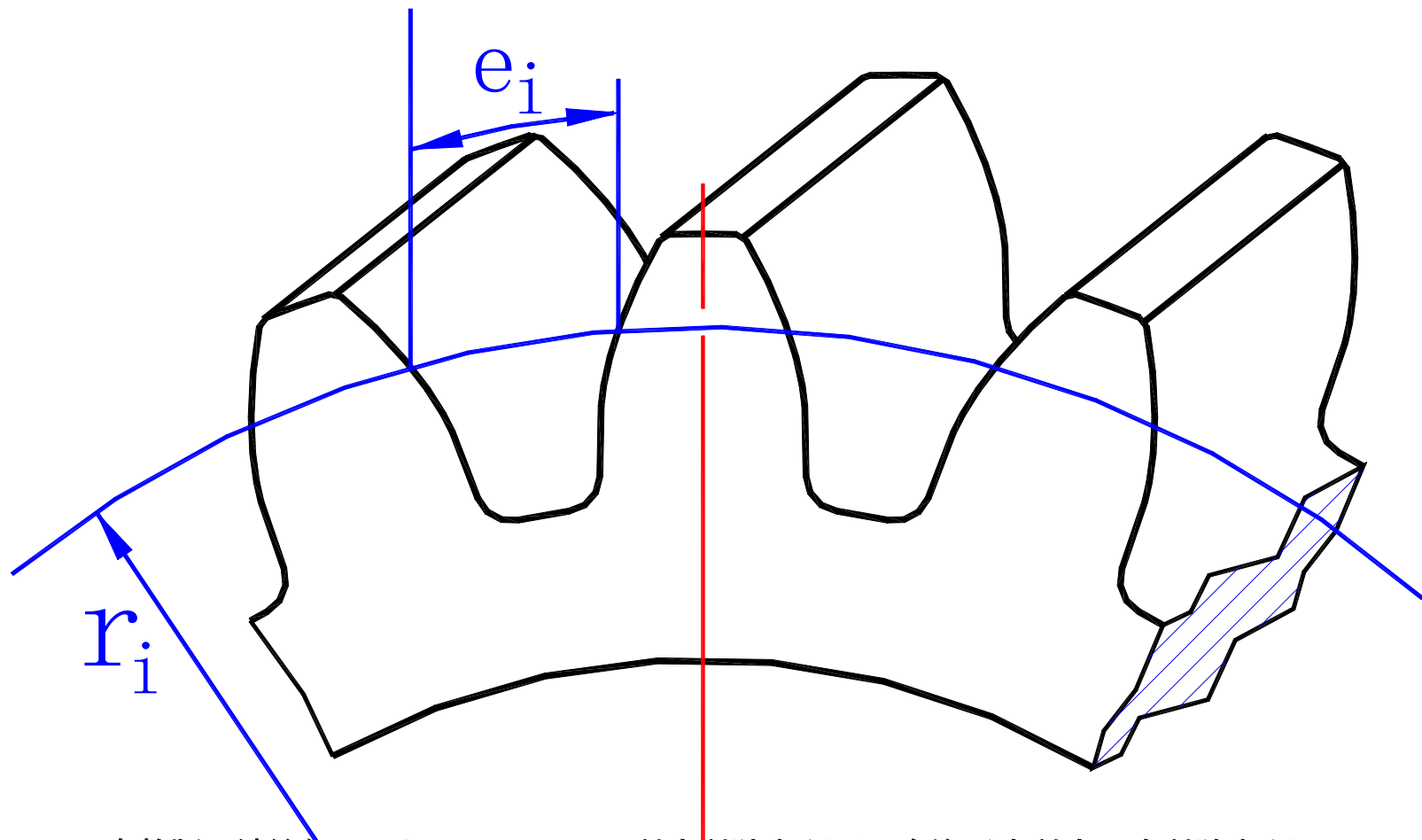


The tooth space (齿槽) ----the space between the profiles of two adjacent(相邻的) teeth.



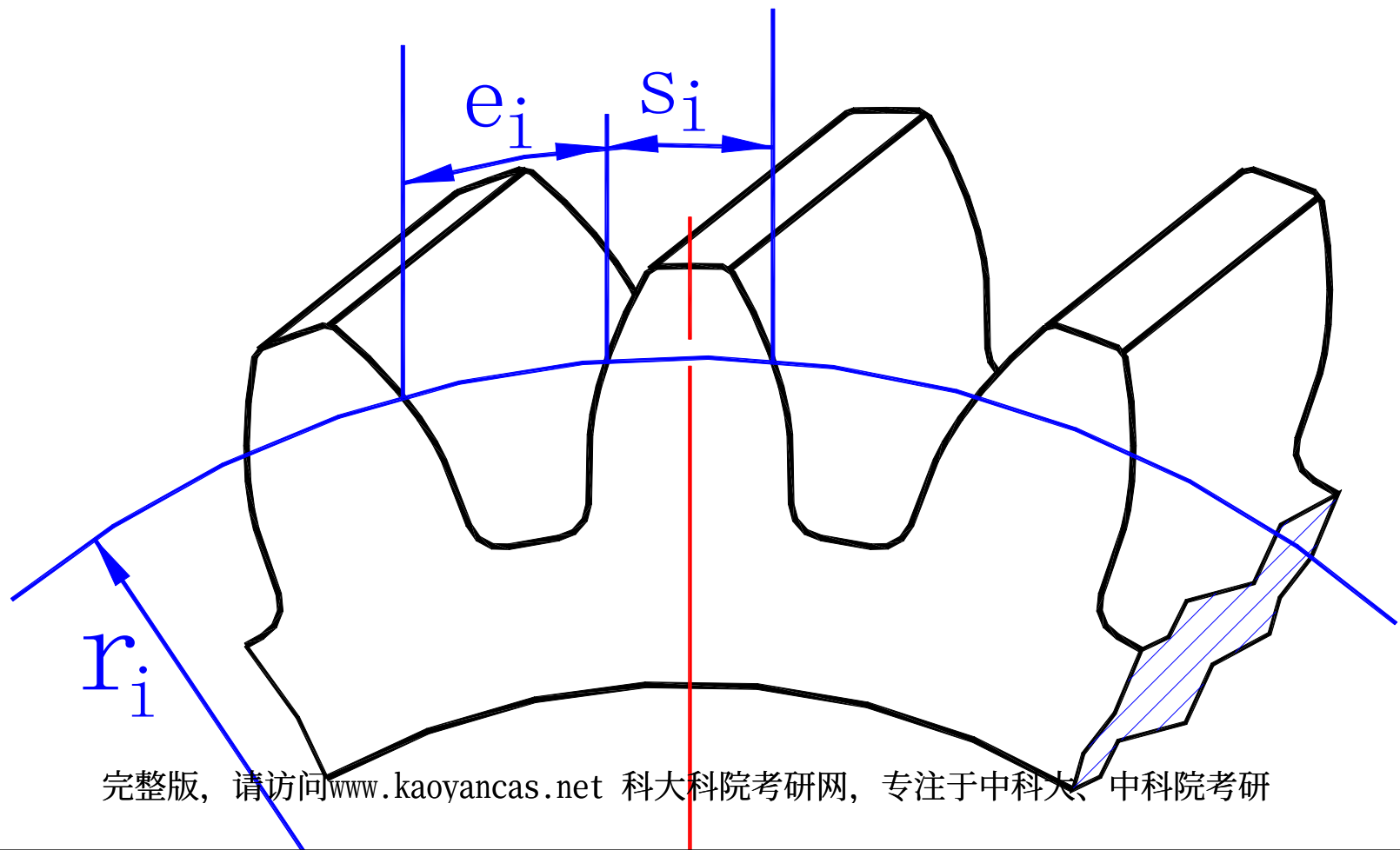


The spacewidth(齿槽宽) $\widehat{e_i}$ ---the arc length of the tooth space along an arbitrary circle.



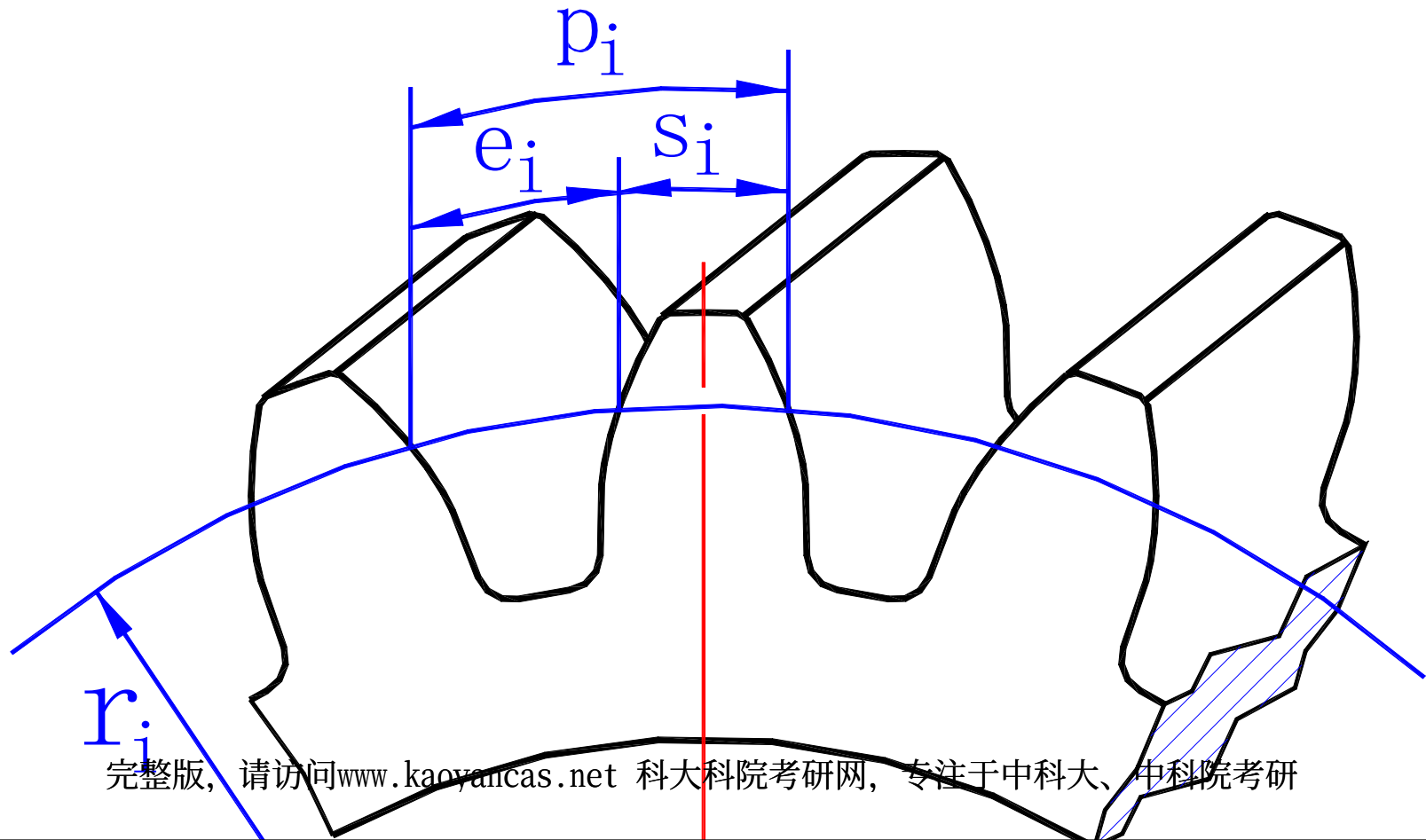


The tooth thickness (齿厚) $\widehat{s_i}$ ----- the arc length of the tooth along an arbitrary circle.





The pitch(齿距, 周节) $\widehat{p_i}$ ---arc distance between corresponding points of adjacent teeth along an arbitrary circle.

$$\widehat{p_i} = \widehat{e_i} + \widehat{s_i}$$


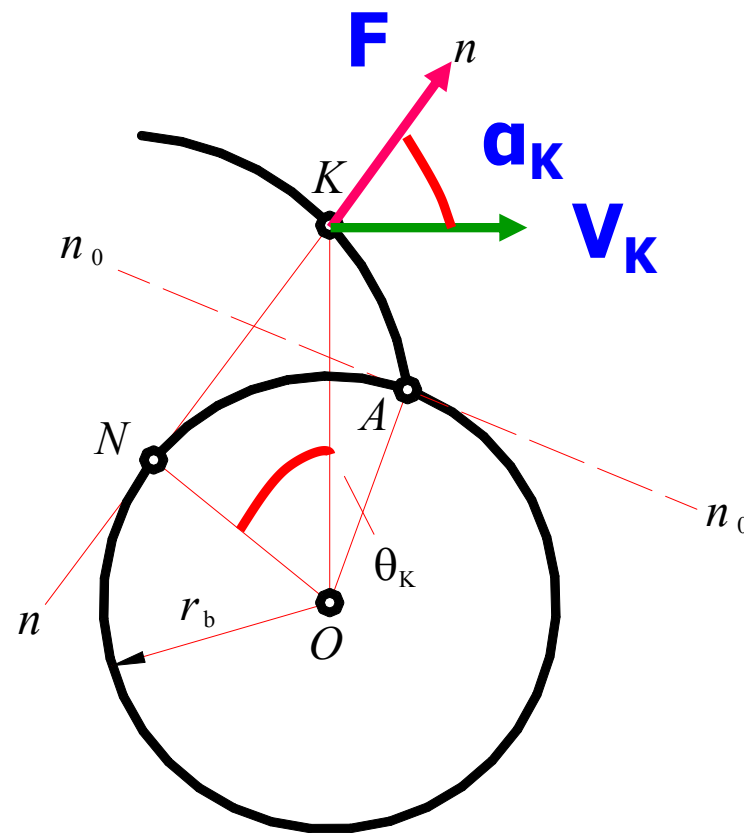


pressure angle α_K

$$r_b = r_K \cos \alpha_K$$

Different point on the involute has different pressure angle α_K .

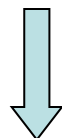
The circle the point on which has 20° pressure angle is defined as reference circle (分度圆).





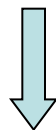
Any parameter on the reference circle has no subscript(下标), $r, e, s, p, \alpha=20^\circ$

$$p_i = e_i + s_i$$

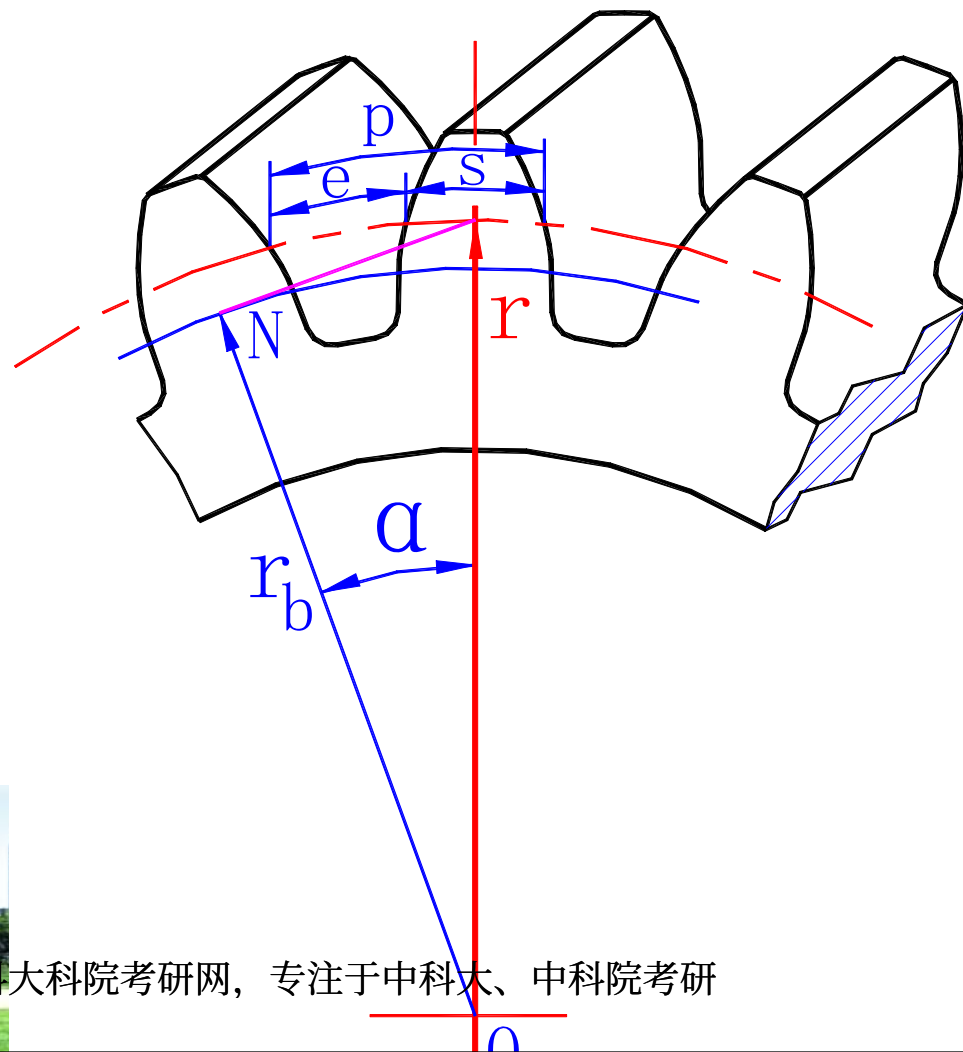


$$p = e + s$$

$$r_b = r_K \cos \alpha_K$$



$$r_b = r \cos \alpha$$



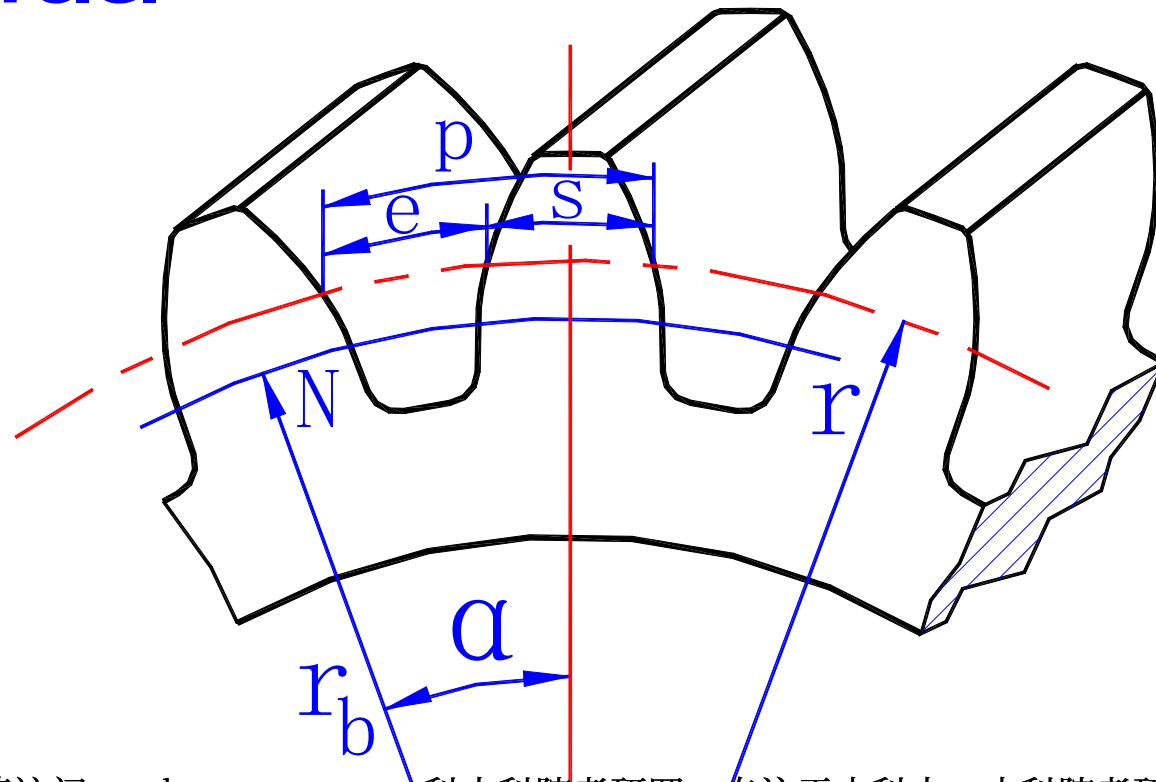


$$\pi d_i = Z p_i \longrightarrow \pi d = Z p$$

$$d = \frac{p}{\pi} Z = m Z$$

$$m = \frac{p}{\pi}$$

The module m (模数)----- a basic parameter on the reference circle.



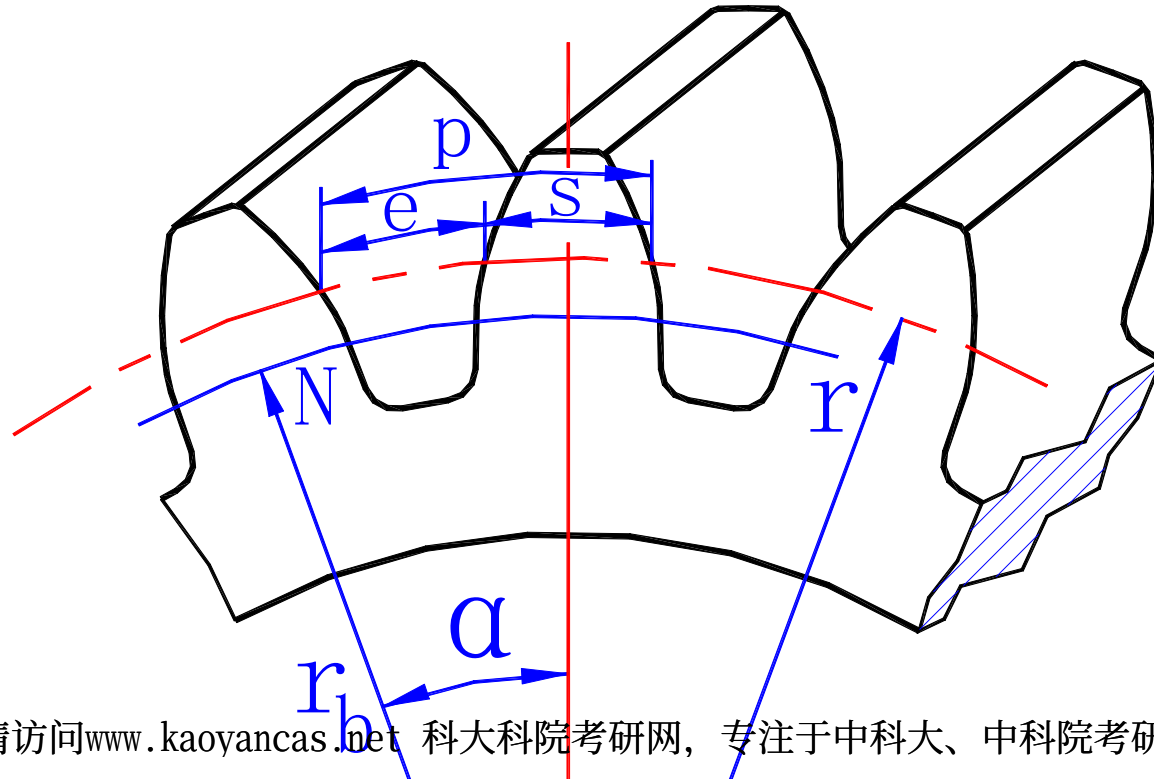


$$d = mZ \quad p = e + s = \pi m$$

m, Z, α are three basic parameters of a gear.

Different module needs different cutter.

The module m has been standardized.

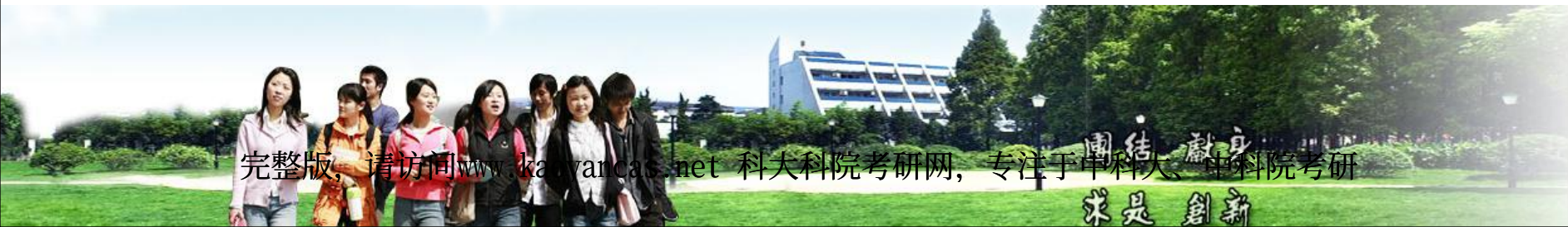




Tab 6-1 modules of Involute Cylindrical Gears (GB/T1357-1987)

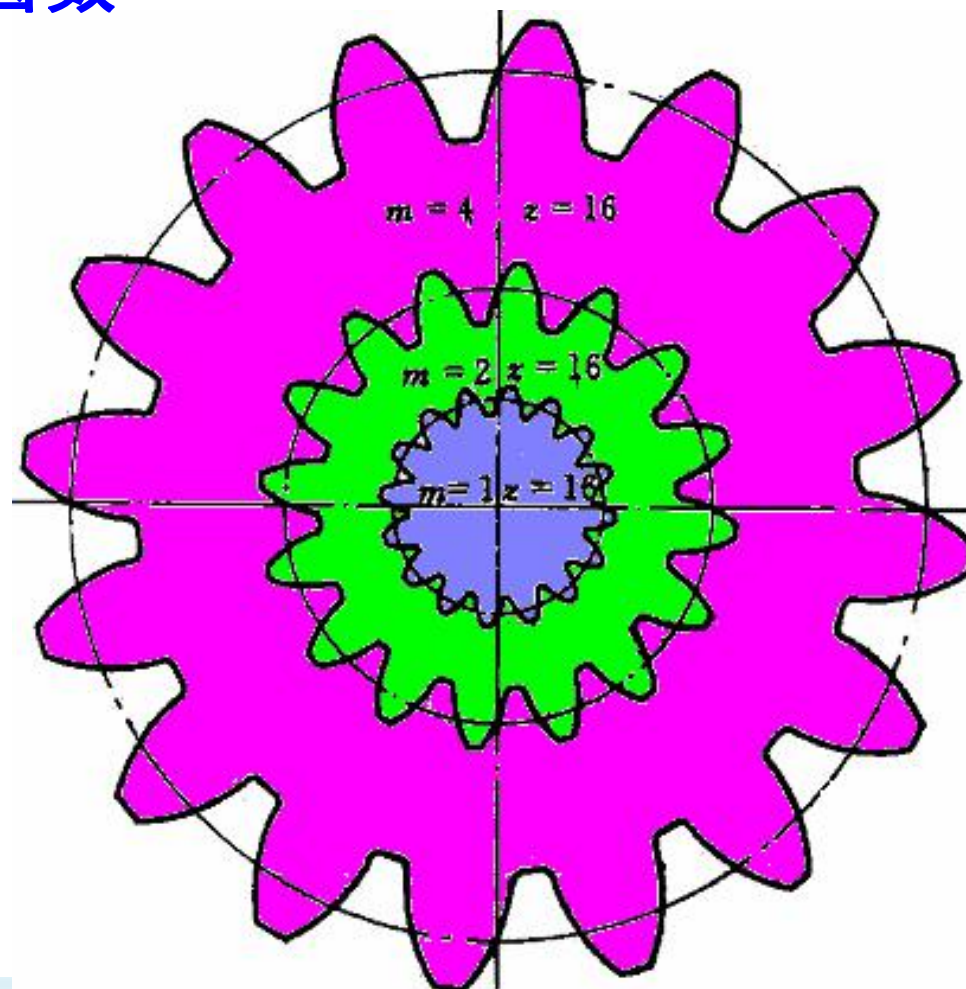
F i r s t series	0.1, 0.12, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.8, 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40,
S e c o n d series	0.35, 0.7, 0.9, 1.75, 2.25, 2.75, (3.25), 3.5, (3.75), 4.5, 5.5, (6.5), 7, 9, (11), 14, 18, 22, 28, (30), 36, 45

Modules of the first series are preferable.



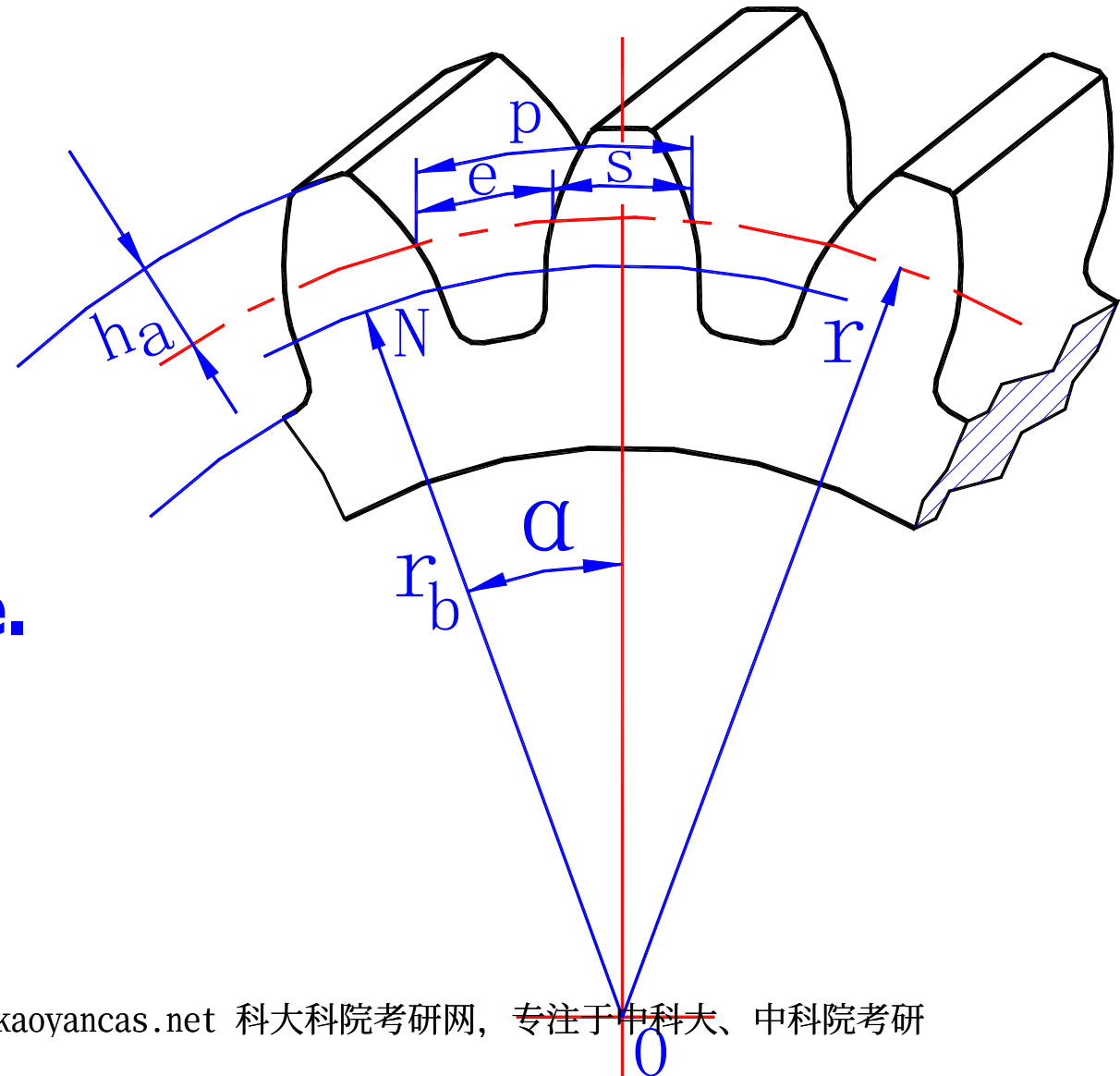


同模数不同齿数



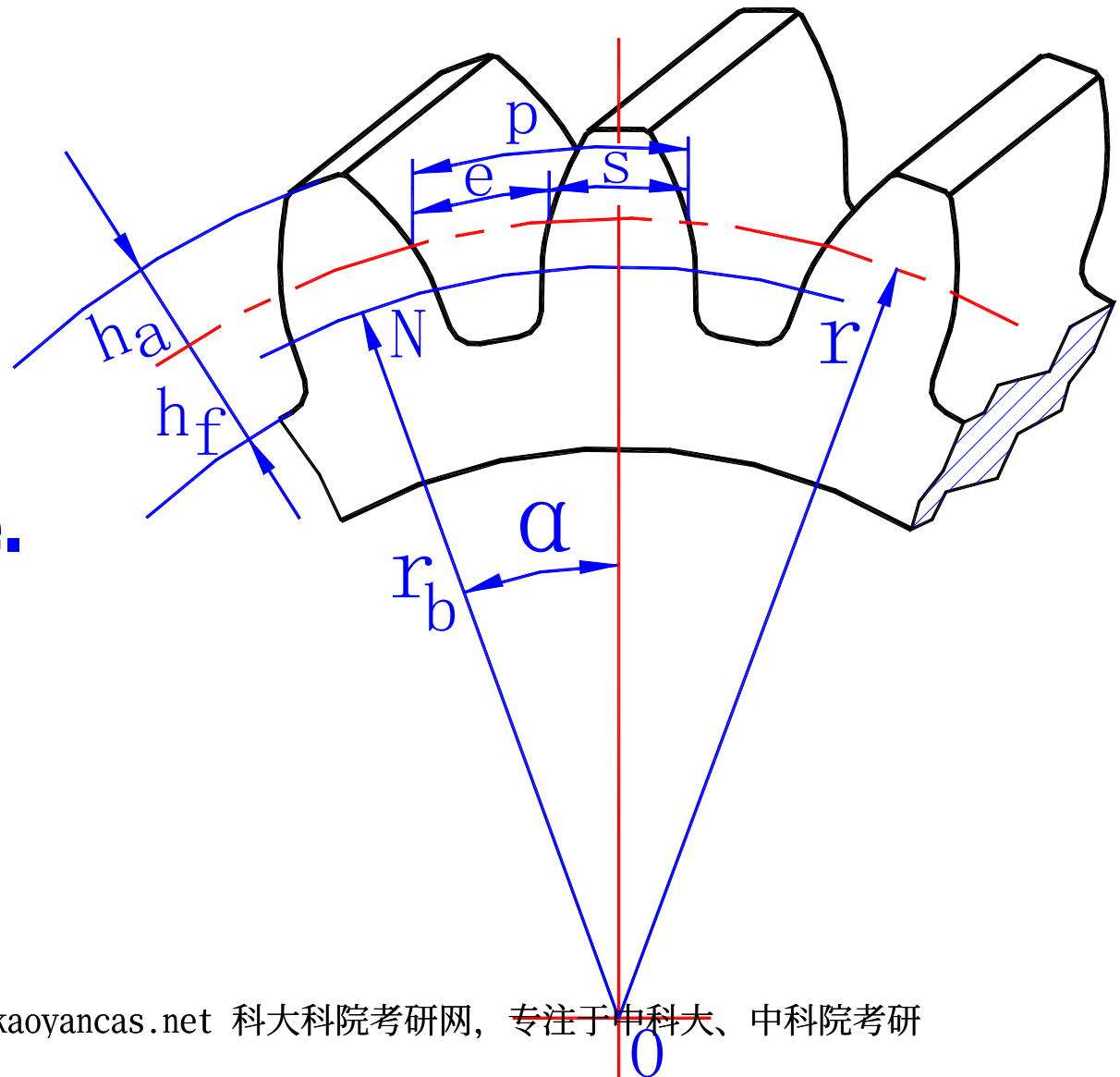


**The addendum
(齿顶高) h_a ----**
**radial distance
between the
reference circle
and the
addendum circle.**



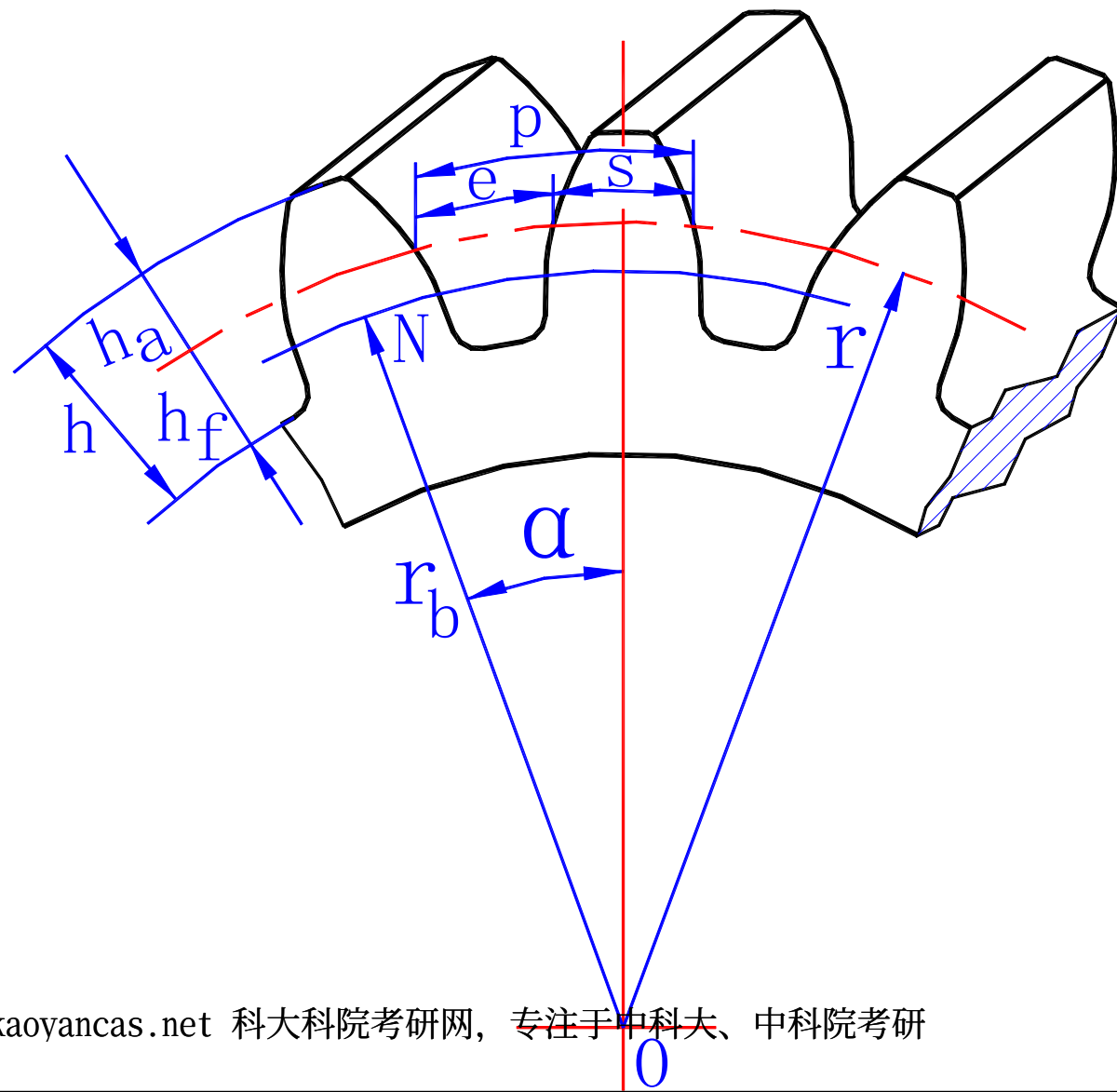


**The dedendum
(齿根高) h_f ----
radial distance
between the
reference circle
and the
dedendum circle.**





**The tooth depth
(齿全高) h ----
radial distance
between the
dedendum circle
and addendum
circle.**





For standard gears:

$$e = s = 0.5 \pi m$$

$$h_a = h_a^* m$$

$$h_f = (h_a^* + c^*) m$$

h_a^* ---- the coefficient of addendum(齿顶高系数)

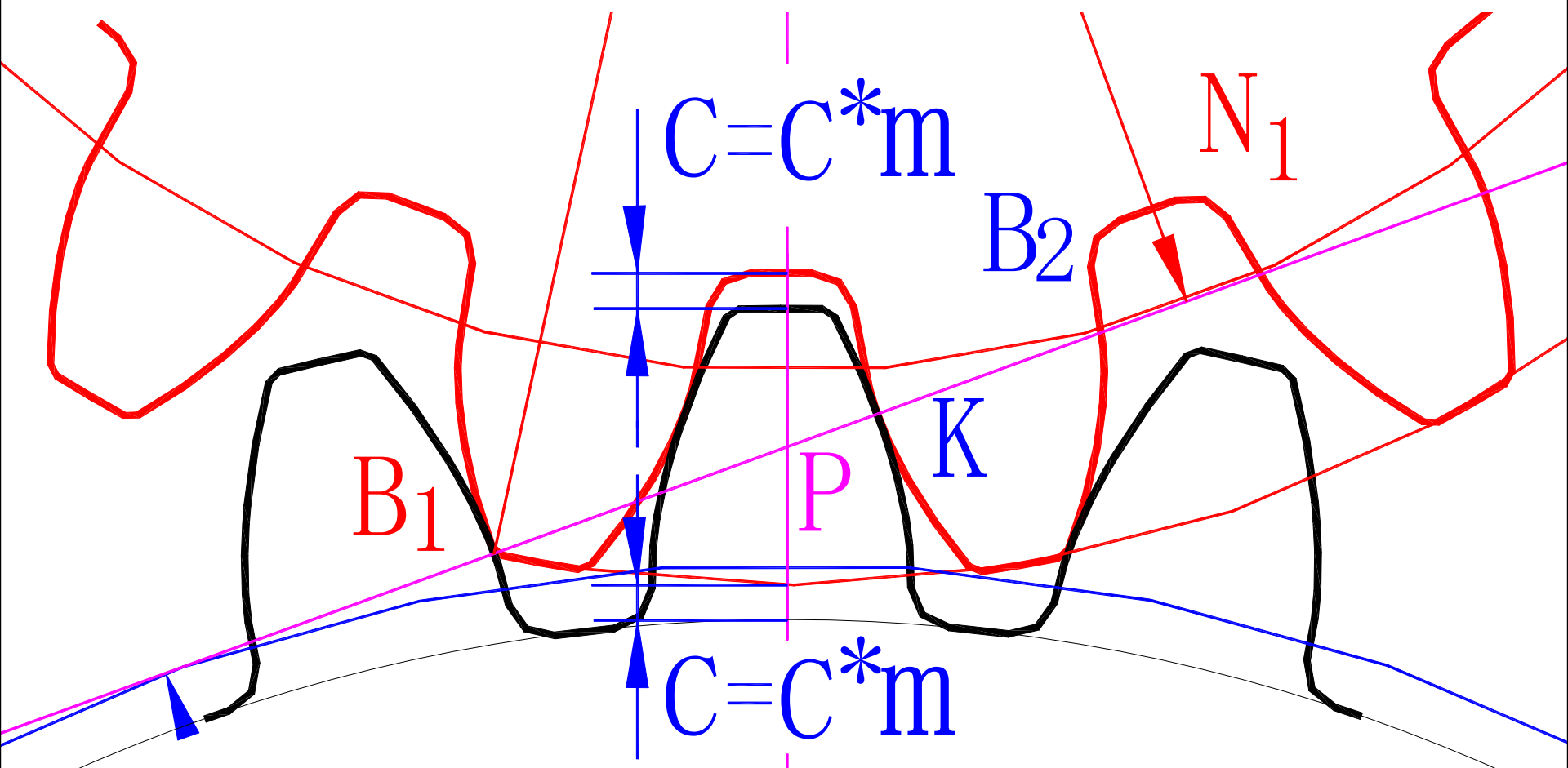
c^* ---- the coefficient of bottom clearance(顶隙系数)

$$h = (2h_a^* + c^*) m$$

$h_a^* = 1$ and $c^* = 0.25$ for the normal tooth(正常齿)

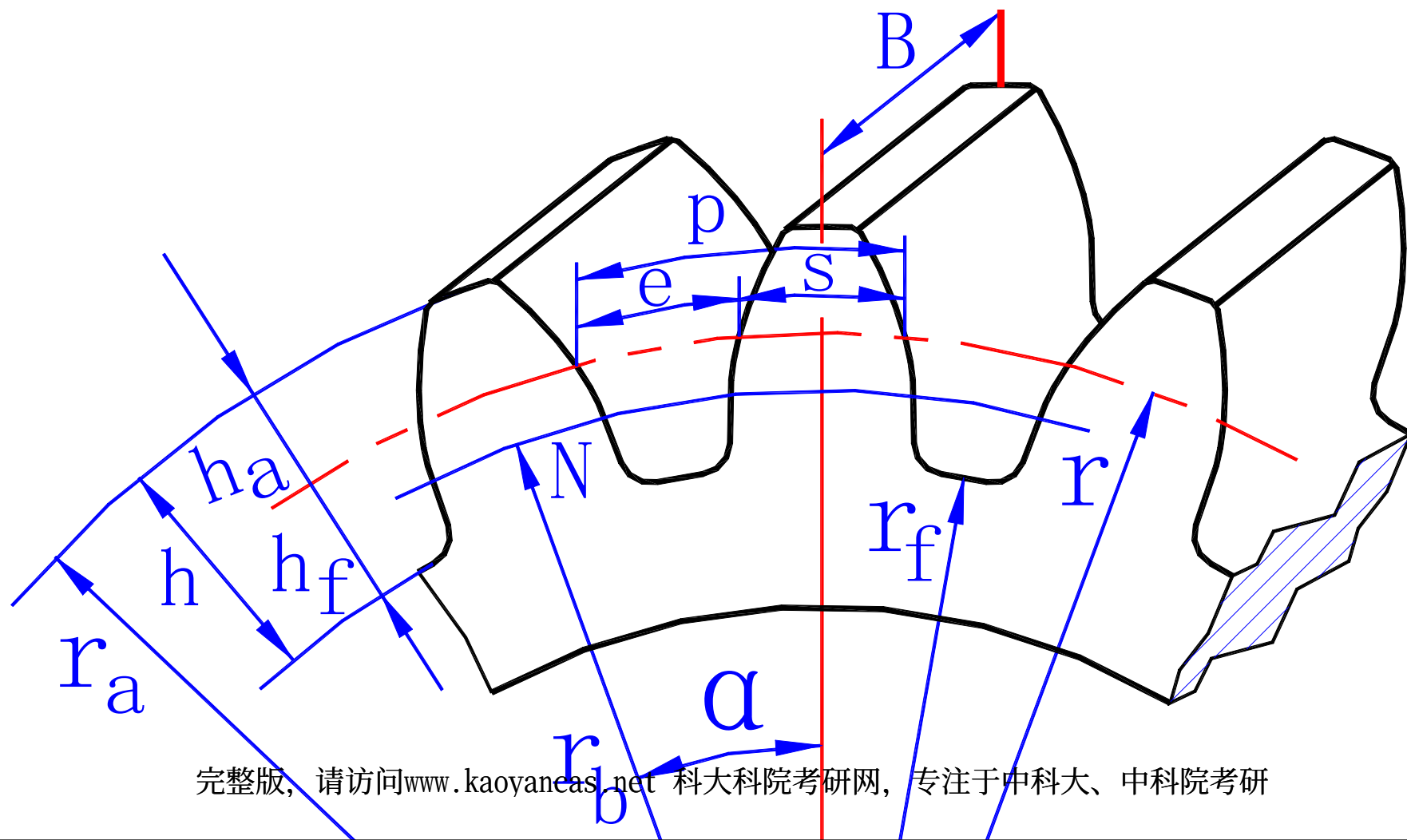
$h_a^* = 0.8$ and $c^* = 0.3$ for the shorter tooth(短齿)

$c=c^* m$ ----- bottom clearance顶隙

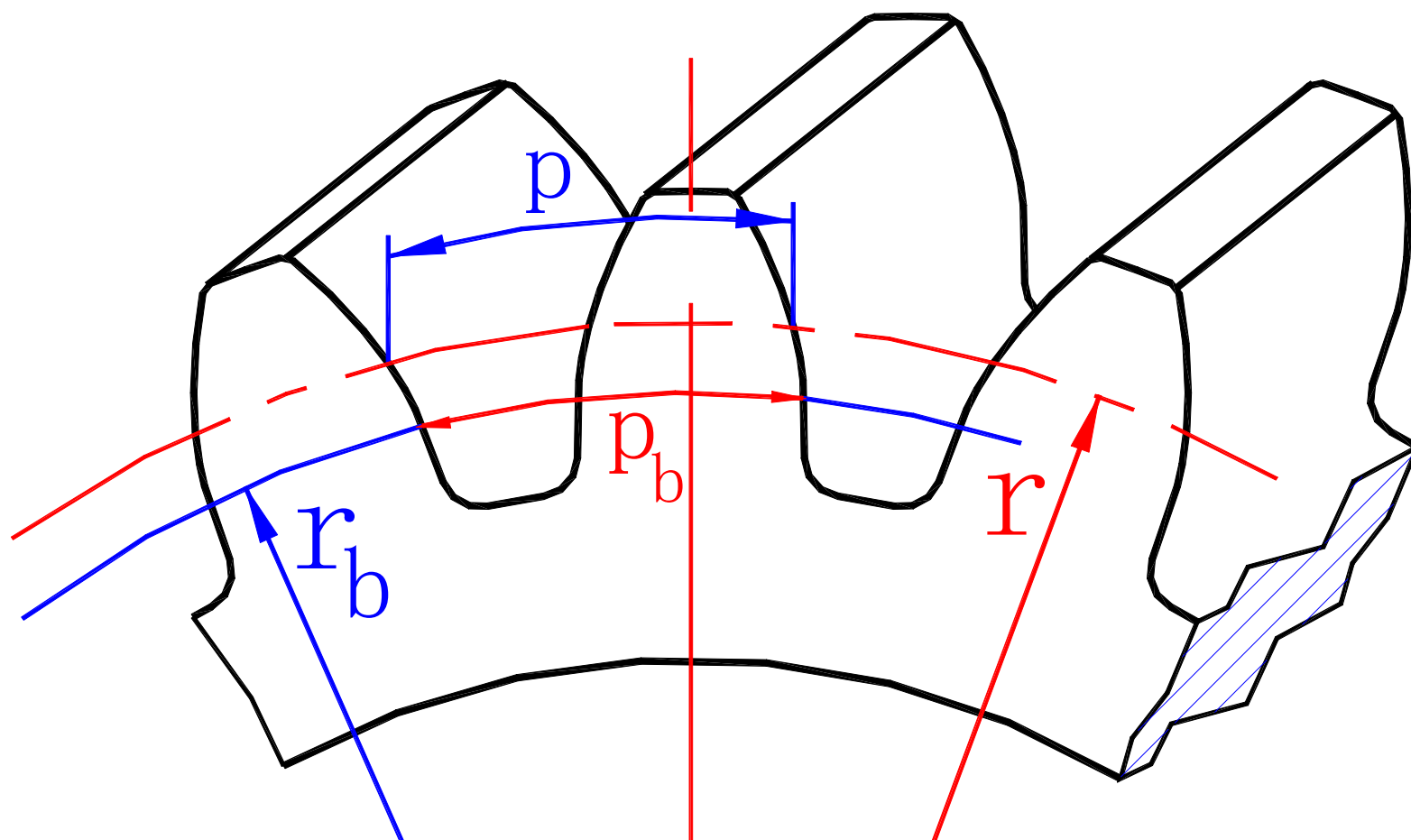


$$d_a = d + 2h_a = (Z + 2h_a^*)m$$

$$d_f = d - 2h_f = (Z - 2h_a^* - 2c^*)m$$



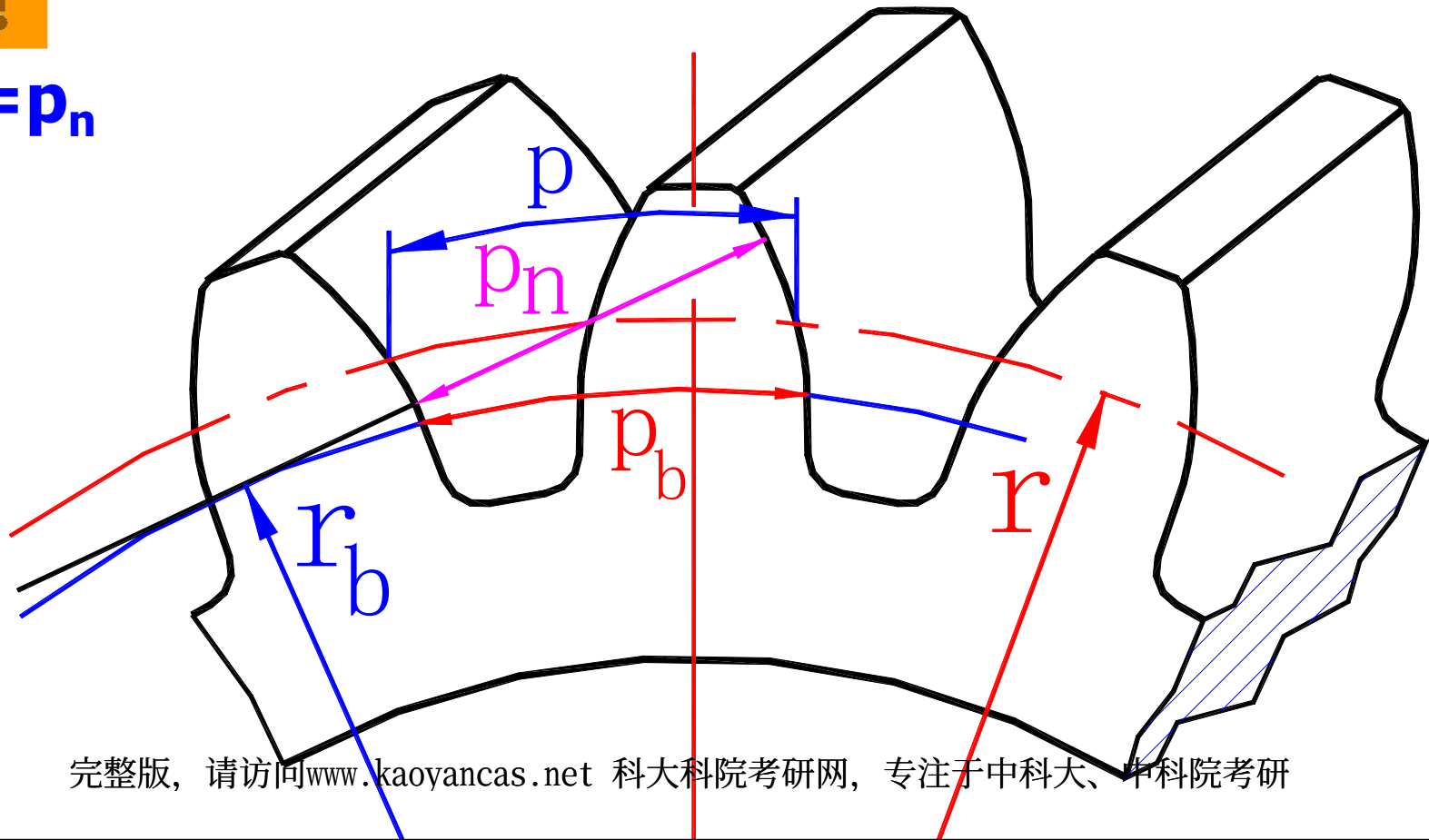
Base pitch(基节) p_b --- the pitch on the base circle.



Normal pitch(法节) p_n -----The distance between corresponding sides of adjacent tooth profiles along the common normal.



$$p_b = p_n$$



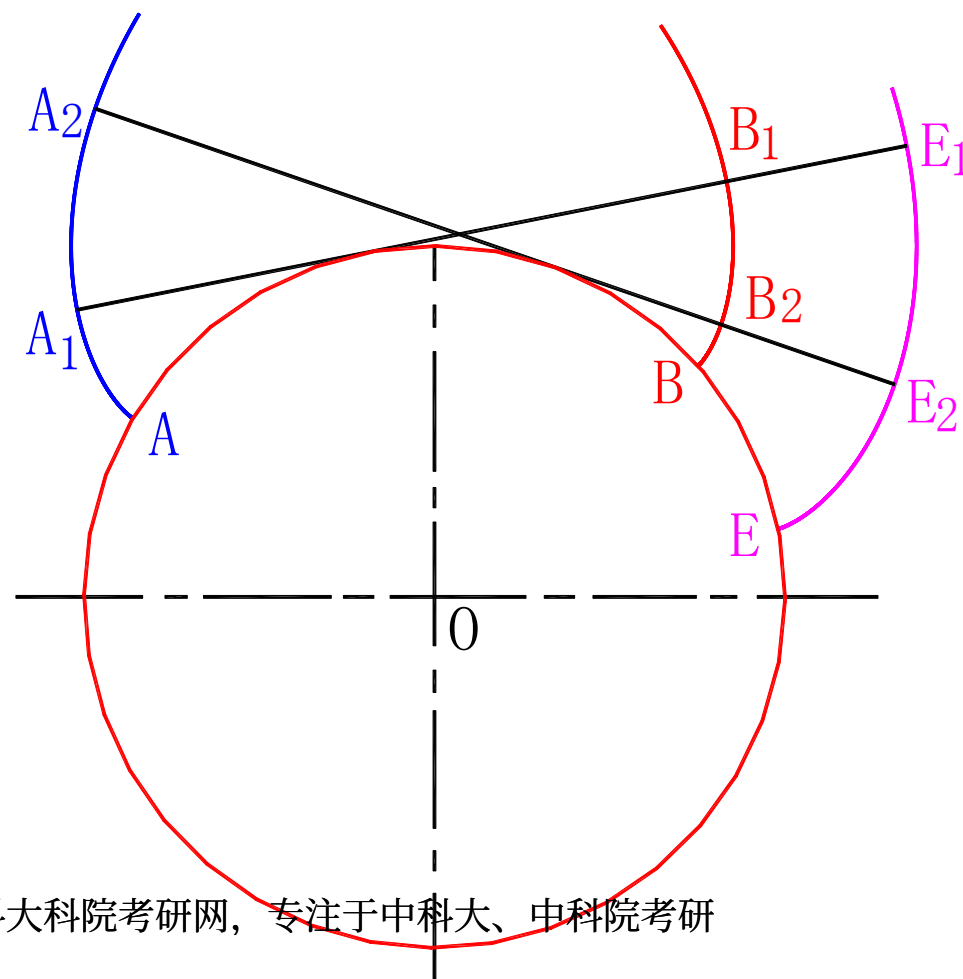


Review: The normal distance between two involutes of the same base circle remains the same.

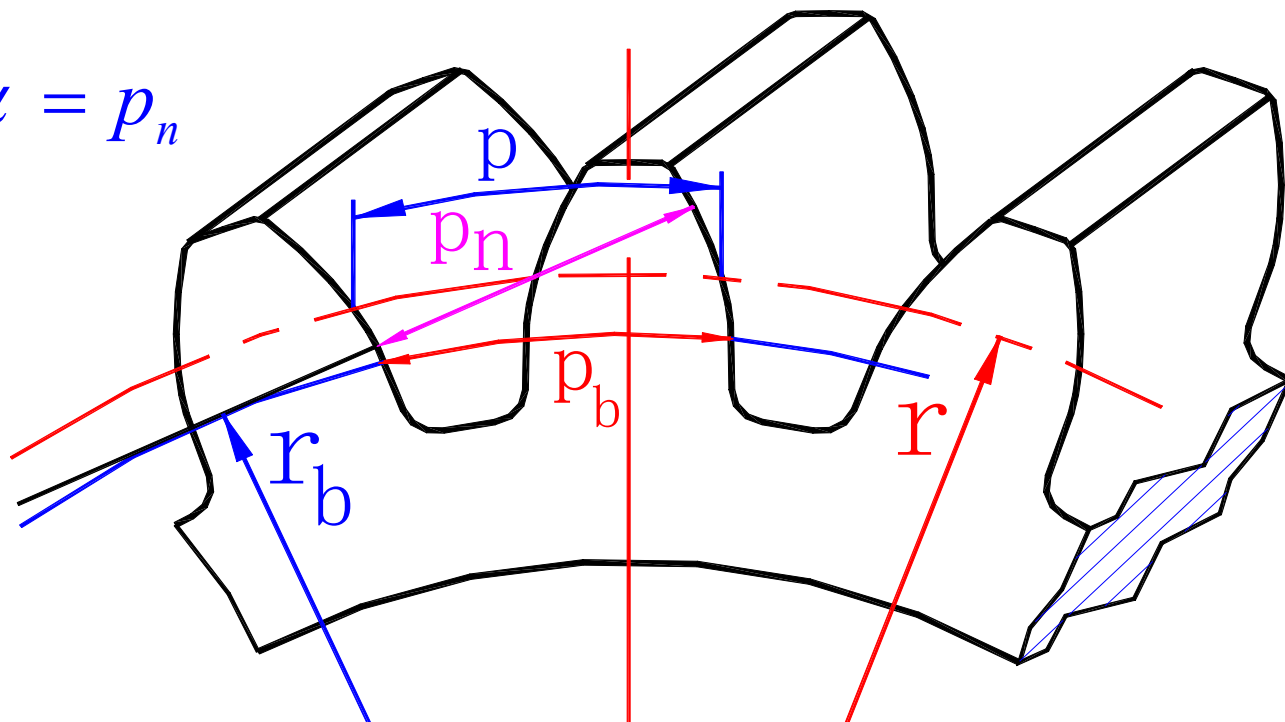
$$\overline{B_1E_1} = \overline{B_2E_2} = \widehat{BE}$$



$$p_b = p_n =$$



$$p_b = p \cos \alpha = p_n$$

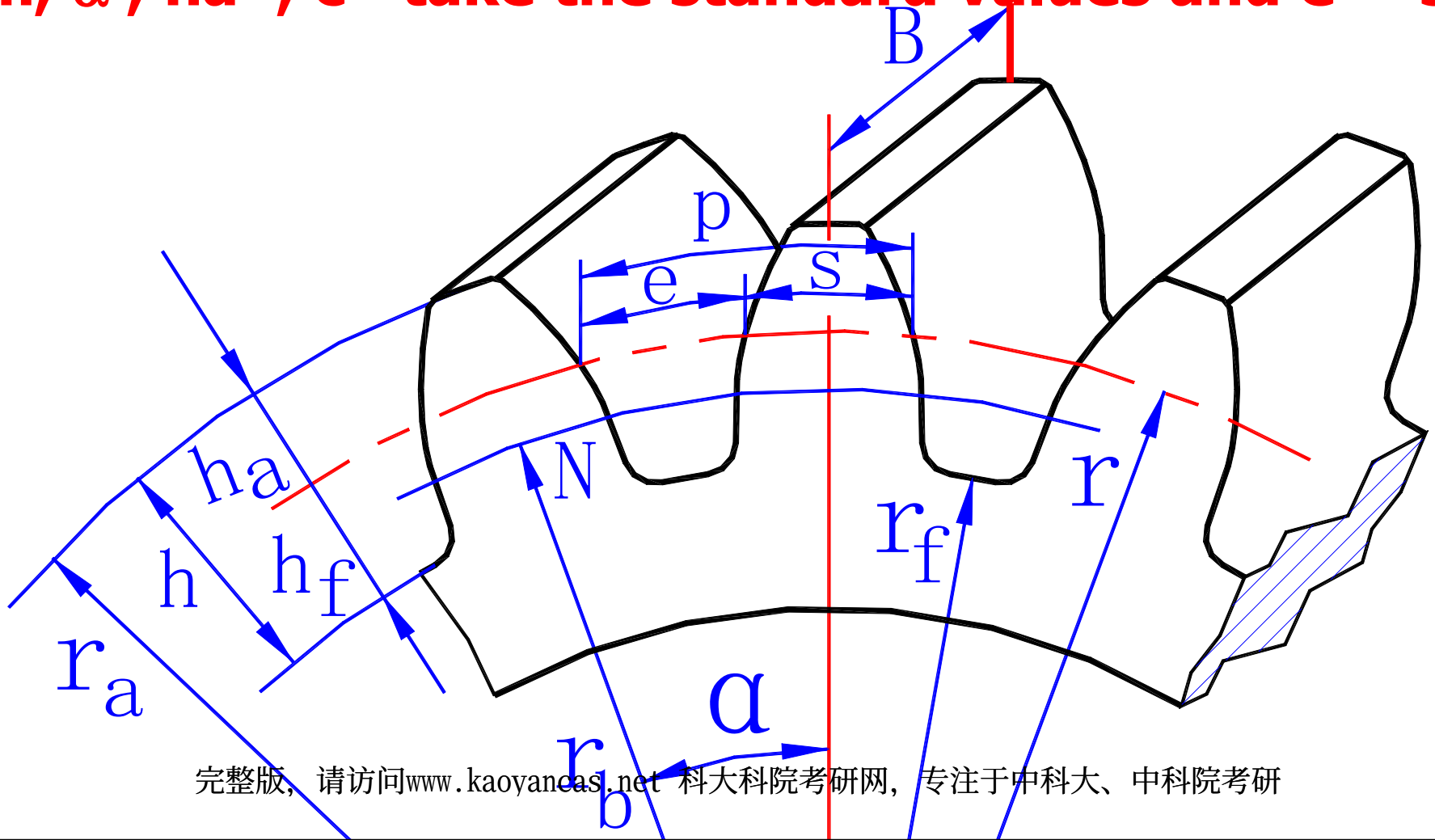


The fundamental parameters:

$$Z, m, \alpha, h_a^*, c^*$$

A standard gear means:

m, α, h_a^*, c^* take the standard values and $e = s$



一个齿轮：

$$d=mz$$

$$d_a=d+2h_a=(z+2h_a^*)m$$

$$d_f=d-2h_f=(z-2h_a^*-2c^*)m$$

$$d_b=d\cos\alpha$$

$$h_a=h_a^*m$$

$$h_f=(h_a^*+c^*)m$$

$$h=h_a+h_f=(2h_a^*+c^*)m$$

$$P=\pi m$$

$$S=e=\frac{1}{2}\pi m$$

一对标准齿轮正常安装：

$$a=\frac{1}{2}(d_2\pm d_1)=\frac{1}{2}m(z_2\pm z_1)$$

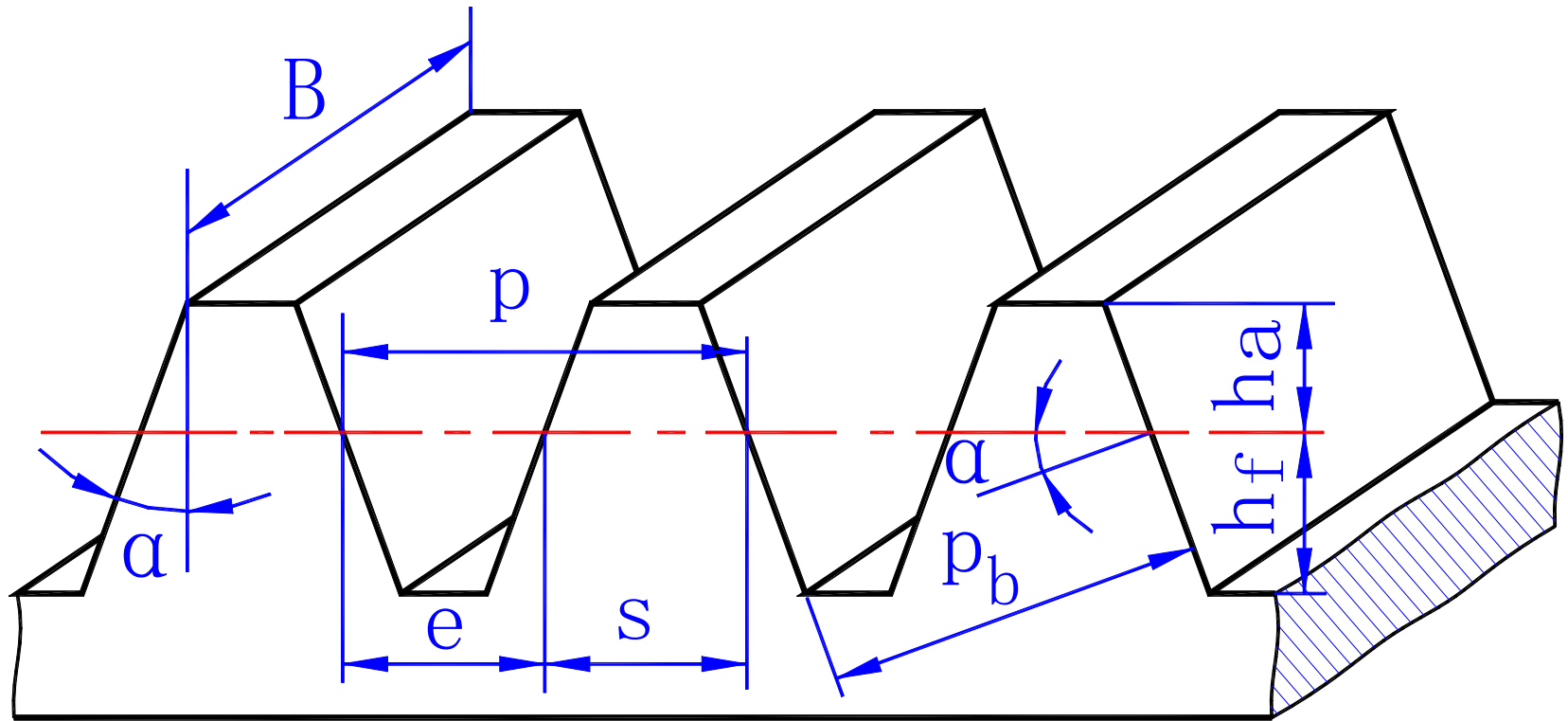
① m 、 z 决定了分度圆的大小，而齿轮的大小主要取决于分度圆，因此 m 、 z 是决定齿轮大小的主要参数

② 轮齿的尺寸与 m ， h_a^* ， c^* 有关与 z 无关

③ 至于齿形，与 m ， z ， α 有关

模数制齿轮

6.4.2 The Rack (齿条)



Involutes $\longrightarrow$ **straight lines**

$$\alpha_i \equiv \alpha \quad p_i \equiv p \equiv \pi m$$

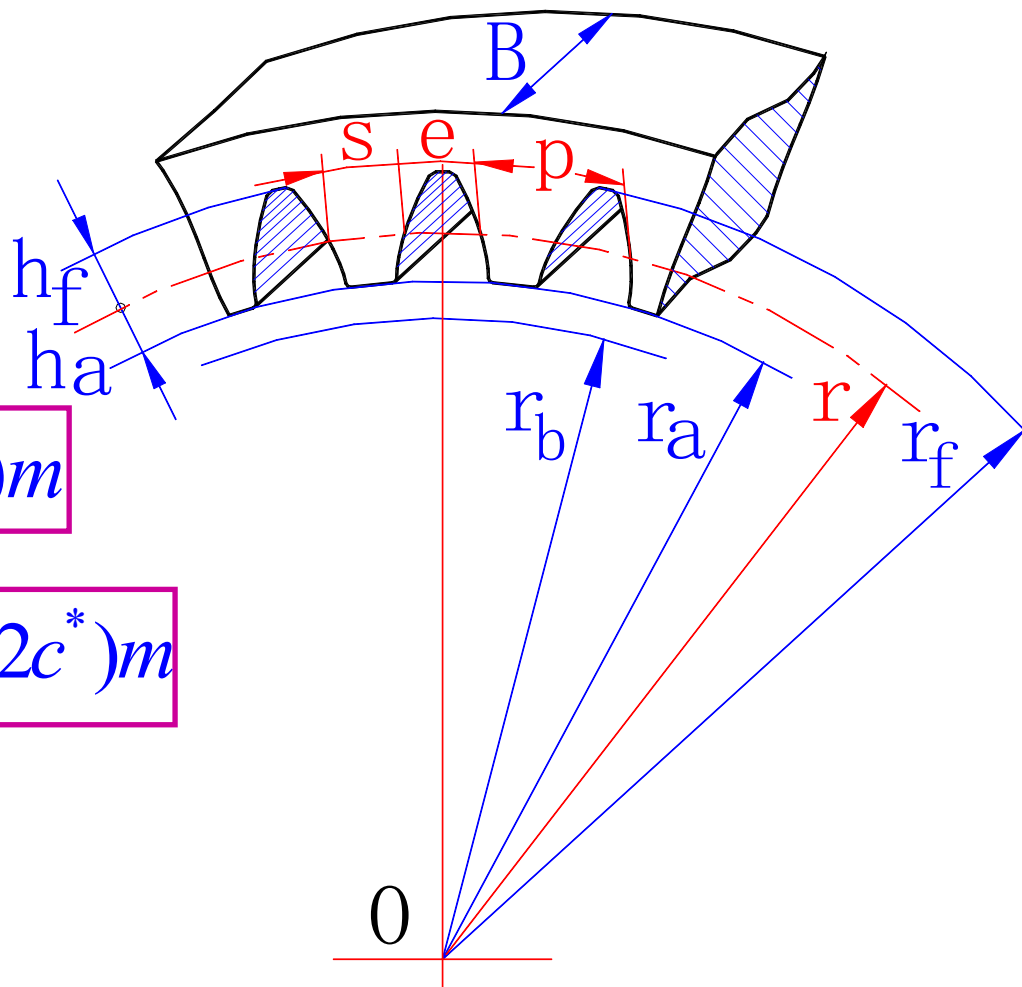
On the reference line(分度线), $e=s$.

For standard rack, $h_a = h_a^* m$, $h_f = (h_a^* + c^*) m$



6.4.3 Internal Gears

The tooth thickness $\leftrightarrow$ the tooth space



$$d_a = d - 2h_a = (Z - 2h_a^*)m$$

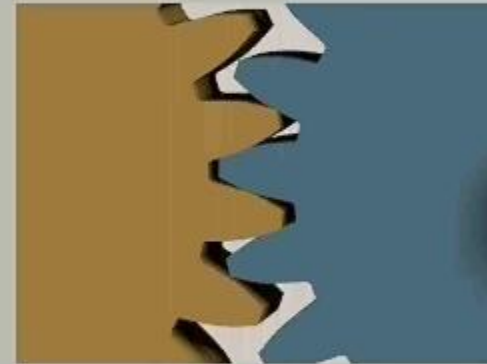
$$d_f = d + 2h_f = (Z + 2h_a^* + 2c^*)m$$



6.5 Gearing of Involute Spur Gears

6.5.1 Proper Meshing Conditions(正确啮合条件) for Involute Gears

To ensure a continuous transmission, there must exist such situation that two pairs of teeth contact simultaneously(同时地).



两对齿分别在 K_1 、 K_2 点啮合 根据啮合基本定律

K_1 在 N_1N_2 上
 K_2 在 N_1N_2 上

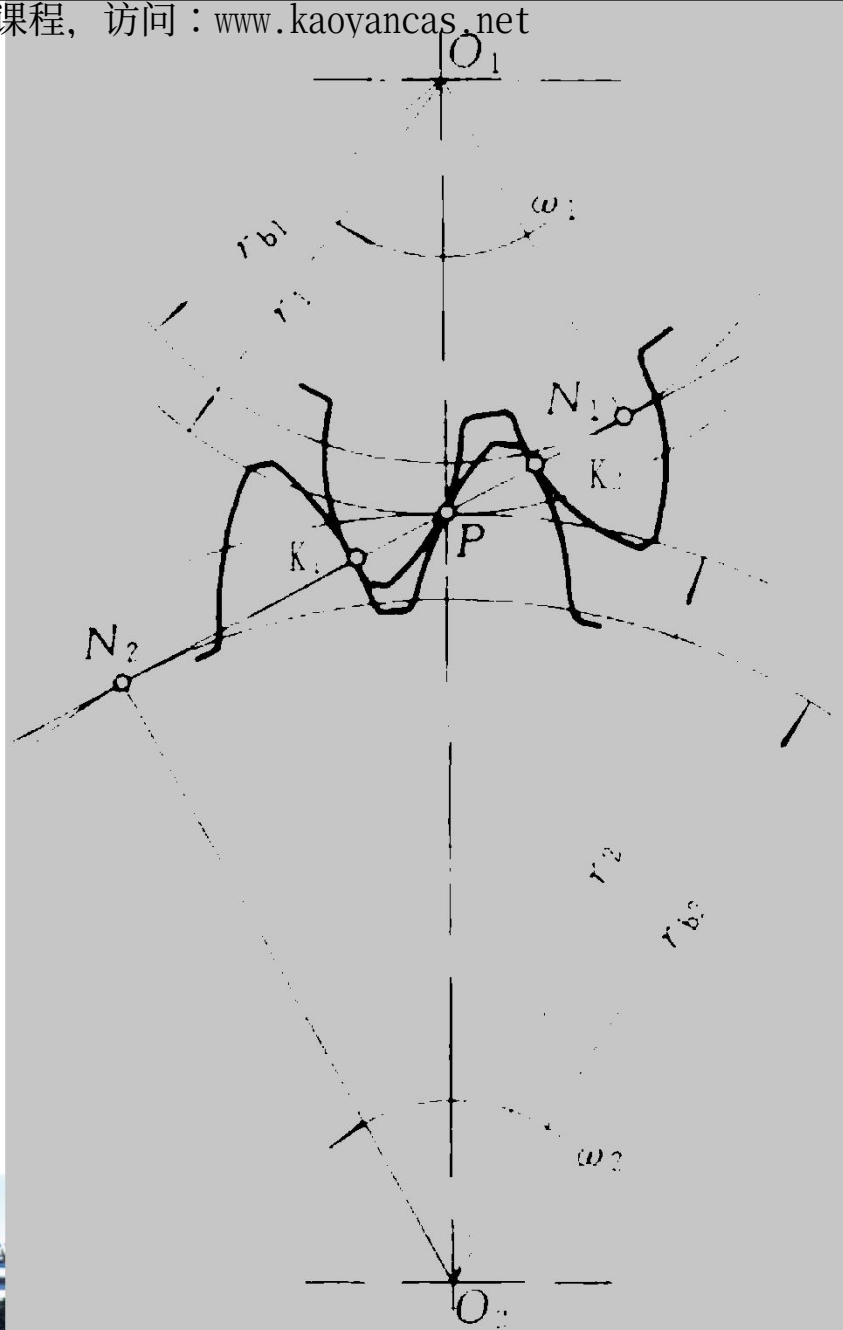
K_1K_2 ——法向齿距

在齿轮1上： $K_1K_2 = P_{b1}$

在齿轮2上： $K_1K_2 = P_{b2}$

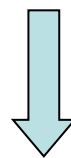
$$\therefore P_{b1} = P_{b2}$$

$$\pi m_1 \cos \alpha_1 = \pi m_2 \cos \alpha_2$$





$$\pi m_1 \cos \alpha_1 = \pi m_2 \cos \alpha_2$$



$$\begin{cases} m_1 = m_2 = m = \text{标准值} \\ \alpha_1 = \alpha_2 = \alpha = 20^\circ \end{cases}$$

The proper meshing condition for involute gears:

The modules and pressure angles of two meshing gears are the same respectively.

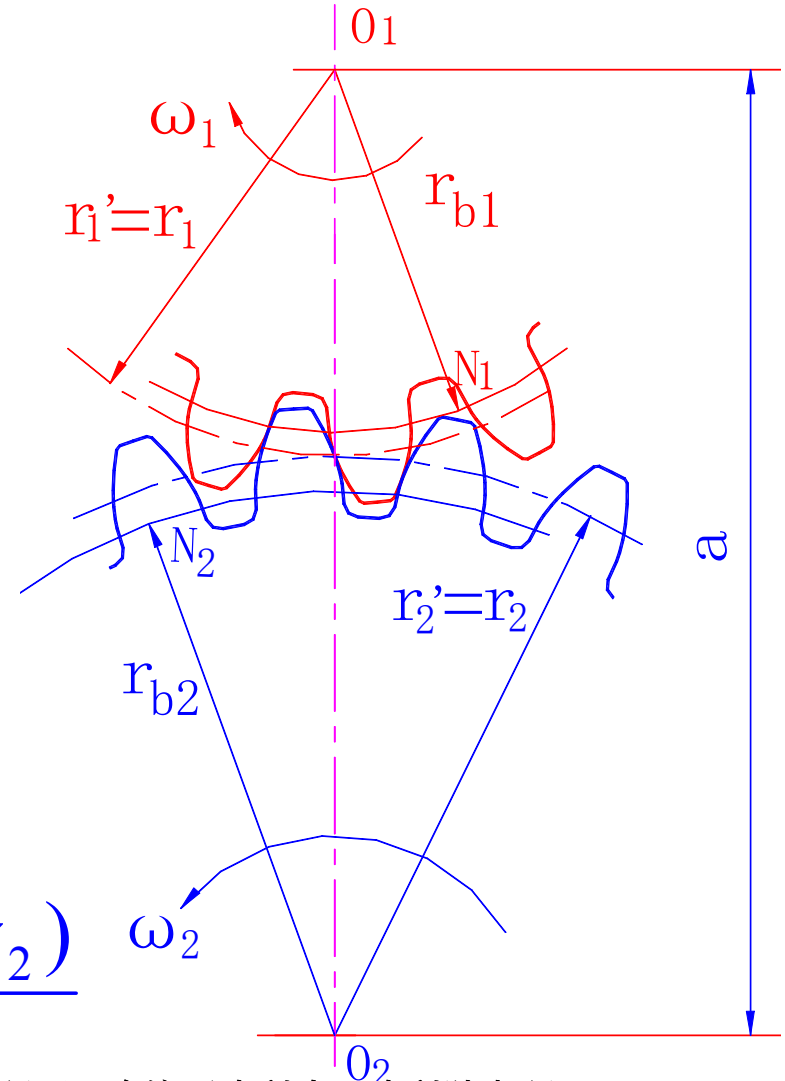
6.5.2 Centre Distance and Working Pressure Angle of a Gear Pair

要求：一、侧隙应该为零
二、顶隙为标准值

The two reference circles are tangent and roll without slipping.

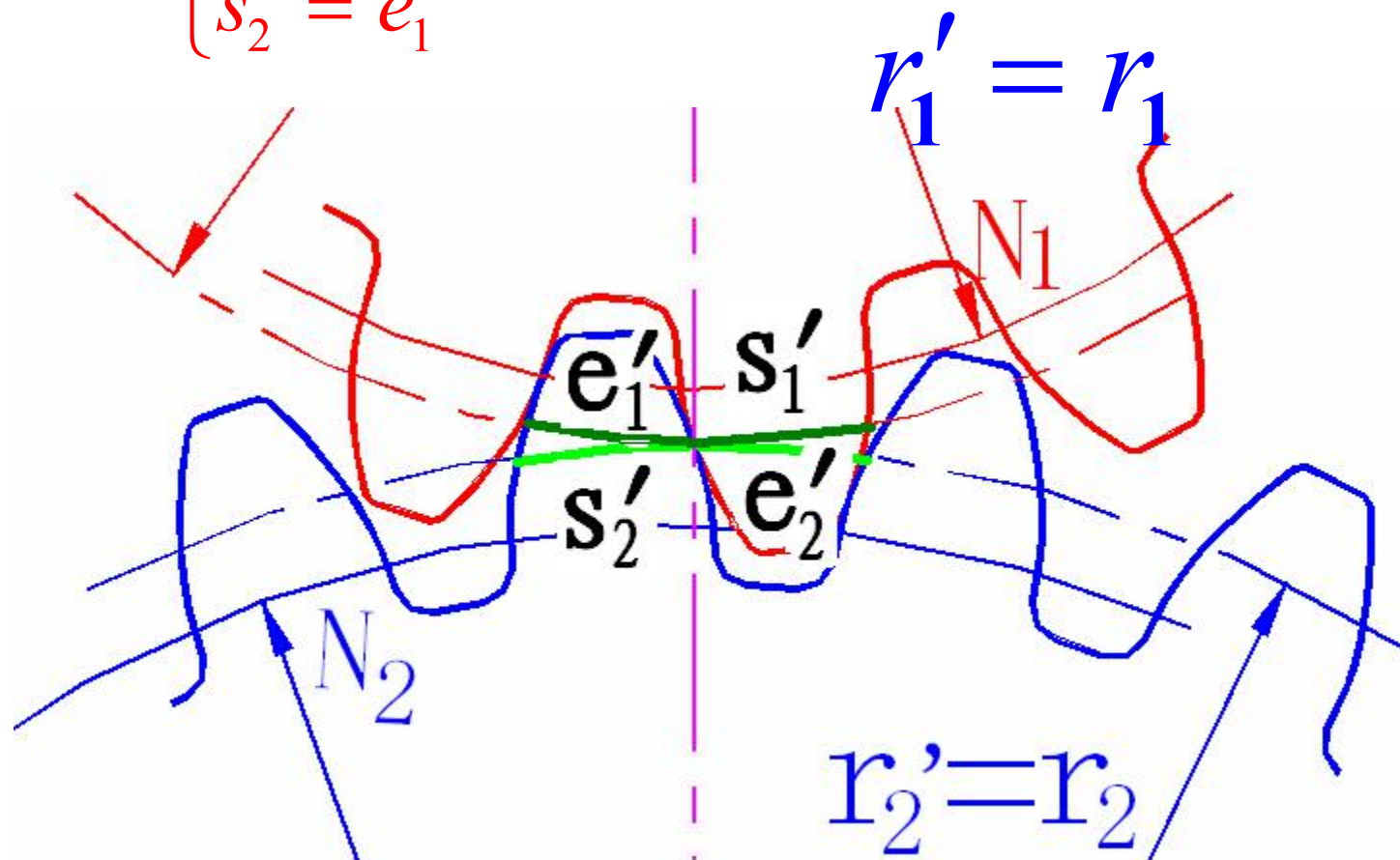
The centre distance a is equal to $(r_1 + r_2)$, the reference circles coincide with their pitch circles. 节圆

$$a = r_1' + r_2' = r_1 + r_2 = \frac{m(z_1 + z_2)}{2}$$



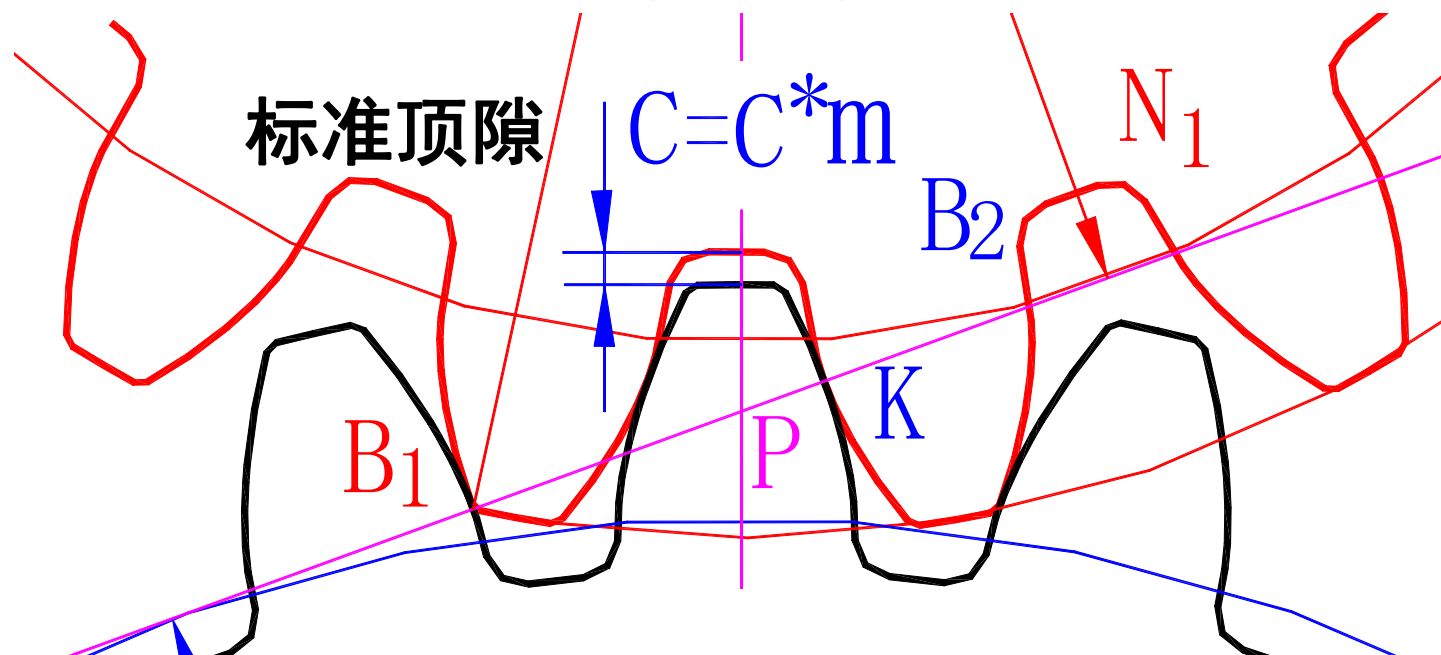
To keep zero backlash, it holds: 无侧隙啮合条件

$$\begin{cases} s_1' = e_2' \\ s_2' = e_1' \end{cases}$$



标准安装

the bottom clearance(顶隙) c : the radial distance from the dedendum circle of one gear to the addendum circle of another



Therefore, when two standard gears are assembled at a centre distance $a=r_1+r_2$, not only the backlash =0, but also bottom clearance

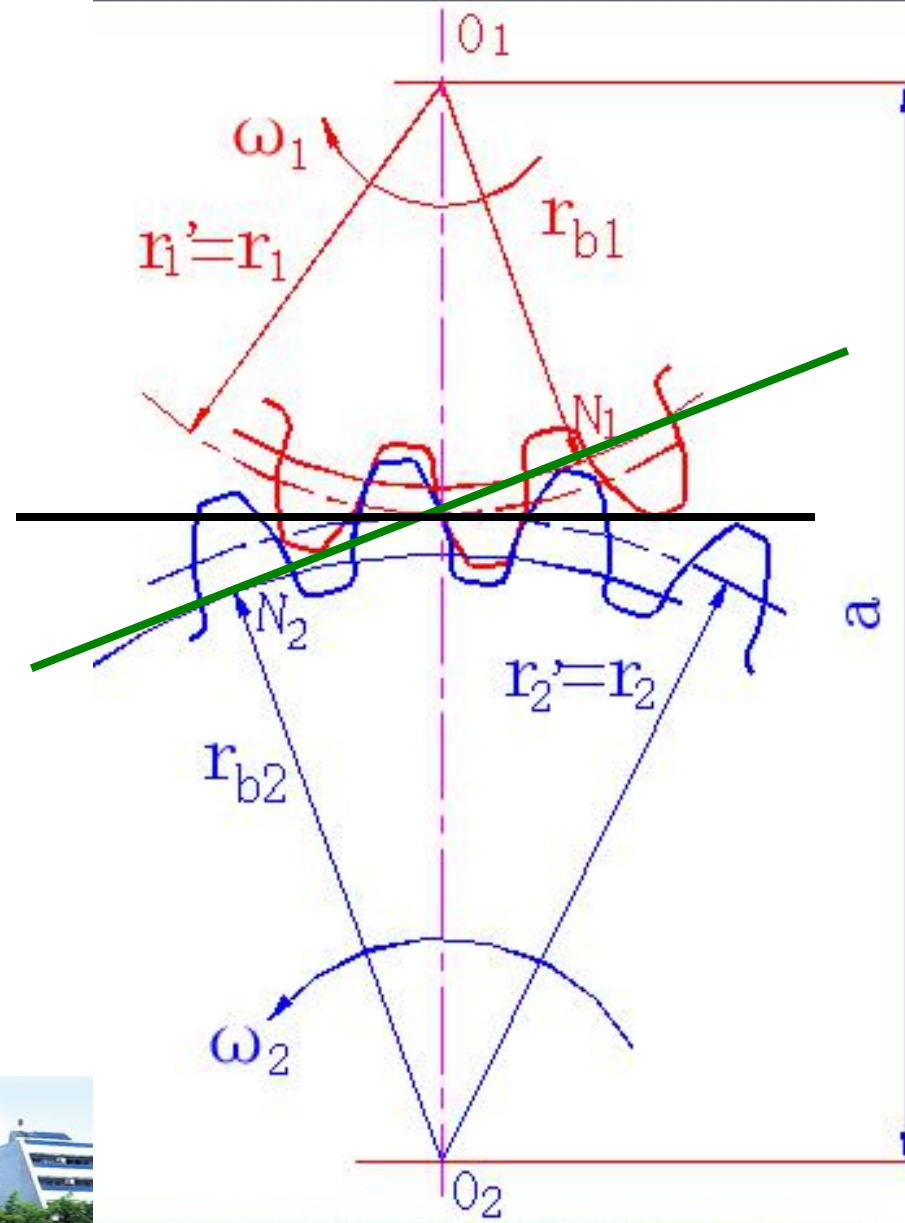
$c=c^*m$. 完整版, 请访问www.kaoyancas.net 科大科院考研网, 专注于中科大、中科院考研



标准安装

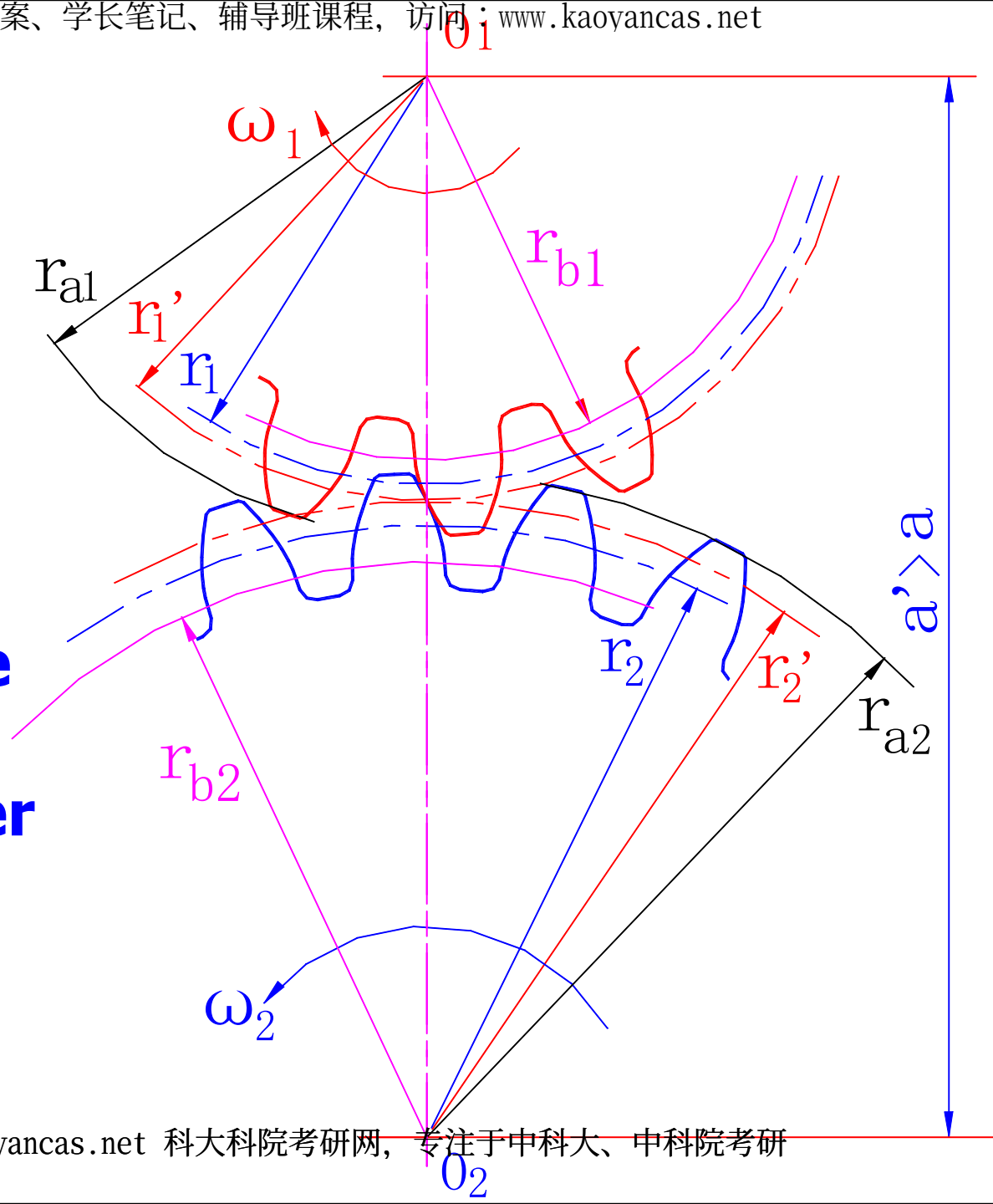
**Working pressure
angle α' (啮合角)**

$$\alpha' = \alpha$$



非标准安装

When the actual centre distance a' is larger than $(r_1 + r_2)$, the reference circles will separate. The two pitch circles will become larger and remain tangent.



非标准安装

a' 只有增大

由图可知：

$$\underline{r'_1 = \frac{r_{b1}}{\cos \alpha'} = \frac{r_1 \cos \alpha}{\cos \alpha'}}$$

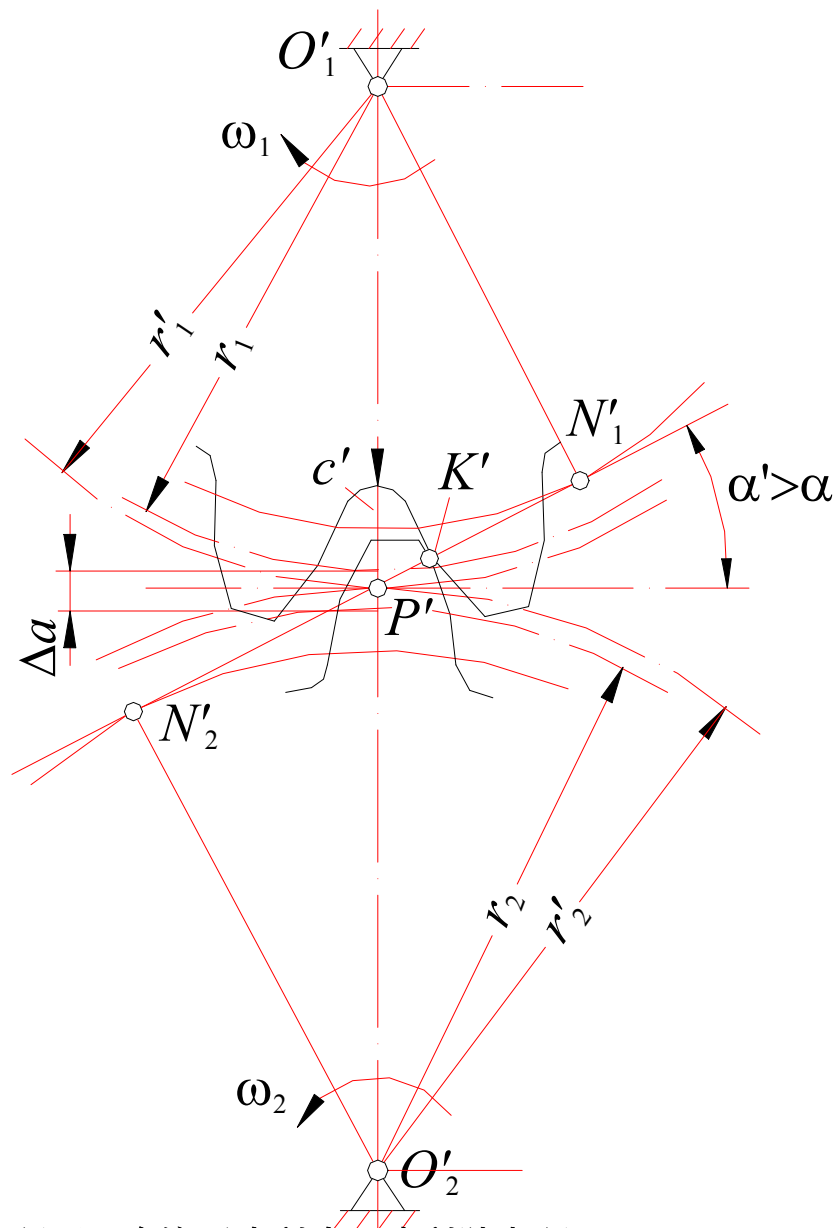
$$\underline{r'_2 = \frac{r_{b2}}{\cos \alpha'} = \frac{r_2 \cos \alpha}{\cos \alpha'}}$$

$$\underline{a' = r'_1 + r'_2 = (r_1 + r_2) \frac{\cos \alpha}{\cos \alpha'} = a \frac{\cos \alpha}{\cos \alpha'}}$$

$$a' > a, \therefore \alpha < \alpha'$$

$$\therefore r'_1 > r_1, r'_2 > r_2, c' > c$$

——有侧隙

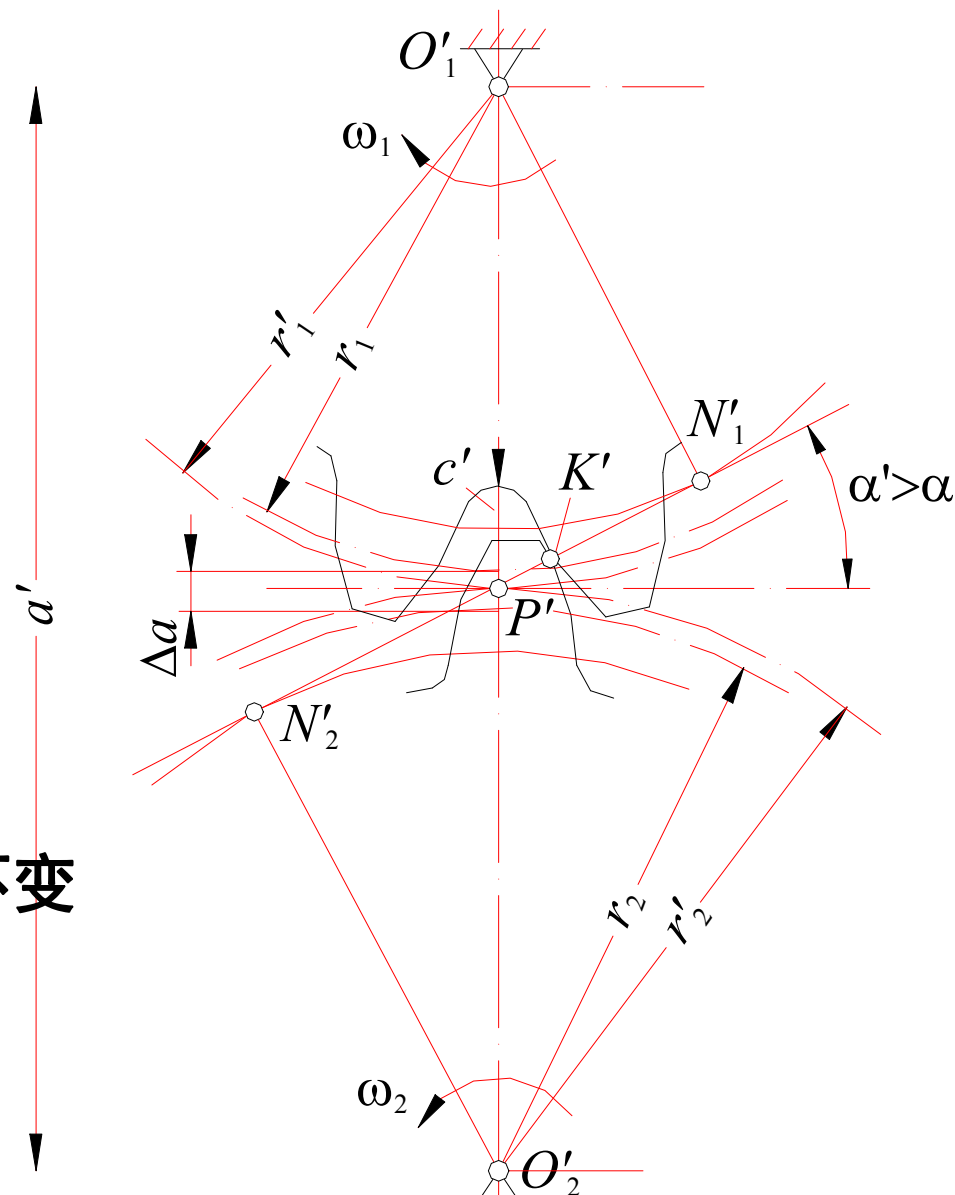




传动比

$$\begin{aligned} i_{12} &= \frac{\omega_1}{\omega_2} = \frac{r_2'}{r_1'} \\ &= \frac{r_{b2}}{r_{b1}} = \frac{r_2}{r_1} \\ &= \frac{z_2}{z_1} = \text{constant} \end{aligned}$$

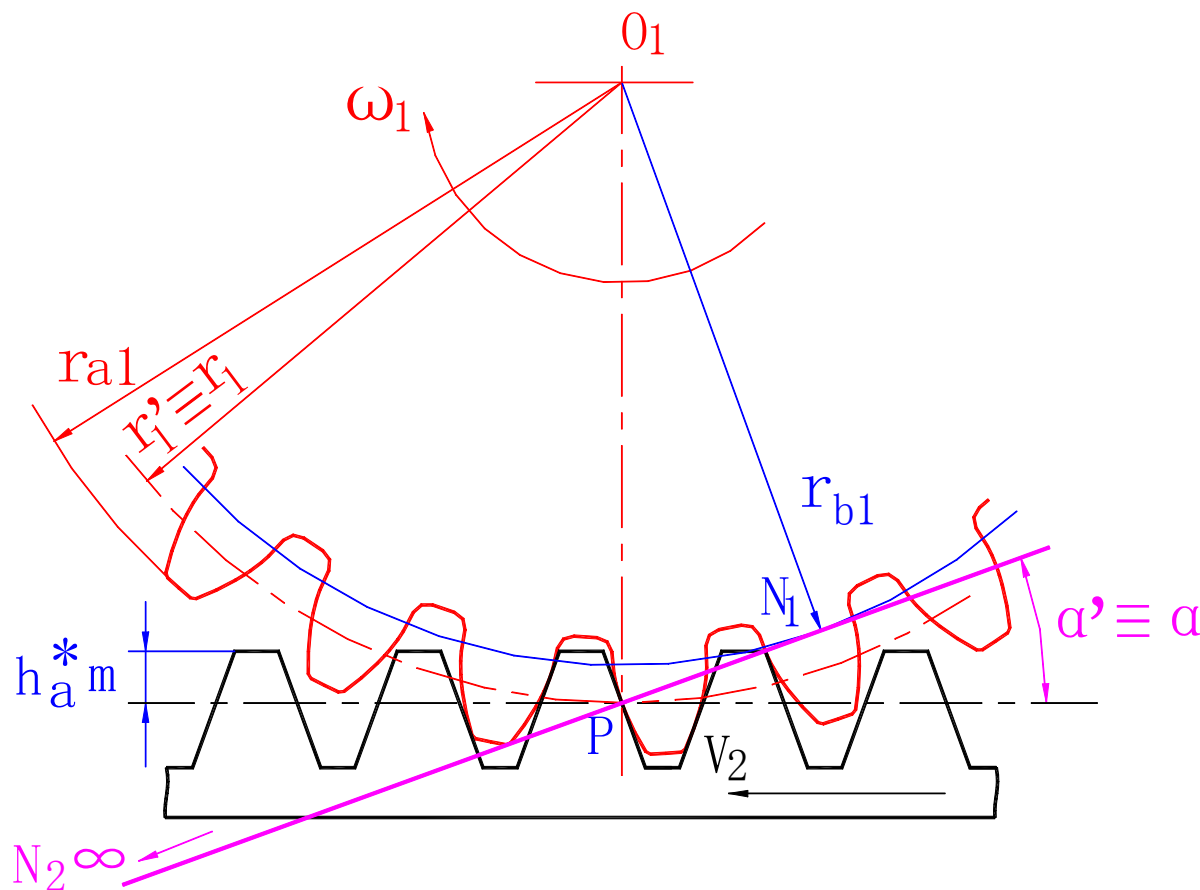
中心距变大，传动比仍保持不变





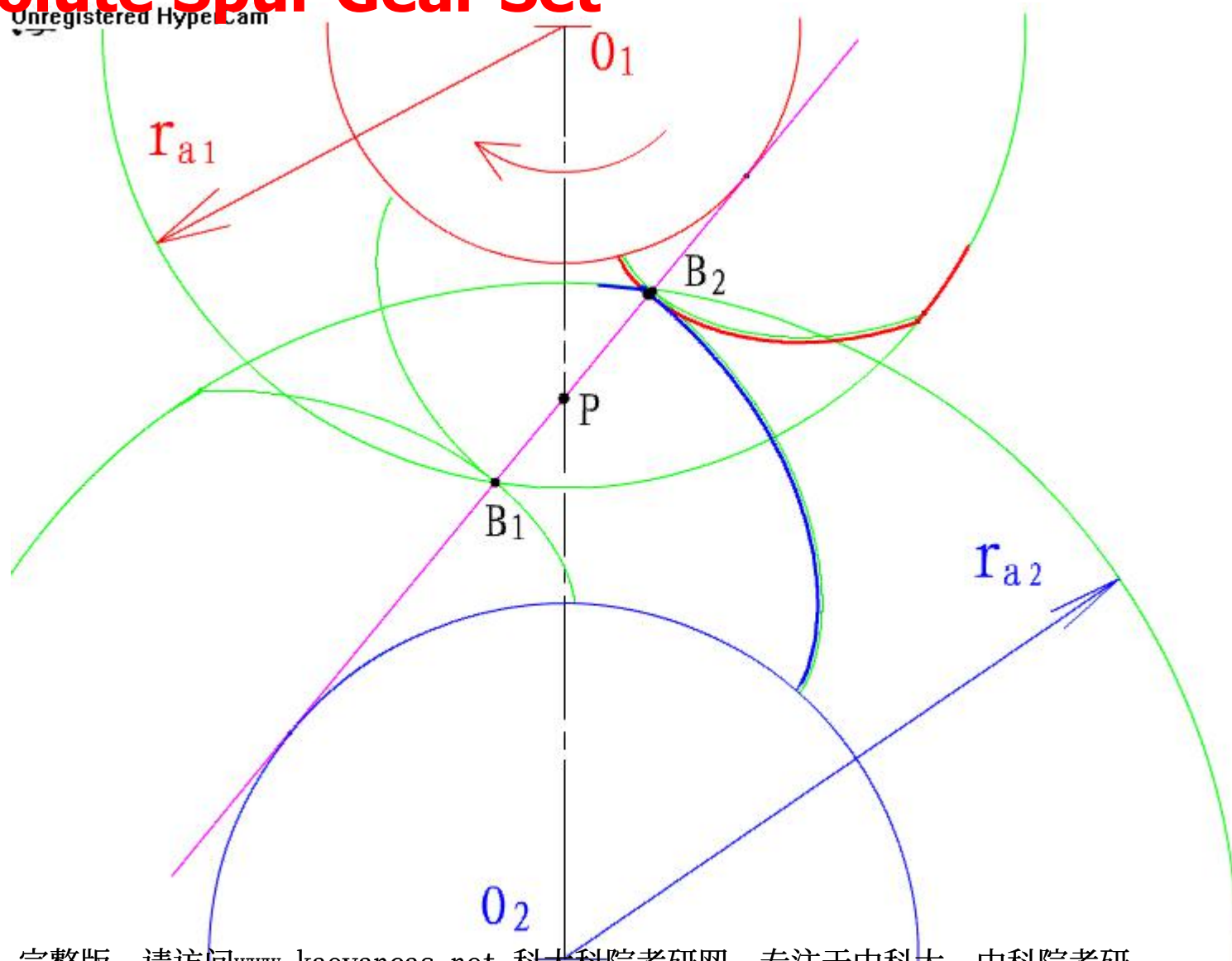
6.5.3 Meshing of r rack and pinion

无论齿条离齿轮近或远，啮合角始终等于压力角；
节圆半径始终等于分度圆半径。



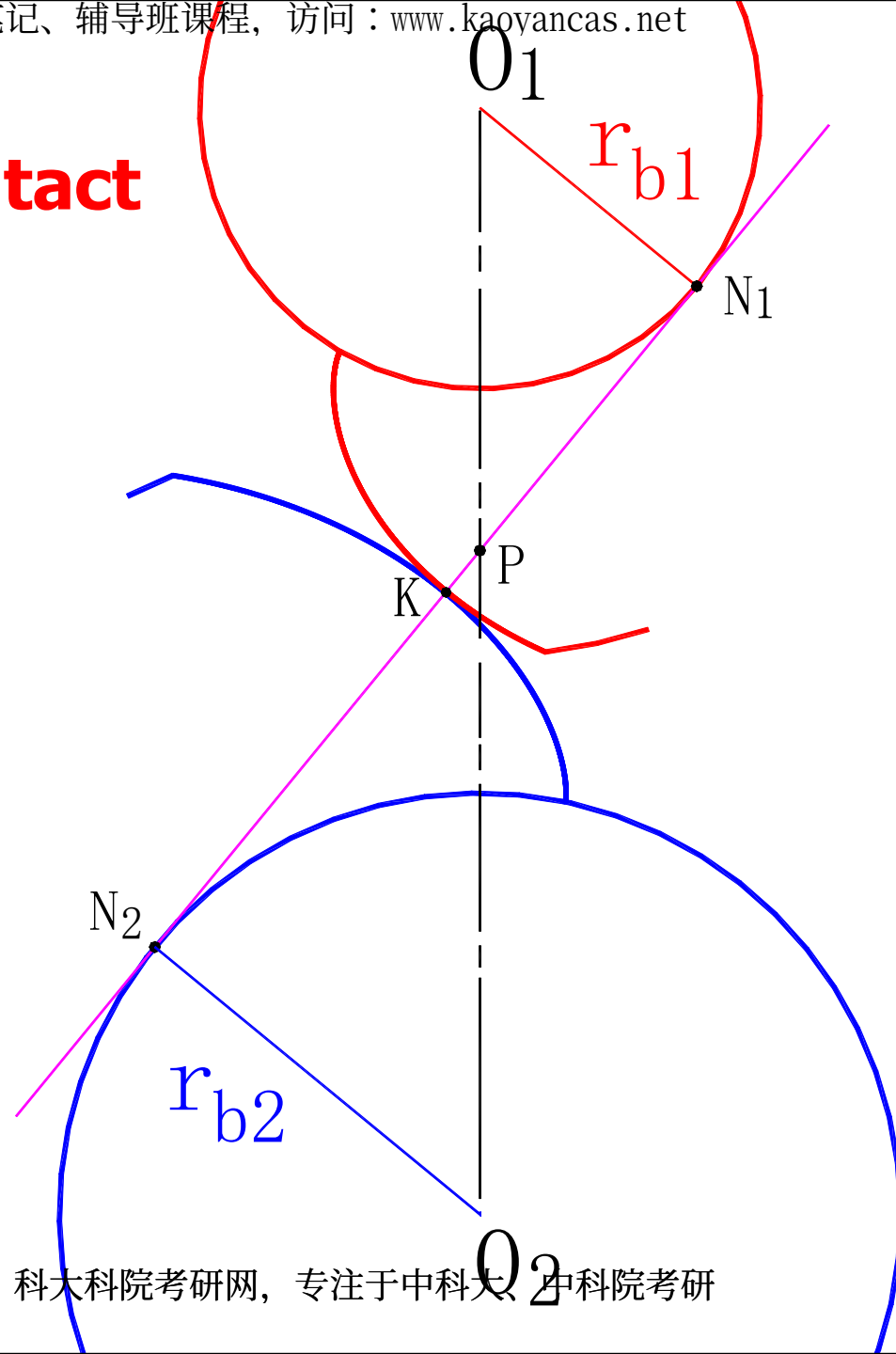
6.6 Contact Ratio(重合度) of an Involute Spur Gear Set

齿轮啮合全过程

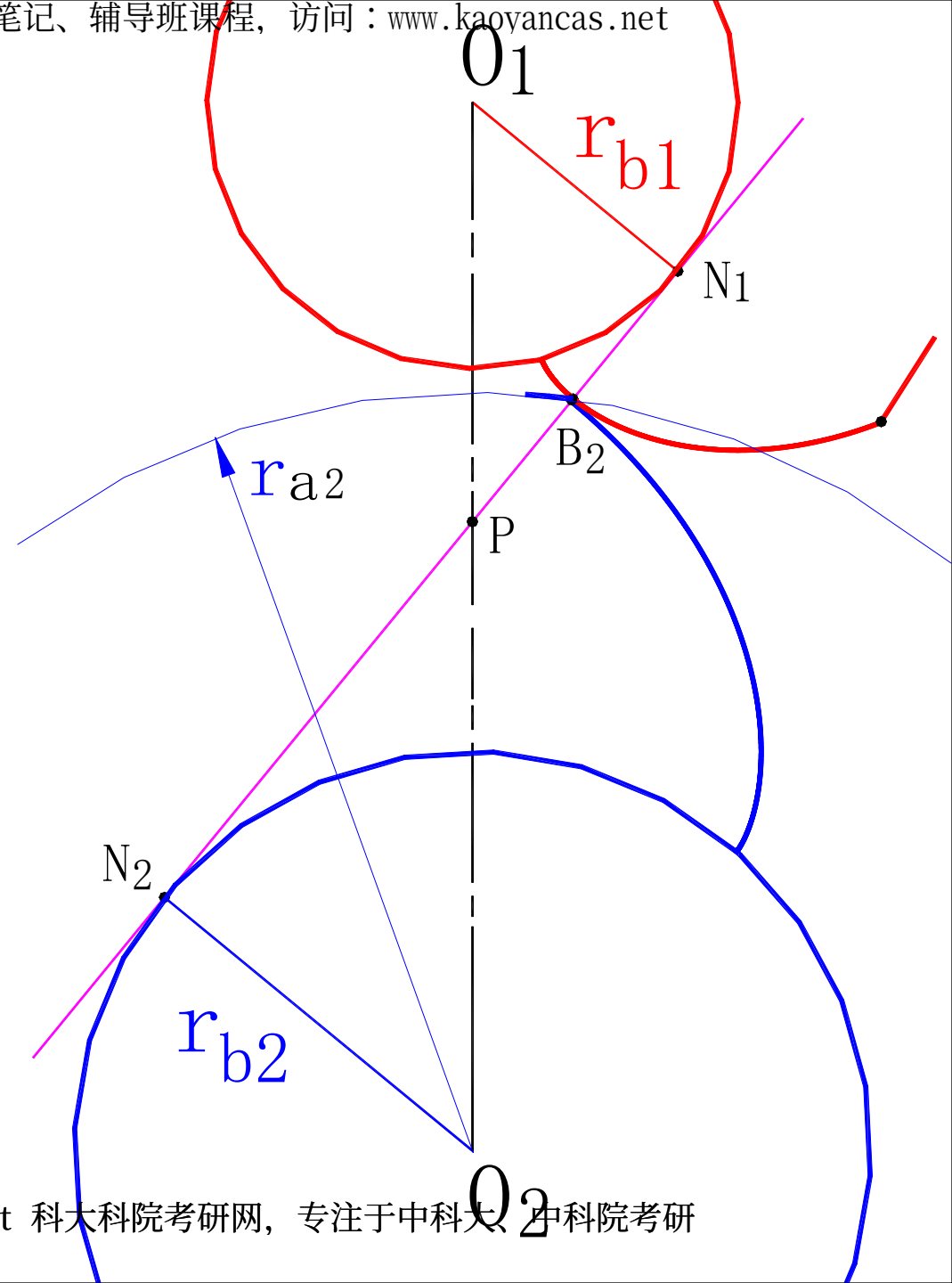


6.6.1 definition of Contact Ratio(重合度)

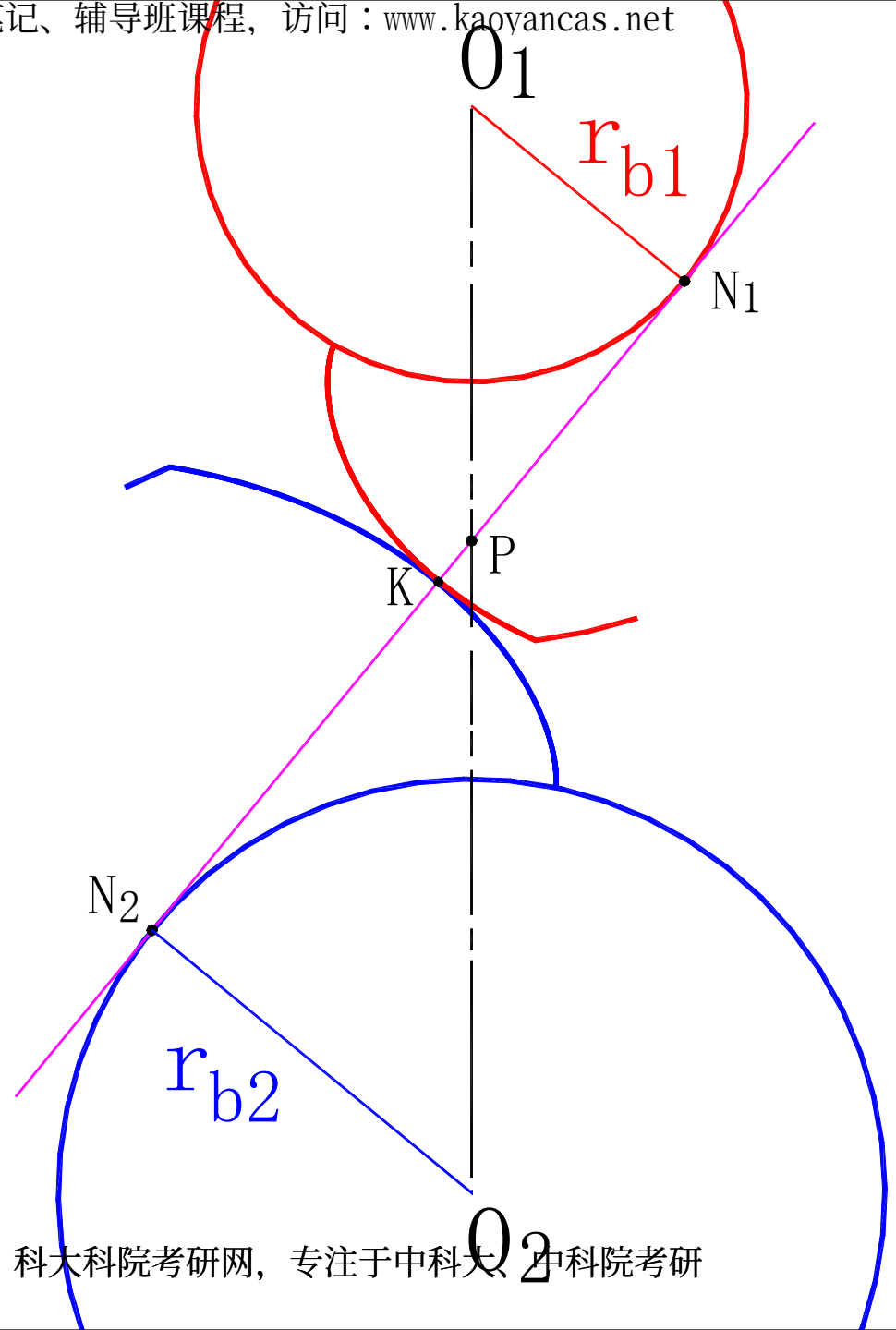
Gear 1 is a driver



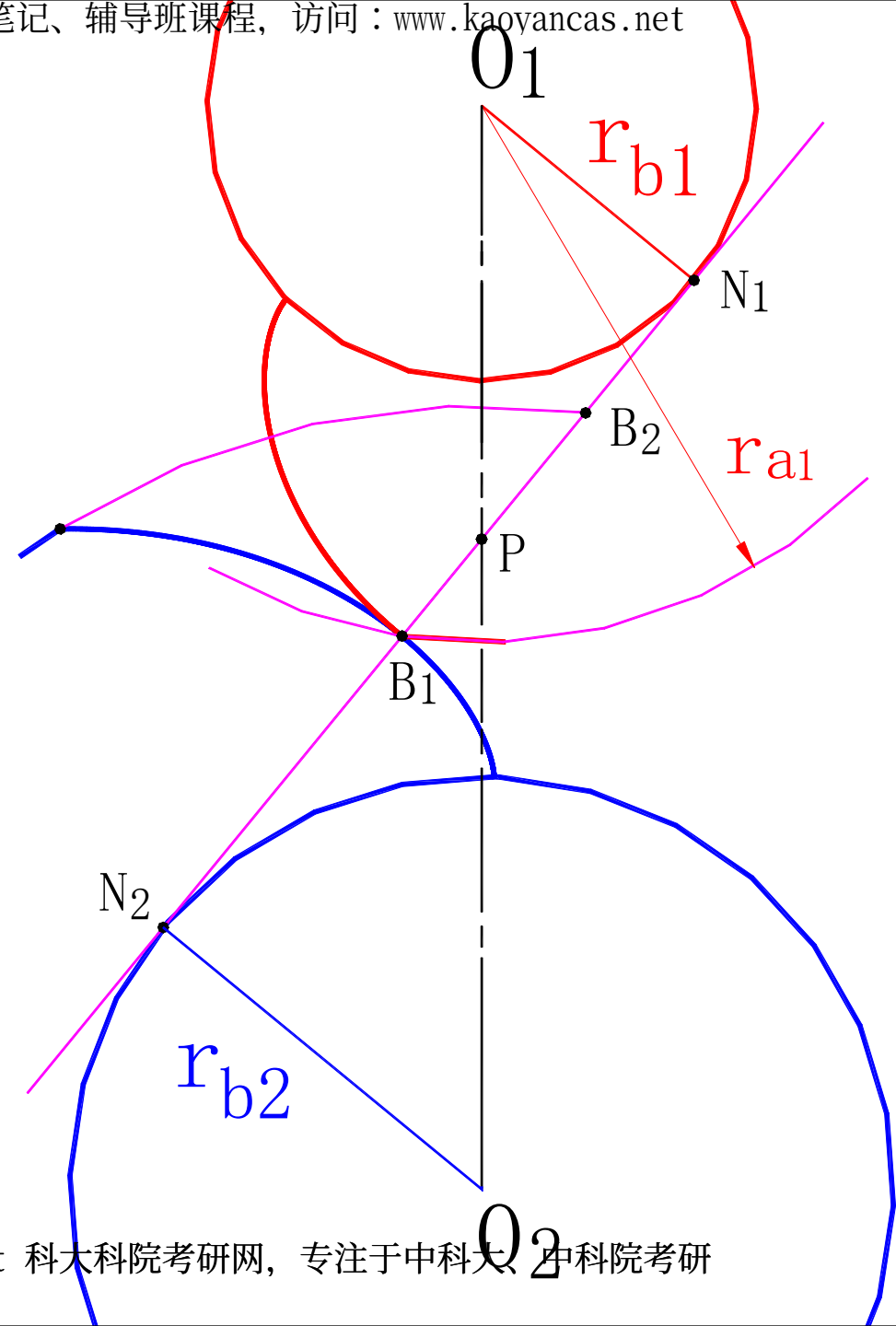
The meshing begins at point B2,



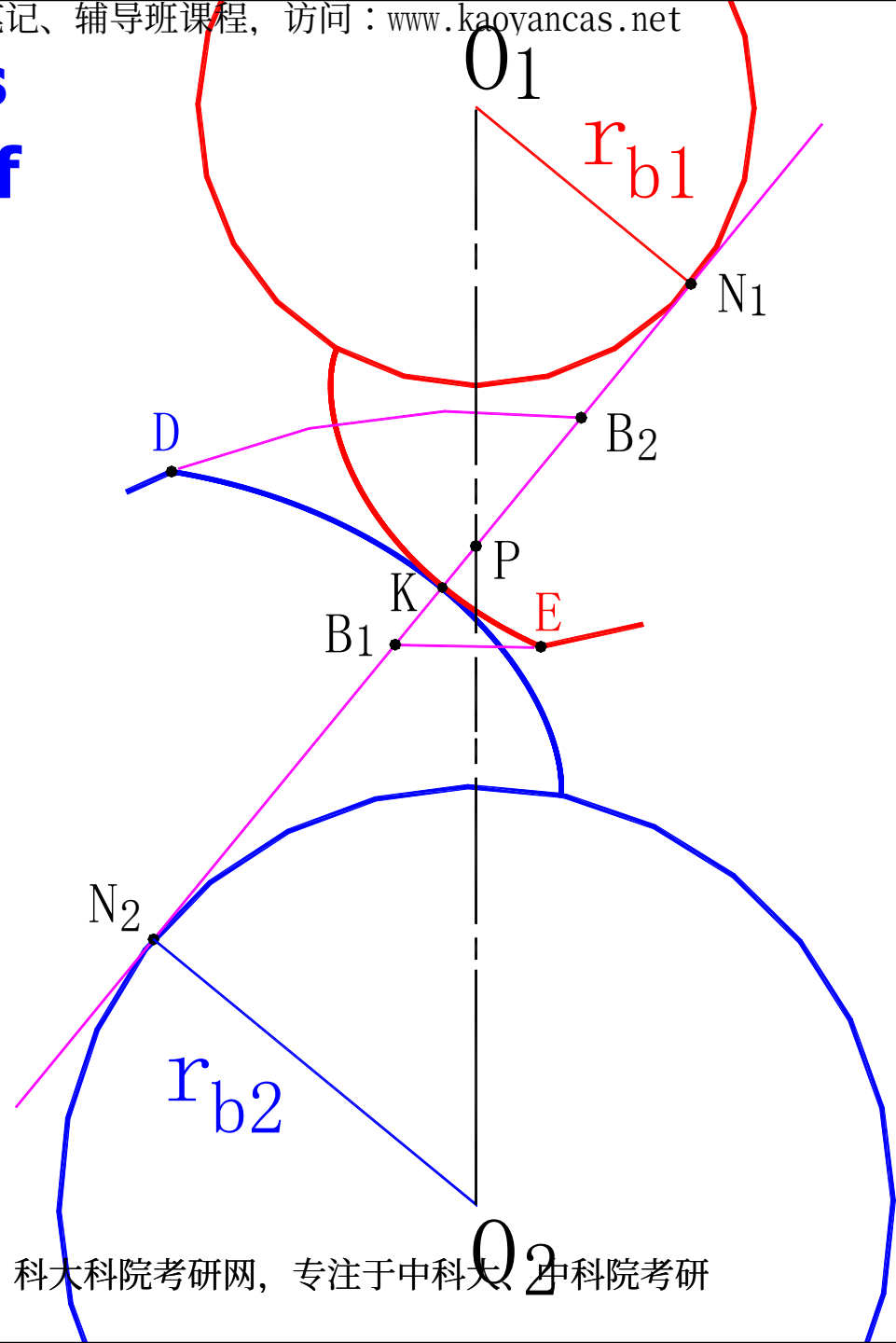
K moves along the line of action



and ends at point **B1**



The segment B_1B_2 is called the actual line of action(实际啮合线).

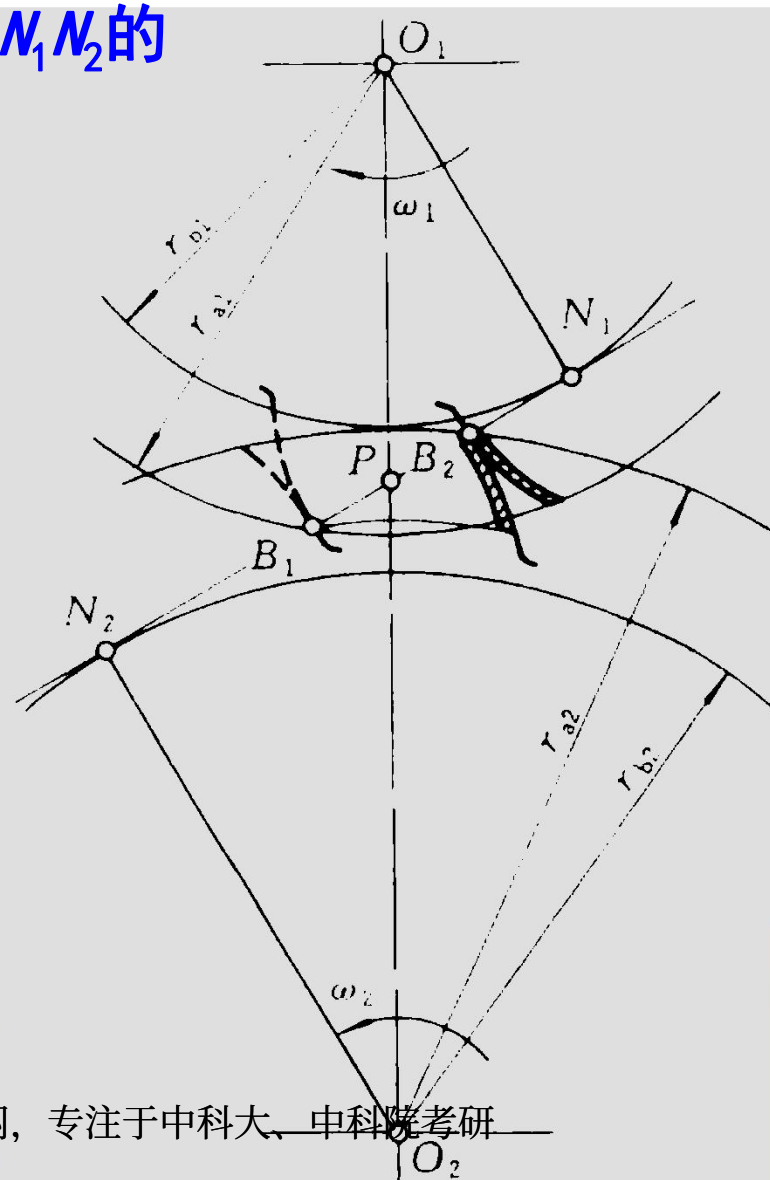


起始啮合点：

从动轮的齿顶点与主动轮的齿根处某点接触，在啮合线上为从动轮的齿顶圆与啮合线 N_1N_2 的交点 B_2 。

终止啮合点：

主动轮的齿顶点与从动轮的齿根处某点接触，在啮合线 N_1N_2 上为主动轮的齿顶圆与啮合线 N_1N_2 的交点 B_1 。



$\overline{B_1B_2}$

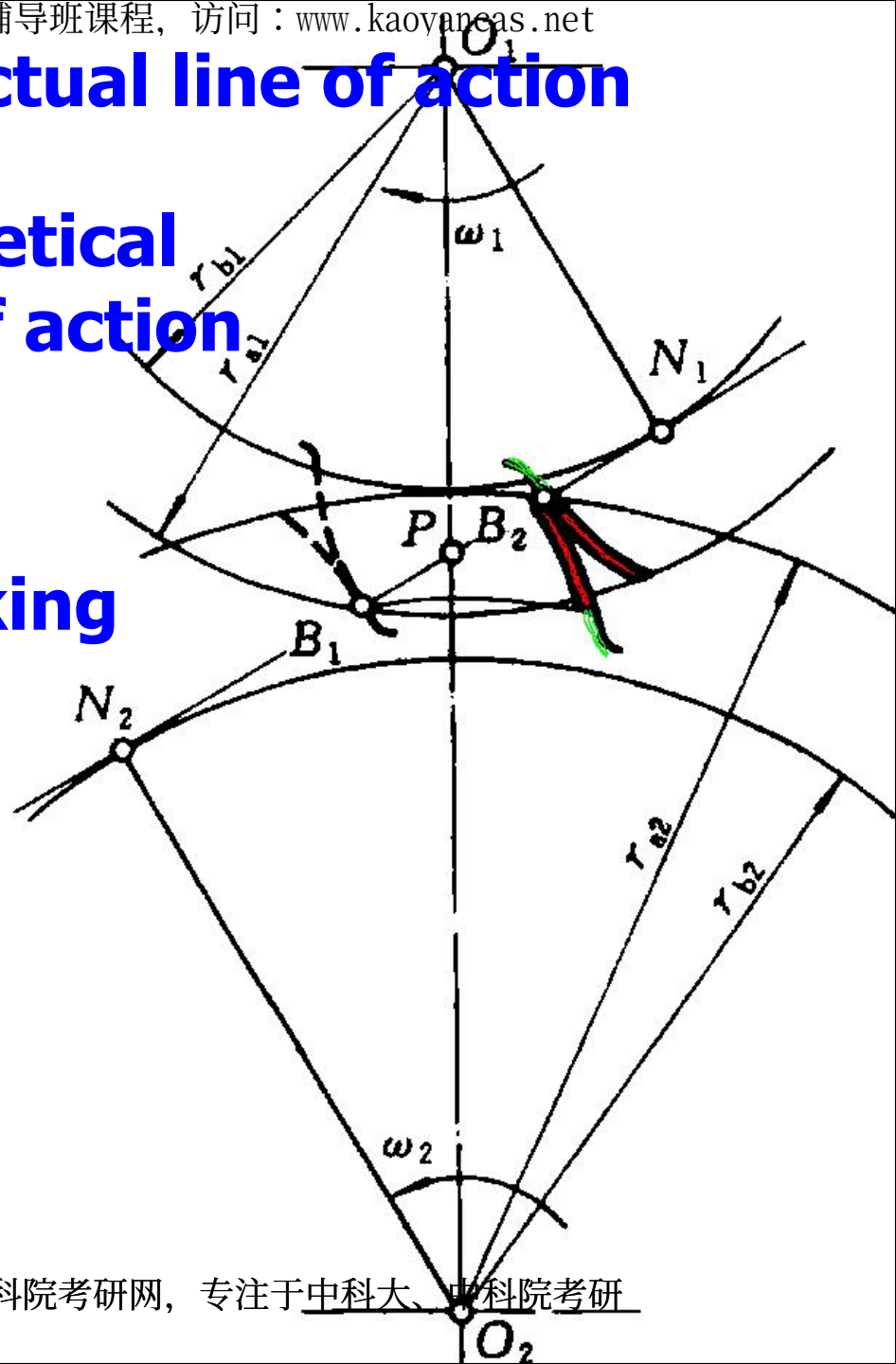
——实际啮合线the actual line of action

$\overline{N_1N_2}$

——理论啮合线theoretical
line of action

齿廓工作段 (the actual working
profile)

齿廓非工作段



If Transmission is continuous

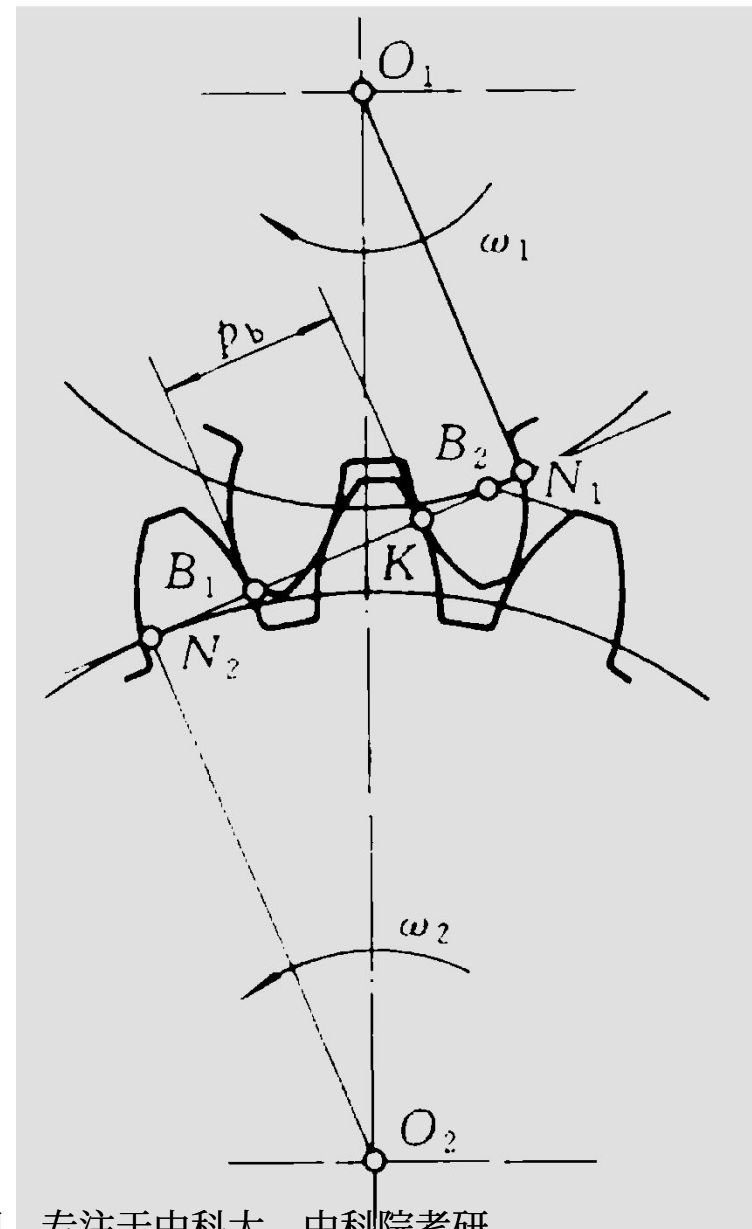
$$B_1 B_2 \geq p_b$$

Define the contact ratio as :

$$\varepsilon_\alpha = \frac{\overline{B_1 B_2}}{p_b} \geq 1$$

齿轮传动的连续性条件

Usually, $\varepsilon > [\varepsilon] = 1.2$.





6.6.2 重合度的意义

重合度表示同时参加啮合的齿对数多或多对齿啮合所占的时间比例大。

重合度不仅是齿轮传动的连续性条件，而且是衡量齿轮承载能力和传动平稳性的重要指标。

6.6.3 calculation of Contact Ratio(重合度)

$$\varepsilon_{\alpha} = \frac{B_1 B_2}{p_b} = \frac{PB_1 + PB_2}{p_b}$$

$$\overline{PB_1} = \overline{B_1 N_1} - \overline{PN_1}$$

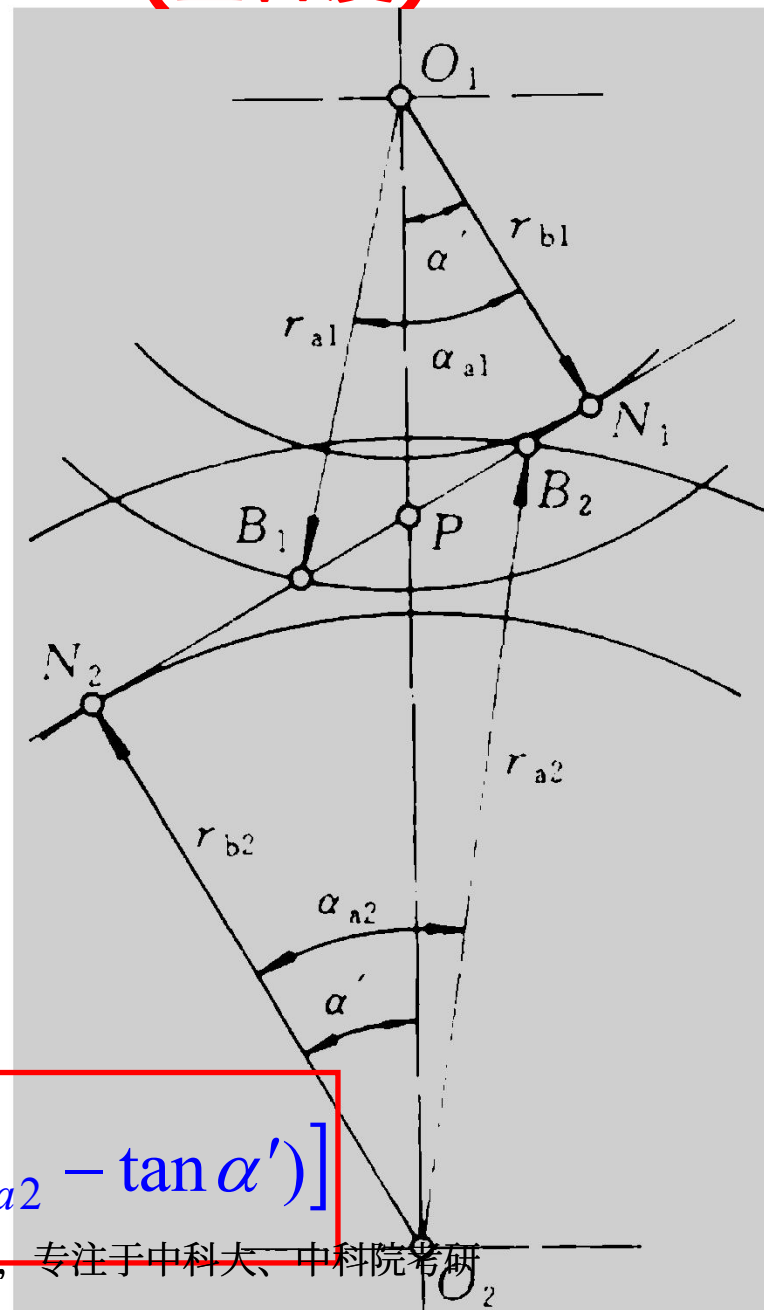
$$= r_{b1} (\tan \alpha_{a1} - \tan \alpha')$$

$$= \frac{1}{2} m z_1 \cos \alpha (\tan \alpha_{a1} - \tan \alpha')$$

$$\overline{PB_2} = \frac{1}{2} m z_2 \cos \alpha (\tan \alpha_{a2} - \tan \alpha')$$

$$p_b = m \pi \cos \alpha$$

$$\varepsilon_{\alpha} = \frac{1}{2\pi} [z_1 (\tan \alpha_{a1} - \tan \alpha') + z_2 (\tan \alpha_{a2} - \tan \alpha')]$$





For rack and pinion gear:

$$\varepsilon_{\alpha} = \frac{1}{2\pi} \left[Z_1 (\tan \alpha_{a1} - \tan \alpha) + \frac{2h_a^*}{\sin \alpha \cdot \cos \alpha} \right]$$

For internal gear pair:

$$\varepsilon_{\alpha} = \frac{1}{2\pi} \left[Z_1 (\tan \alpha_{a1} - \tan \alpha') + Z_2 (\tan \alpha' - \tan \alpha_{a2}) \right]$$

$$\varepsilon_{\alpha} = \frac{1}{2\pi} [Z_1(\tan\alpha_{a1} - \tan\alpha) + Z_2(\tan\alpha_{a2} - \tan\alpha)]$$

$$(1) \frac{a}{a'} = \frac{\cos \alpha'}{\cos \alpha} \Rightarrow \alpha'$$

$$(2) r_{b1} = r_{a1} \cos \alpha_{a1}, \quad r_{b2} = r_{a2} \cos \alpha_{a2}$$

(3) If $r_a \downarrow$, then $\alpha_a \downarrow$ and $\varepsilon_{\alpha} \downarrow$. 短齿制缺点

(4) ε_{α} is independent of module m .

(5) If $Z \uparrow$, then $\varepsilon_{\alpha} \uparrow$. Try to use smaller m .

(6) If $a' \uparrow$, then $\tan \alpha' \uparrow$ and $\varepsilon_{\alpha} \downarrow$.

Example. A pair of standard spur involute gears have the parameters as follow: $Z_1=20$, $Z_2=50$, $m=10\text{mm}$, actual centre distance $a'=350\text{mm}$, $\alpha=20^\circ$, $h_a^*=1$, $c^*=0.25$. Calculate the contact ratio ε_α

Solution:

$$\text{Since } a = \frac{m(Z_1 + Z_2)}{2} = \frac{10(20 + 50)}{2} = 350 = a',$$

$$\alpha' = \alpha = 20^\circ.$$

$$r_{a1} = r_1 + h_a^* m = 20 + 1 * 10 = 30$$

$$r_{a2} = r_2 + h_a^* m = 50 + 1 * 10 = 60$$

$$r_{b1} = r_1 \cos \alpha = 20 * \cos 20^\circ = 18.7938$$

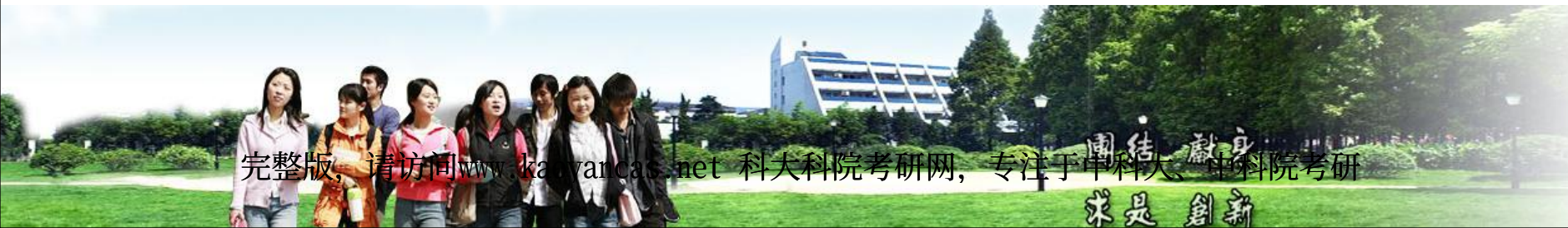
$$r_{b2} = r_2 \cos \alpha = 50 * \cos 20^\circ = 46.9846$$



$$\alpha_{a1} = \cos^{-1}\left(\frac{r_{b1}}{r_{a1}}\right) = \cos^{-1}\left(\frac{18.7938}{22}\right) = 31.3215^\circ$$

$$\alpha_{a2} = \cos^{-1}\left(\frac{r_{b2}}{r_{a2}}\right) = \cos^{-1}\left(\frac{46.9846}{52}\right) = 25.3713^\circ$$

$$\begin{aligned}\varepsilon_\alpha &= \frac{1}{2\pi} [Z_1(\tan\alpha_{a1} - \tan\alpha) + Z_2(\tan\alpha_{a2} - \tan\alpha)] \\ &= \frac{1}{2\pi} [20 \times (\tan 31.3215^\circ - \tan 20^\circ) + 50 \times (\tan 25.3713^\circ - \tan 20^\circ)] \\ &= 1.6558\end{aligned}$$



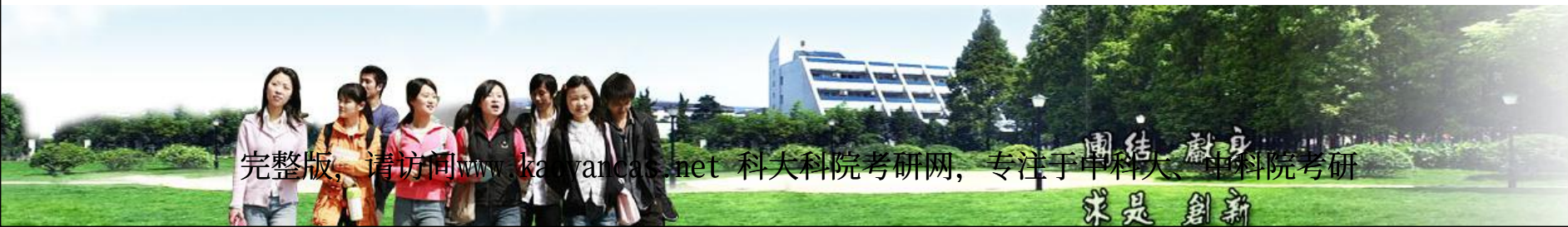
6.7 Manufacturing Methods of Involute Profiles

There are various methods for manufacturing tooth profiles, such as die casting(模铸), precision forging(锻造), punch冲压, powder(粉末) process, cutting切削法 and so on.

Cutting is the most common method for manufacture of gears.

6.7.1 Cutting of Tooth Profiles

- (1) Form (仿形法) Cutting
- (2) Generating (展成法或范成法) Cutting



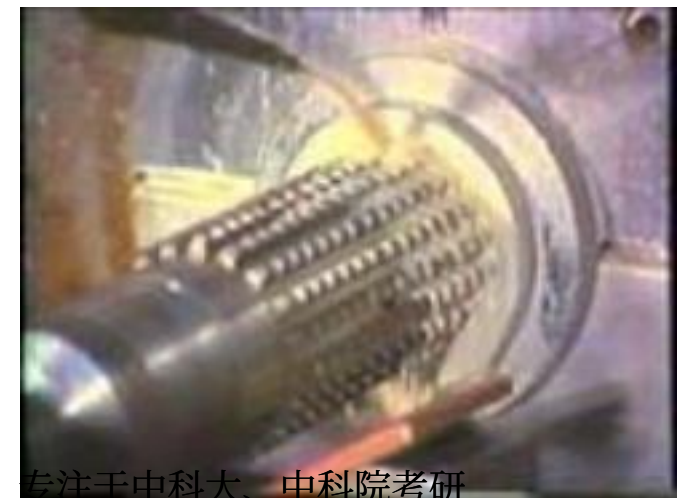
((1))Form Cutting

Disk milling cutter(盘形铣刀)



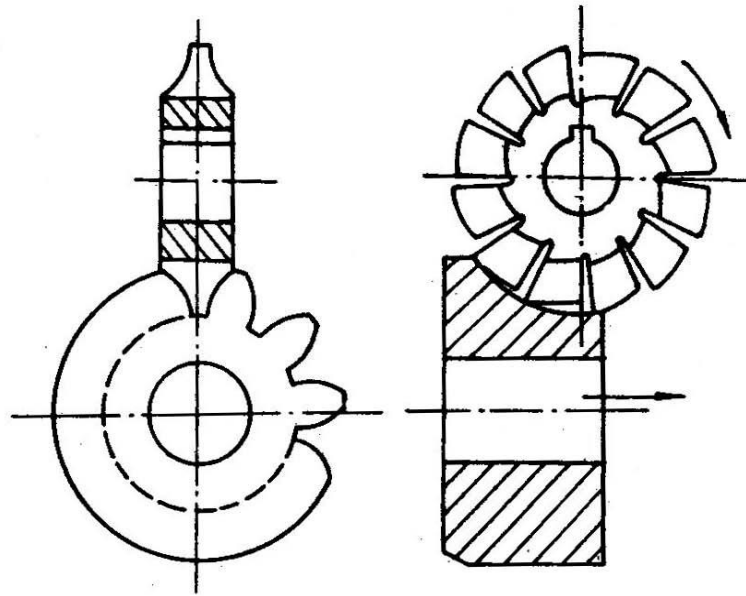
End milling cutter(指状铣刀)

Broach(拉刀)



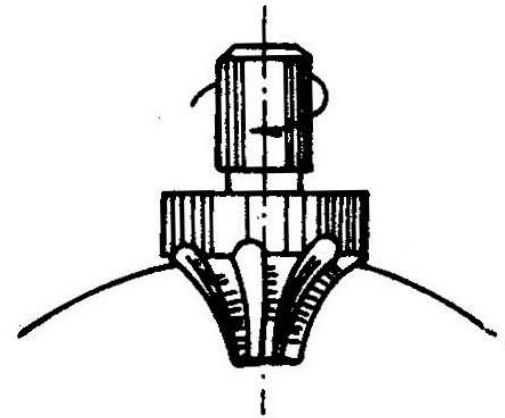


Disk milling cutter



(a) 盘状铣刀

End milling cutters(端铣刀) are used for large modules and double helical gears.



(b) 指状铣刀



broach(拉刀)



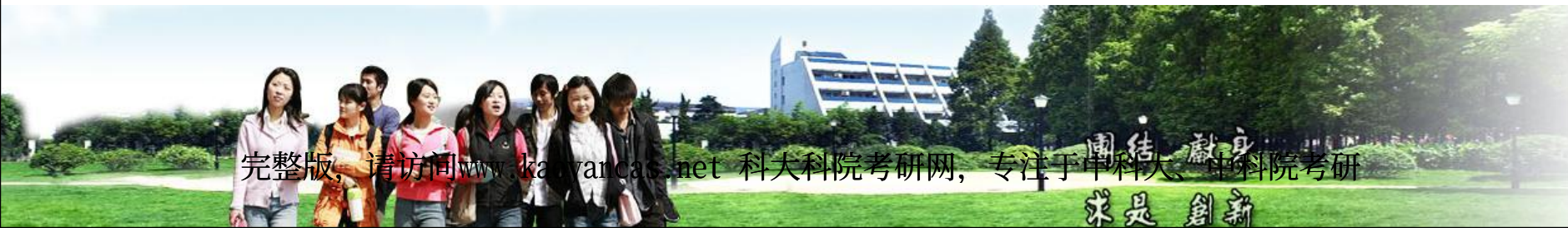


Characteristics of the form cutting:

As the shape of an involute depends on its base radius, a different cutter is required for each number of teeth even for the same module and same pressure angle.

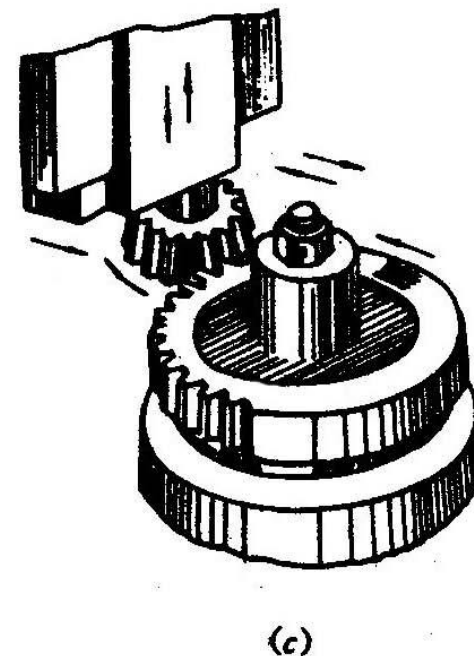
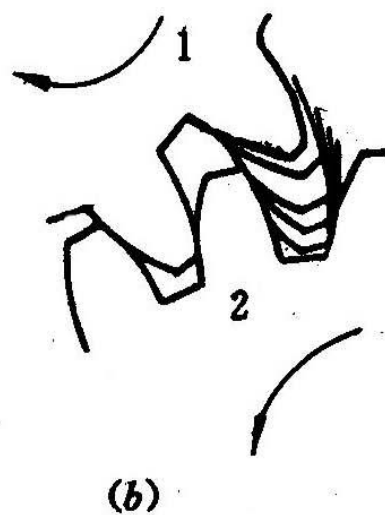
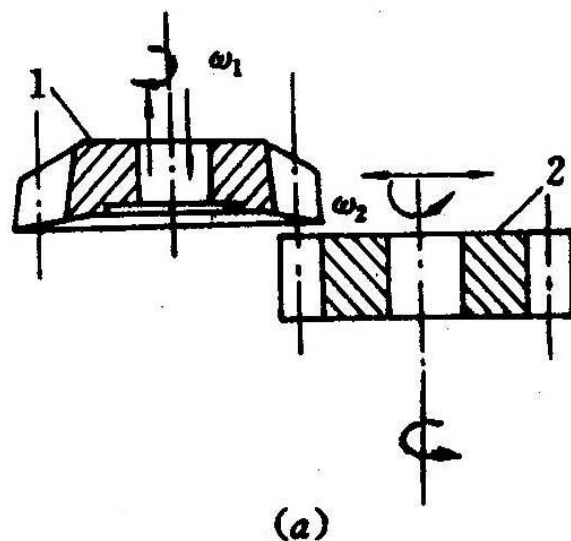
Form cutting is not a good method.

Its main advantage is that it can be accomplished on commonly available milling machines. (铣床)



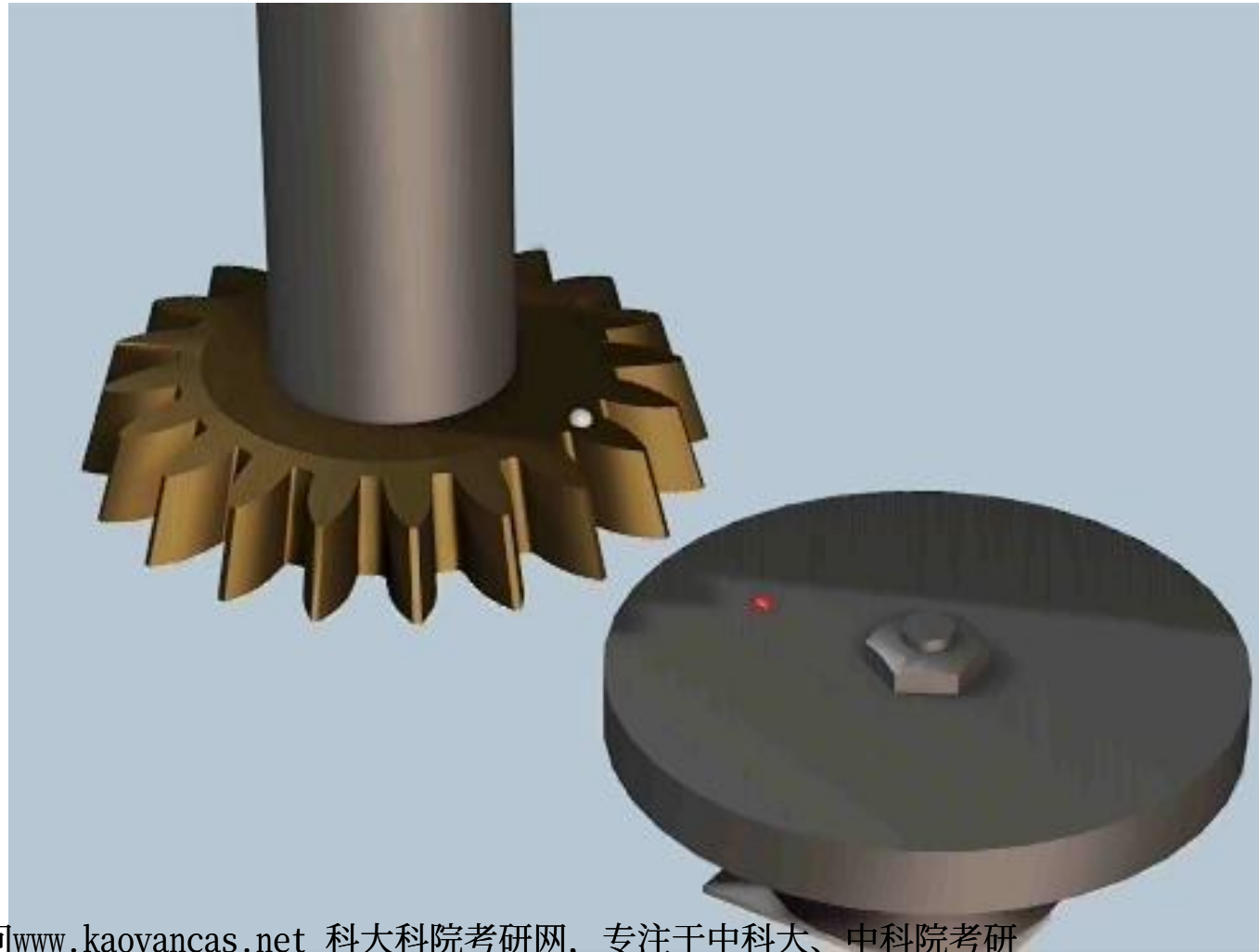


(2) Generating Cutting (展成法或范成法加工)

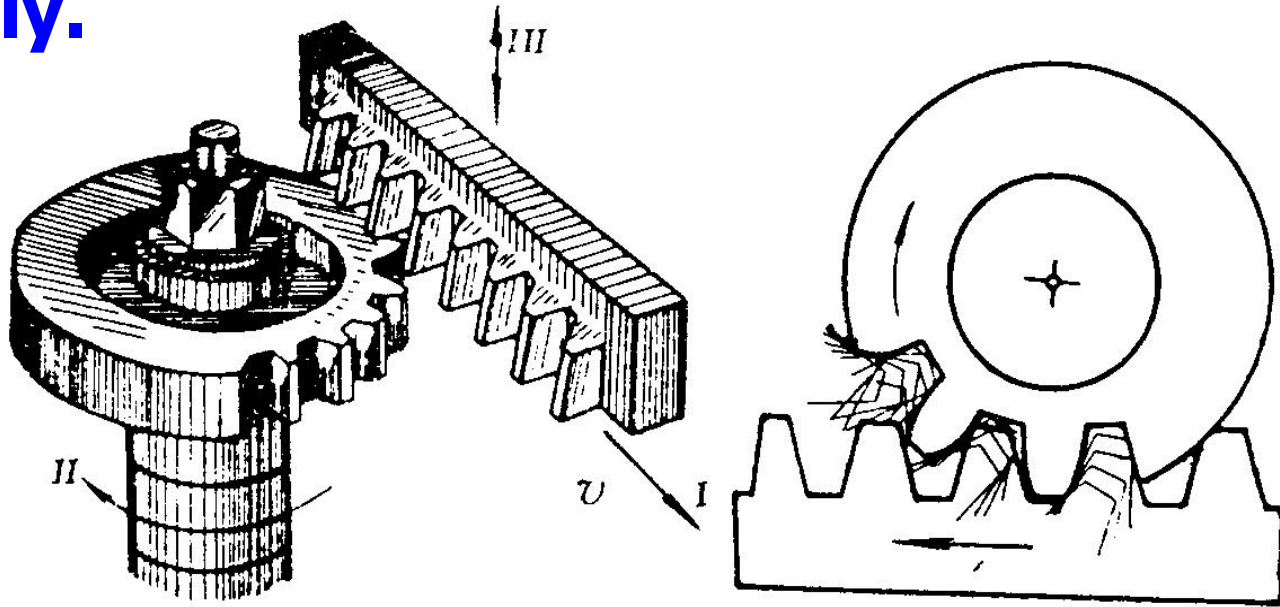


In a generating cutting method, the edges of a cutter take the form of a gear (or rack) with the same module and pressure angle as the gear to be cut.

- 1. Cutting motion**
- 2. Retreat motion** 退刀
- 3. Generating motion** 展成
- 4. Feed motion** 进给



External gears can be cut with a rack-shaped shaper cutter(齿条插刀). The edges of the cutter are now straight lines which can be made accurately.

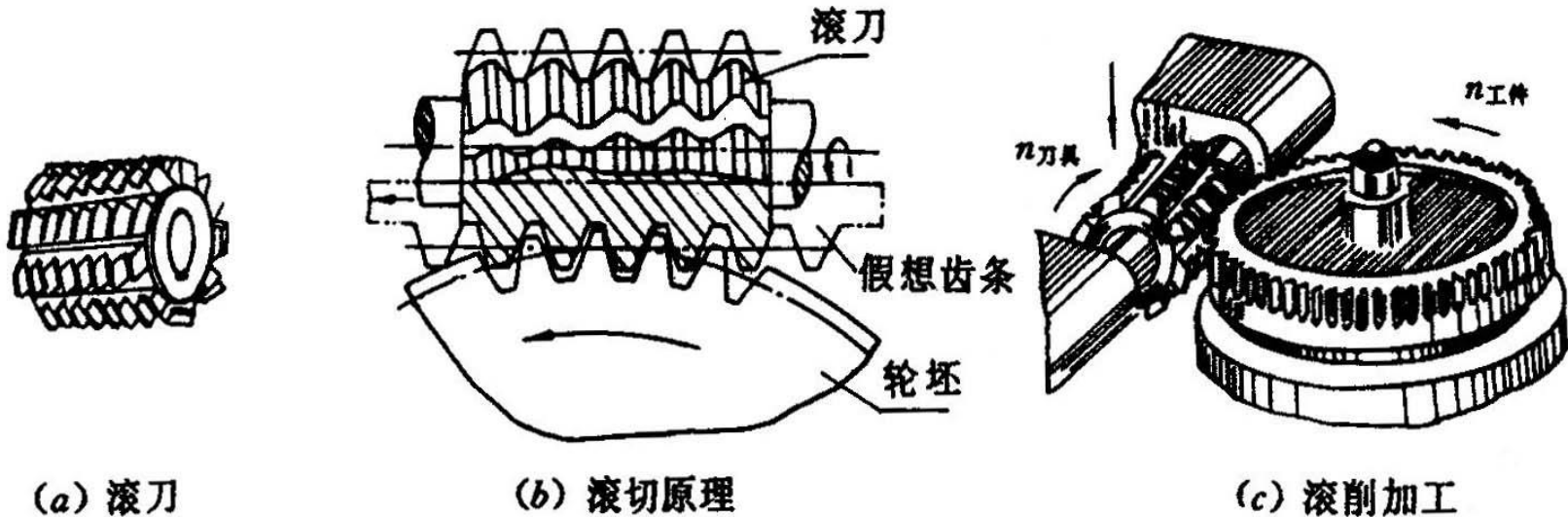


The cutting process in the shaping(插齿) of a gear is not continuous.

优点：用一把插刀可以加工出 m 、 α 相同而齿数不同的各种齿轮(包括内齿轮)。

缺点：切削不连续，生产效率较低。

In mass production(大规模生产), hobbing(滚齿) is used instead of shaping.



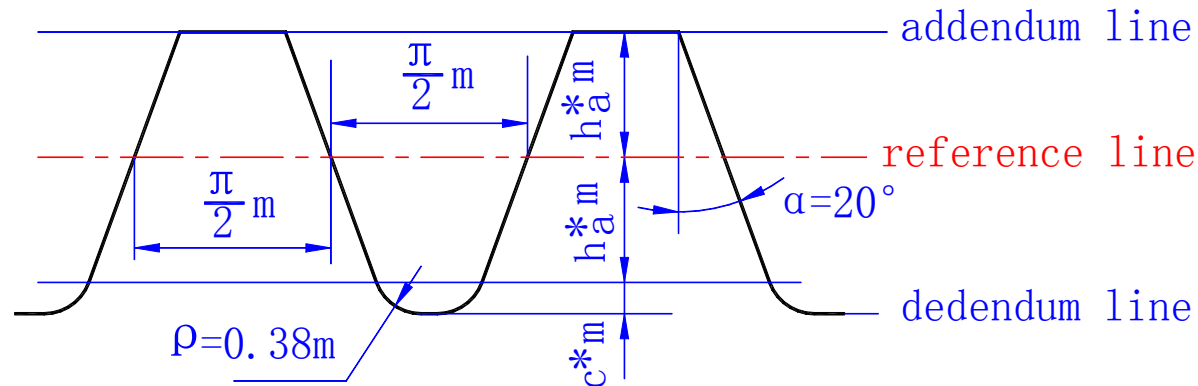
hobbing(滚齿)

优点：用一把滚刀可以加工出 m 、 α 相同而齿数不同的各种齿轮，切削连续，生产效率高。

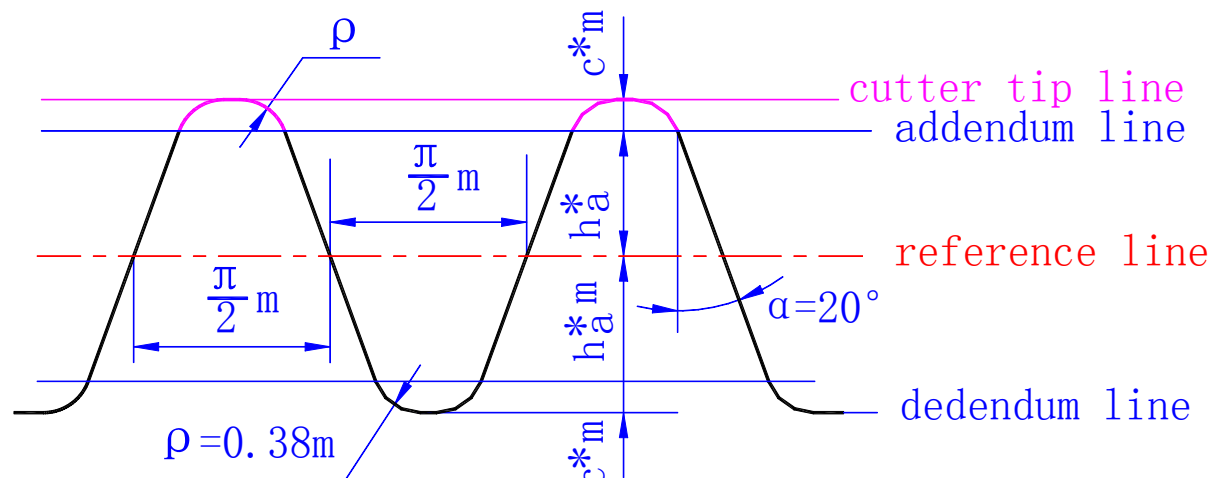
缺点：不能加工内齿轮。

6.7.2 Cutting a Standard Gear with Standard Rack-shaped Cutter

普通齿条

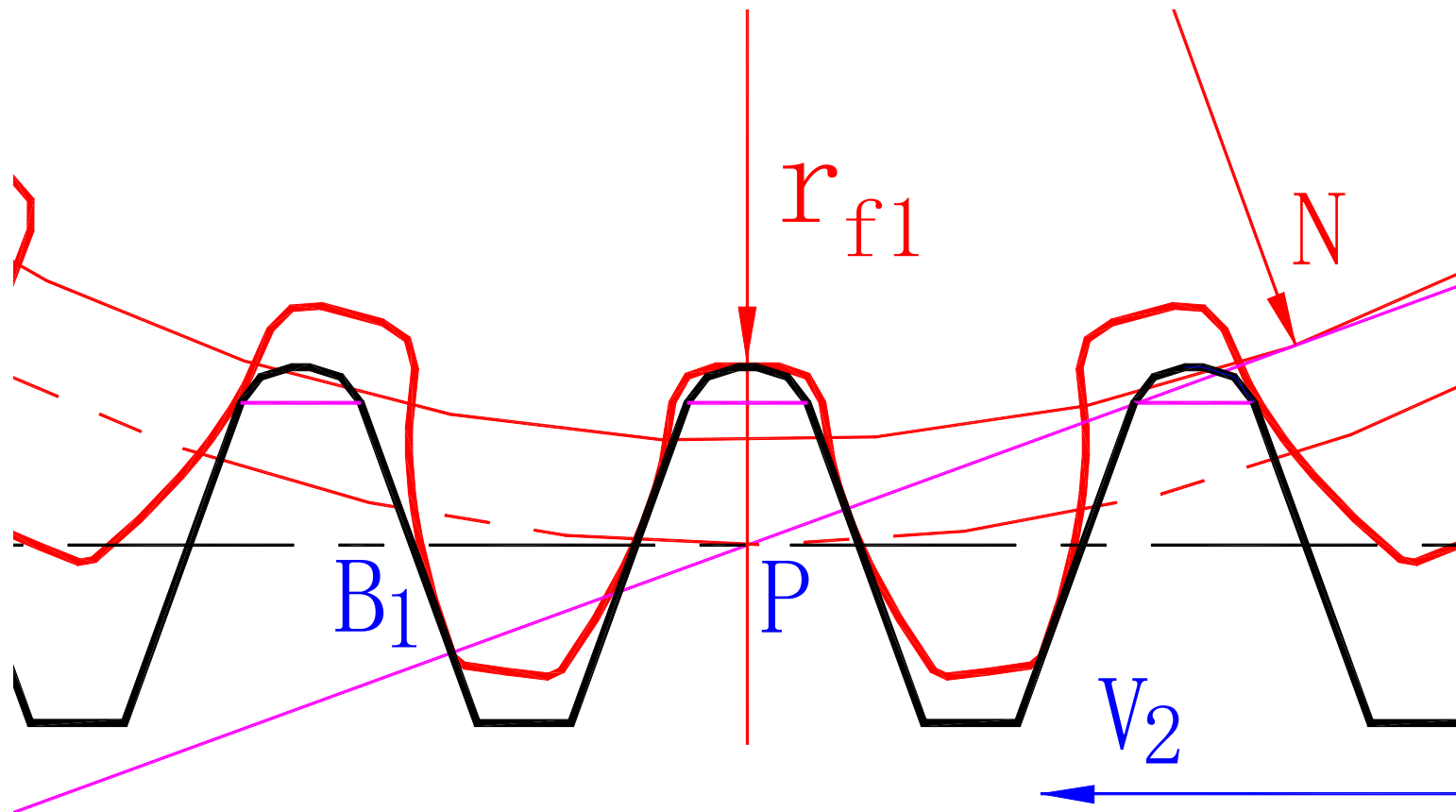


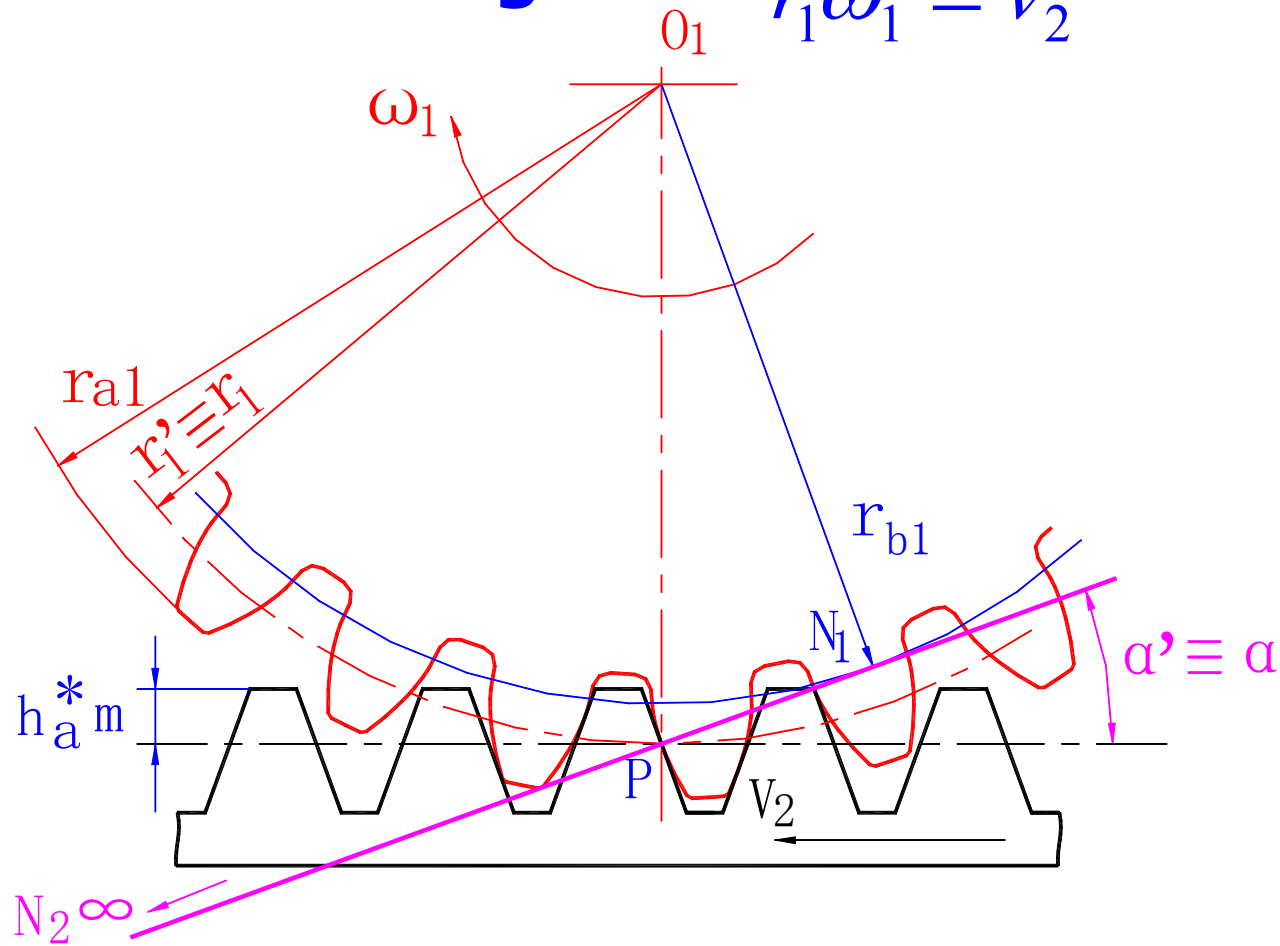
齿条插刀



The tip line is $c*m$ higher than the addendum line for cutting the profile at the root to provide h_f

$$h_f = h_a^* m + c^* m$$

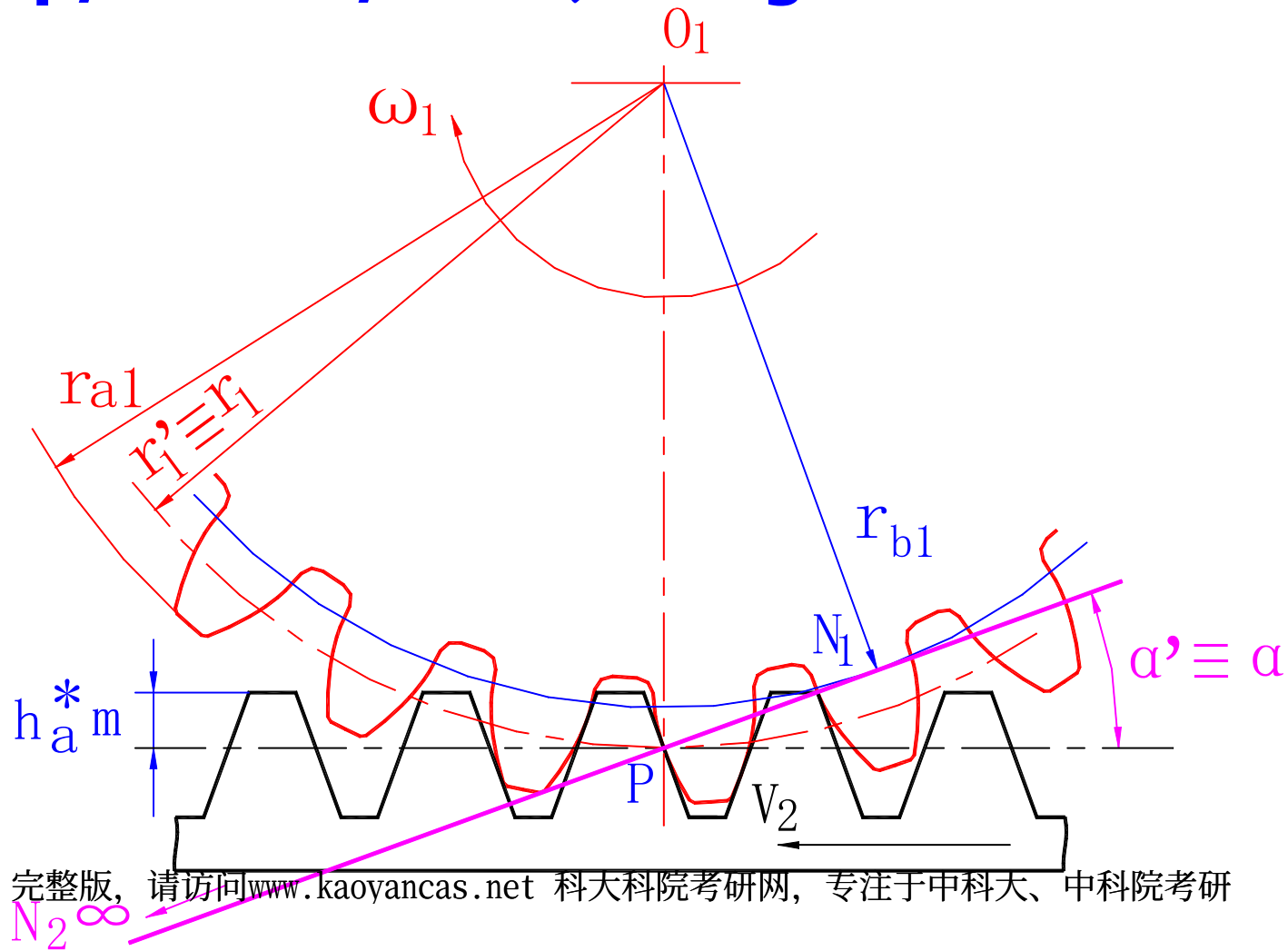


$$r_1 \omega_1 \equiv v_2$$


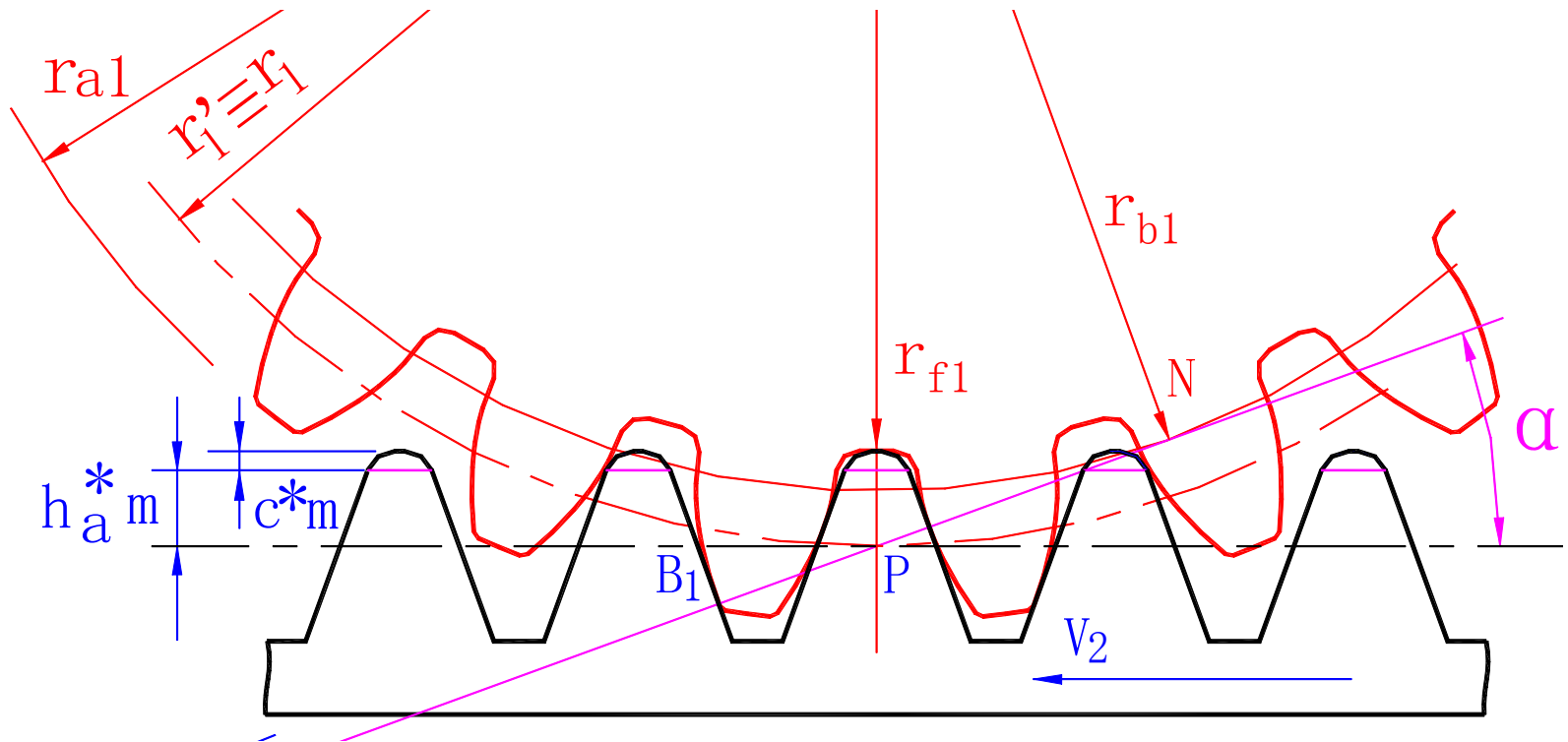
The m, α of the gear on the reference circle

are the same as those on the reference line.

As the s_2 and e_2 of the cutter on the reference line are equal to each other, the gear is cut with $s = e = p/2 = \pi m/2$. So, the gear is standard.



The dedendum circle of the gear is cut by the tip line of the cutter. $r_{f1} = r_1 - (h_a^* + c^*)m$



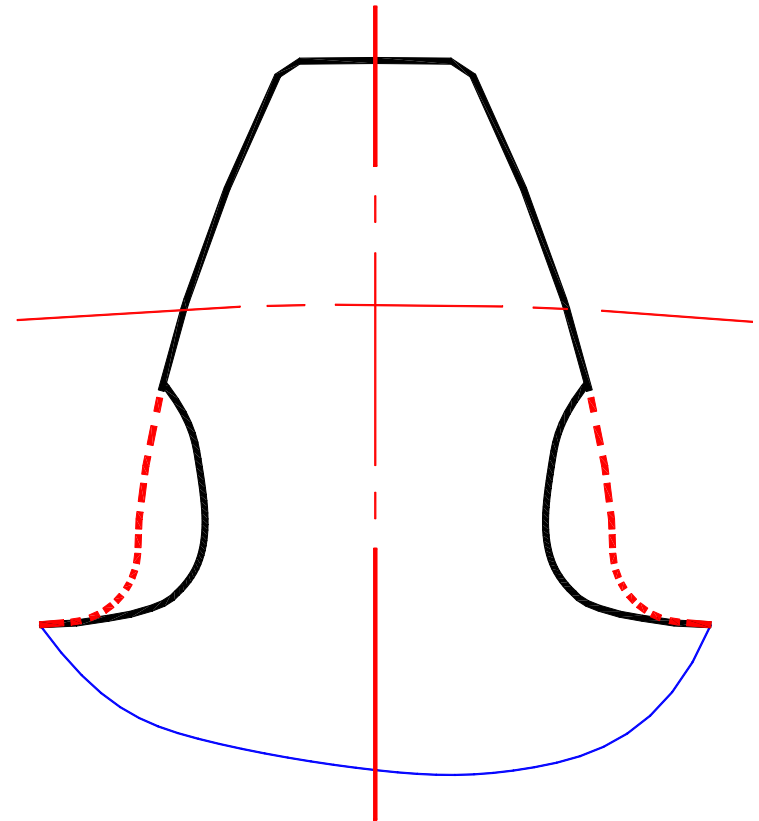
将轮坯的外圆按被切齿轮的齿顶圆直径预先加工好。

The addendum circle of the gear is cut on a lathe(车床). It is not cut by gear cutter!

6.7.3 Cutter Interference(根切现象)

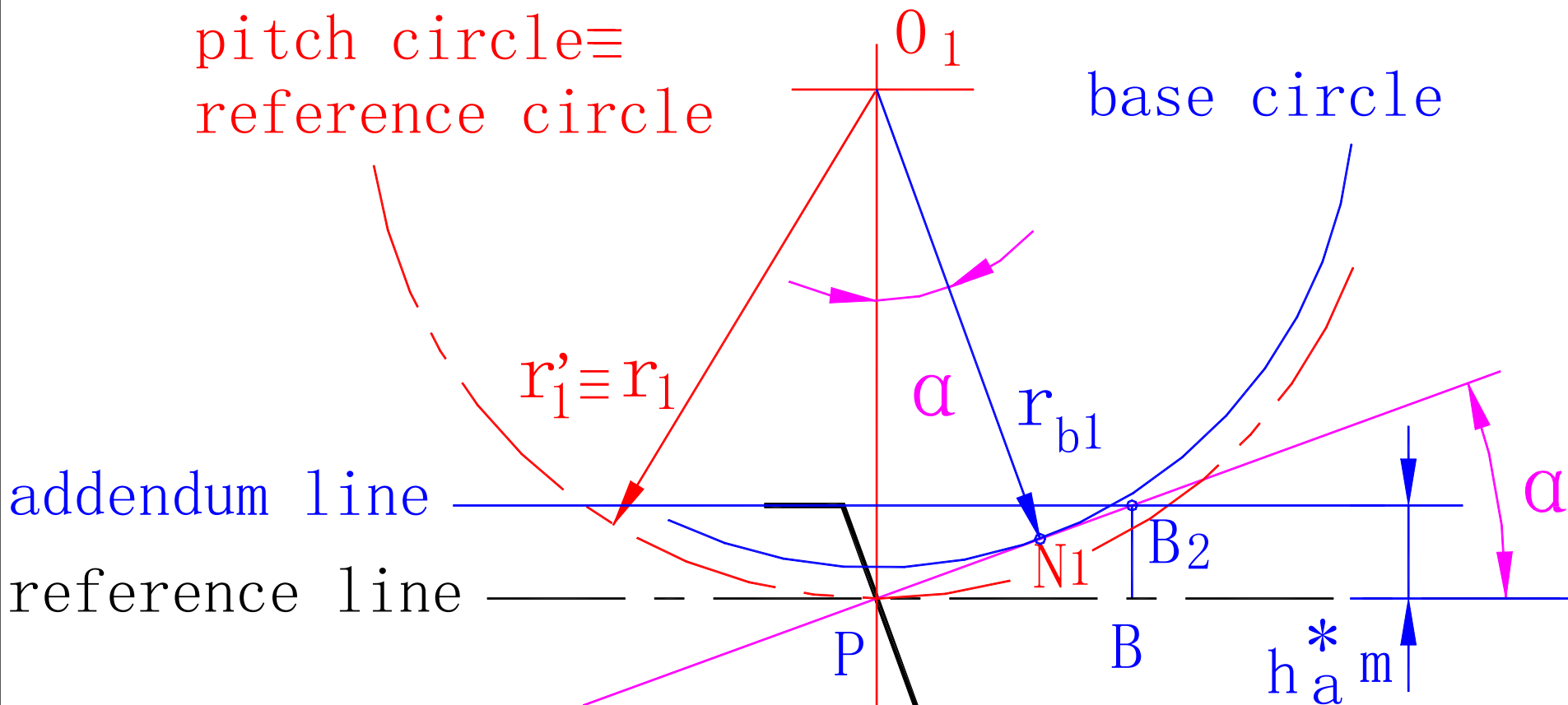
Some part of the involute profile near the root portion is removed in the generating cutting.

This will reduce the contact ratio as well as the strength of the tooth and should be avoided or minimized.



The reason for cutter interference:

The addendum line of the cutter passes the limit point N_1 of the line of action.



To prevent cutter interference :

$$h_a^* m \leq \overline{NM}$$

$$\overline{NM} = \overline{PN} \cdot \sin \alpha = r \sin^2 \alpha = \frac{mz}{2} \sin^2 \alpha$$

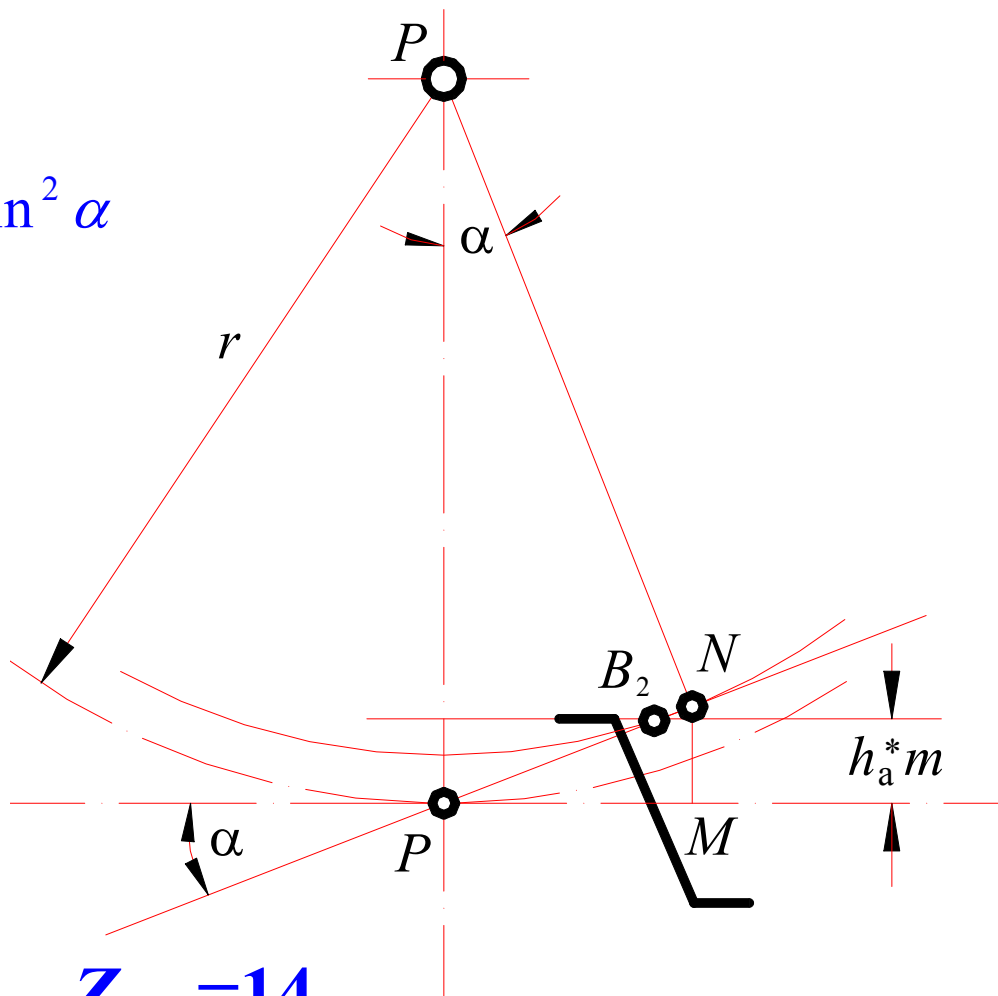
$$h_a^* m \leq \frac{mz}{2} \sin^2 \alpha$$

$$z \geq \frac{2h_a^*}{\sin^2 \alpha}$$

$$\alpha = 20^\circ \quad h_a^* = 1$$

$$Z_{\min} = 17$$

短齿制： $h_a^* = 0.8$, $\alpha = 20^\circ$, $Z_{\min} = 14$



6.8 Addendum Modification on Involute Gears 齿轮变位

6.8.1 Introduction of Addendum Modification

Disadvantages of standard gears:

1. $Z \geq Z_{\min}$

2. $a' = a$ $a' \neq a$ ——外啮合 $a' < a$ 无法安装;

$$a' > a, \varepsilon \downarrow$$

3. The strength of the pinion is much lower than that of the gear.

Corrected gears变位齿轮

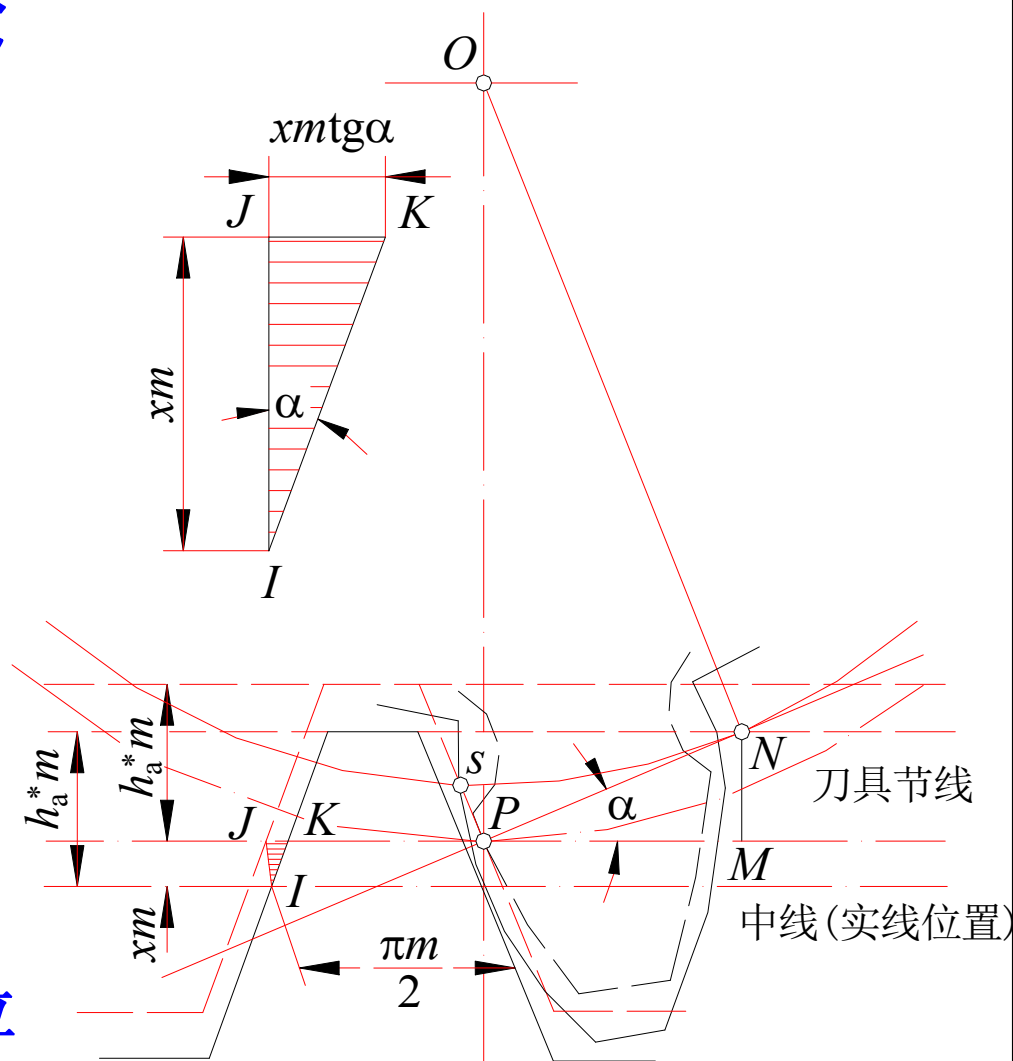
1、用改变刀具与轮坯径向相对位置来切制齿轮的方法称径向变位法。

变位齿轮

xm ——移距或变位

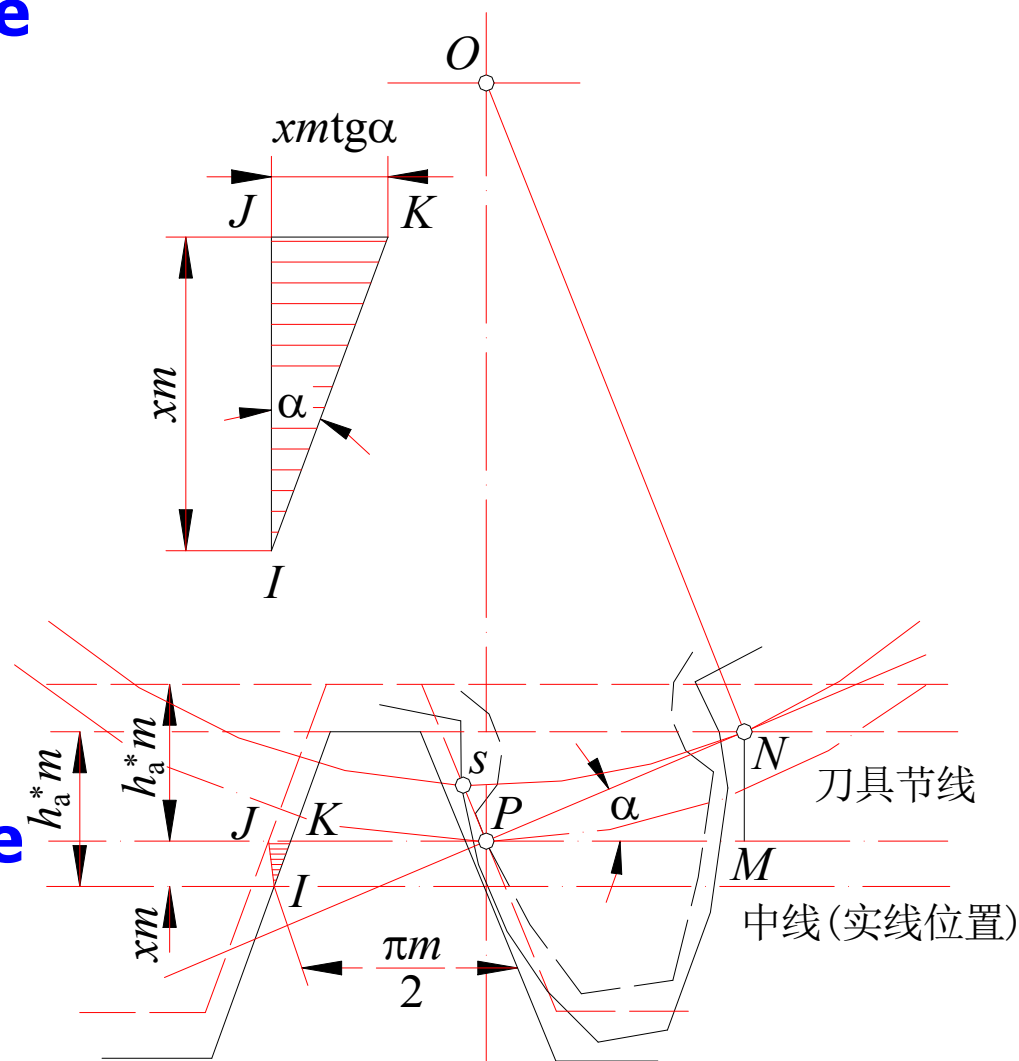
x ——移距系数或变位系数
modification coefficient

规定： $x > 0$ $x = 0$ $x < 0$
正变位 零变位 负变位



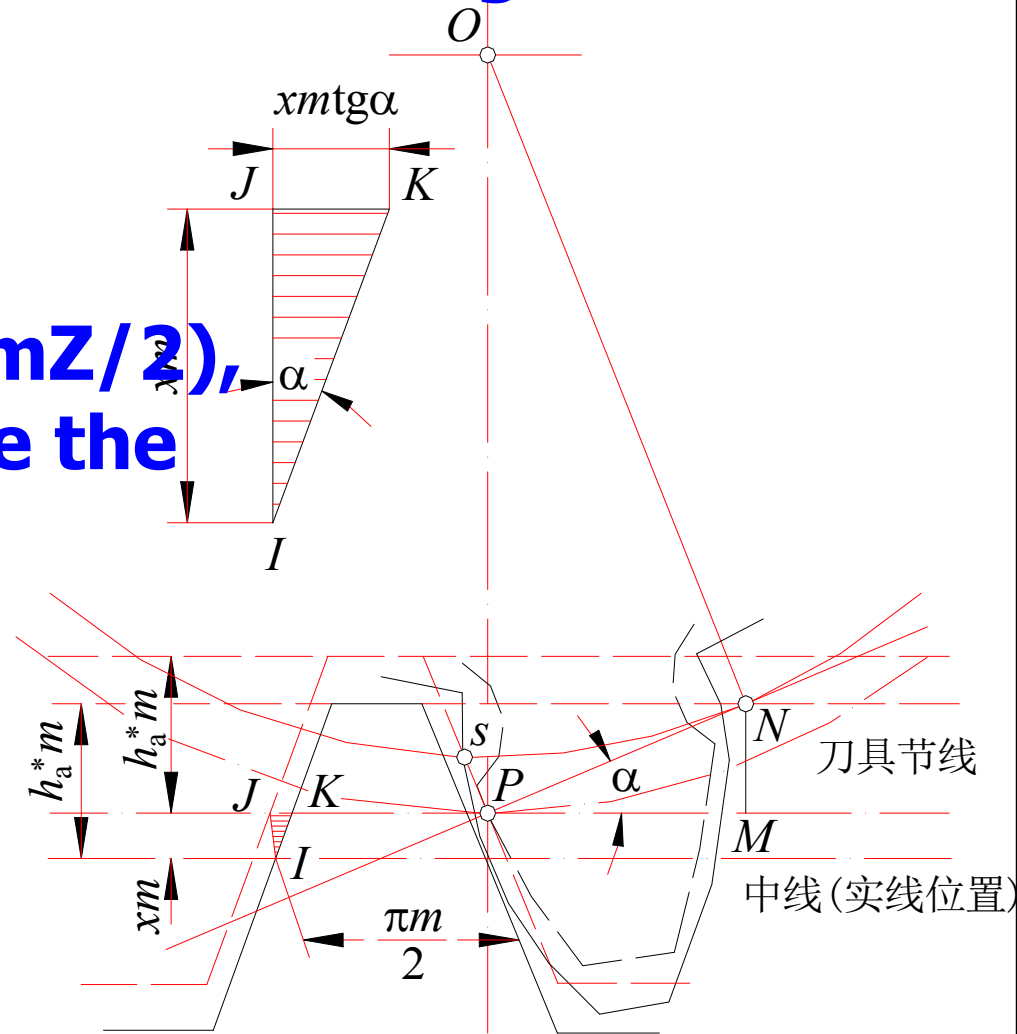
Only the position of the cutter may be changed----the reference line of the cutter may not be tangent to the reference circle of the gear.

The gear-cutting machines, the cutters and the transmission ratio between the cutter and the blank remains unchanged.

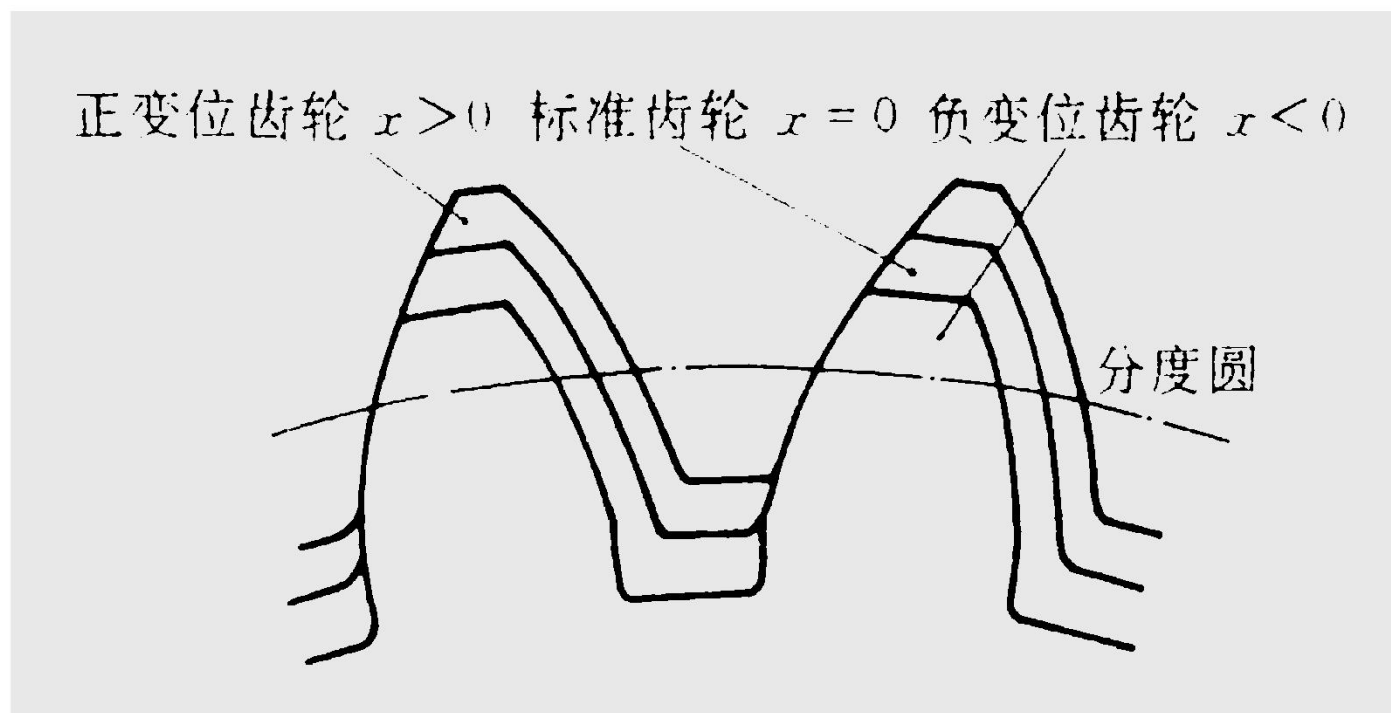


Comparison between a corrected gear and the standard gear:

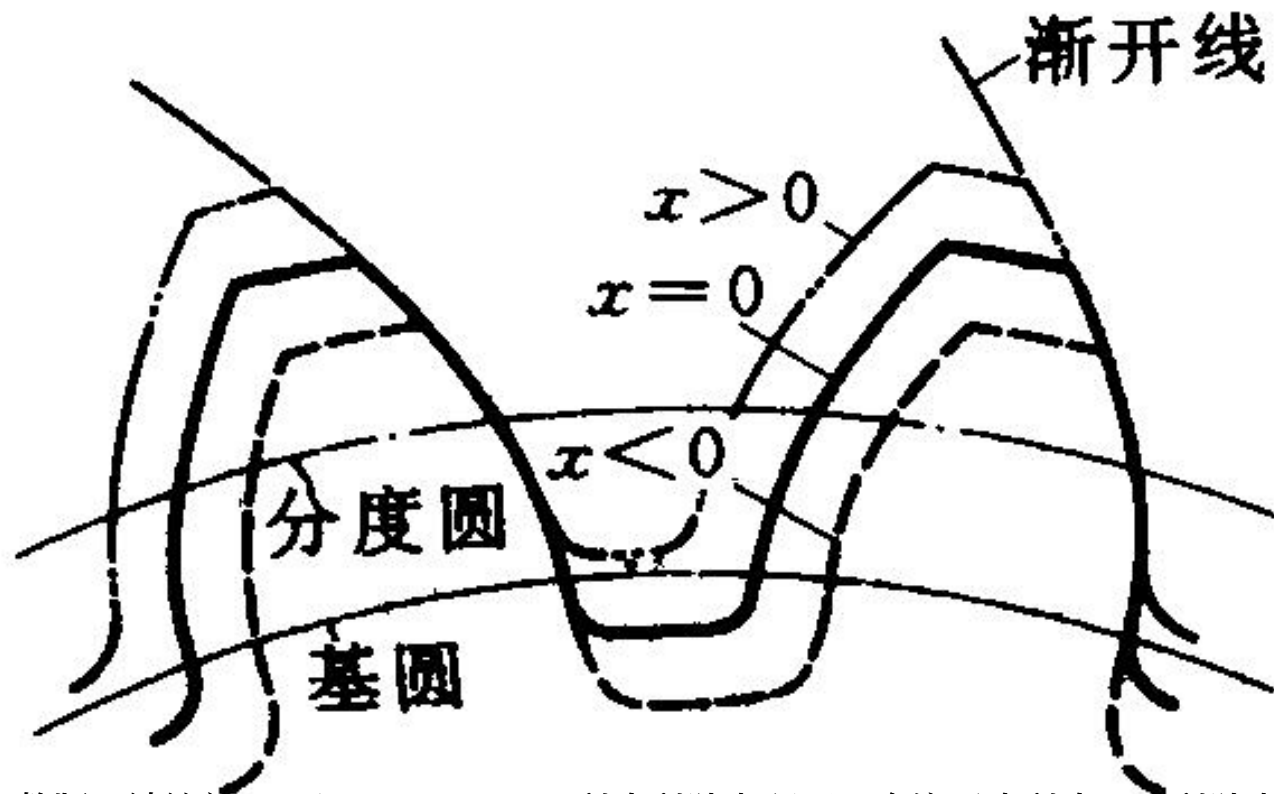
1. Parameters Z , m , $r(=mZ/2)$, α , and $r_b=(r \cos\alpha)$ are the same.



2. Parameters d_f , d_a , s , e may change.



This means that different portions of the same involute are employed for the profiles of the standard gear and the corrected gear



6.8.2 Geometric Dimensions of Corrected Gears

To prevent cutter interference: 最小变位系数

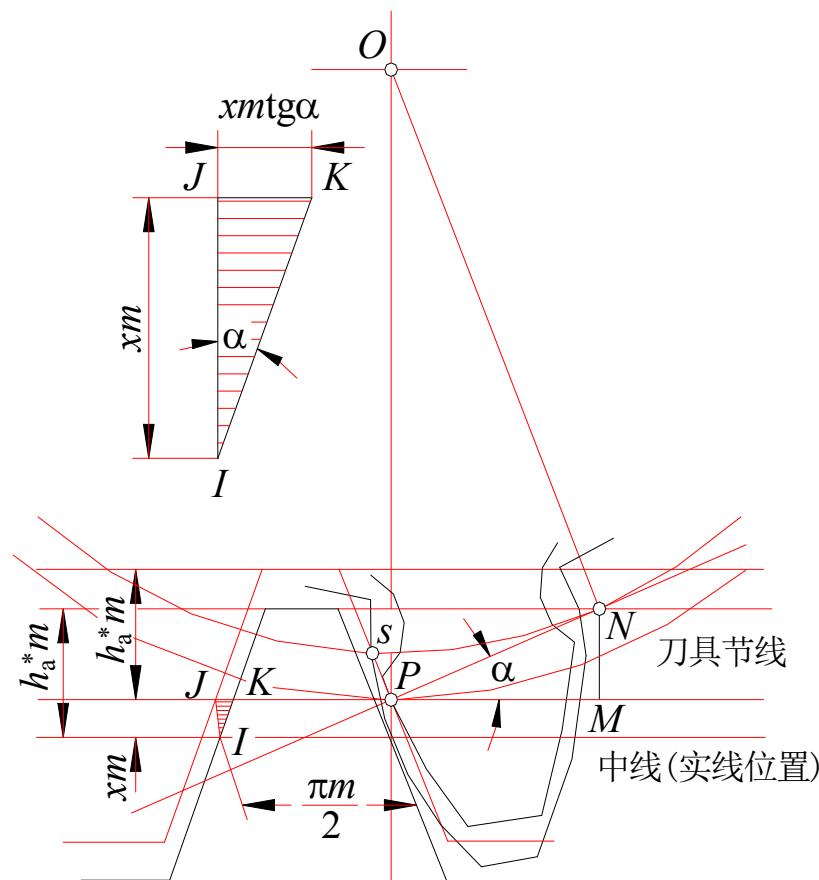
$$\left. \begin{aligned} h_a^* m - xm &\leq \overline{NM} \\ \overline{MN} &= \overline{PN} \sin \alpha = r \sin^2 \alpha = \frac{mZ}{2} \sin^2 \alpha \end{aligned} \right\}$$

$$x \geq h_a^* - \frac{Z}{2} \sin^2 \alpha$$

$$\left(Z_{\min} = \frac{zh_a^*}{\sin^2 \alpha} \right) \longrightarrow \frac{\sin^2 \alpha}{2} = \frac{h_a^*}{Z_{\min}}$$

$$x_{\min} = h_a^* \cdot \frac{Z_{\min} - Z}{Z_{\min}} \quad h_a^* = 1 \quad Z_{\min} = 17$$

$$x_{\min} = \frac{17 - Z}{17}$$



$Z < Z_{\min}$ $x_{\min} > 0$ 正变位 $x \geq x_{\min}$ $Z > Z_{\min}$ $x_{\min} < 0$ 负变位 $x \geq x_{\min}$

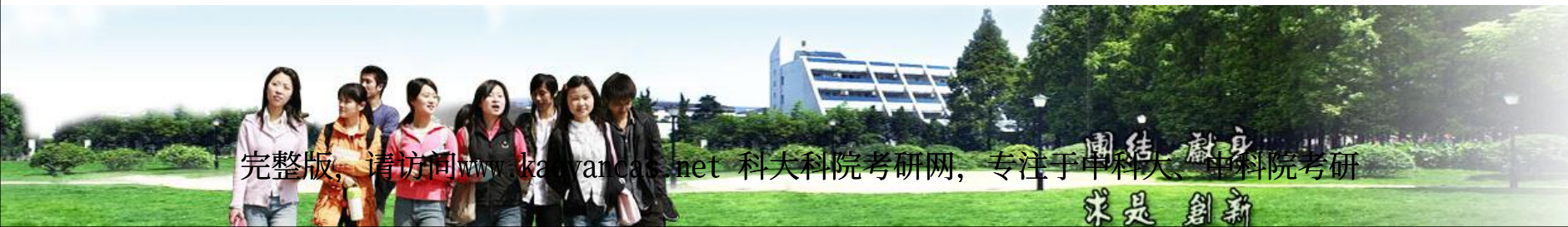
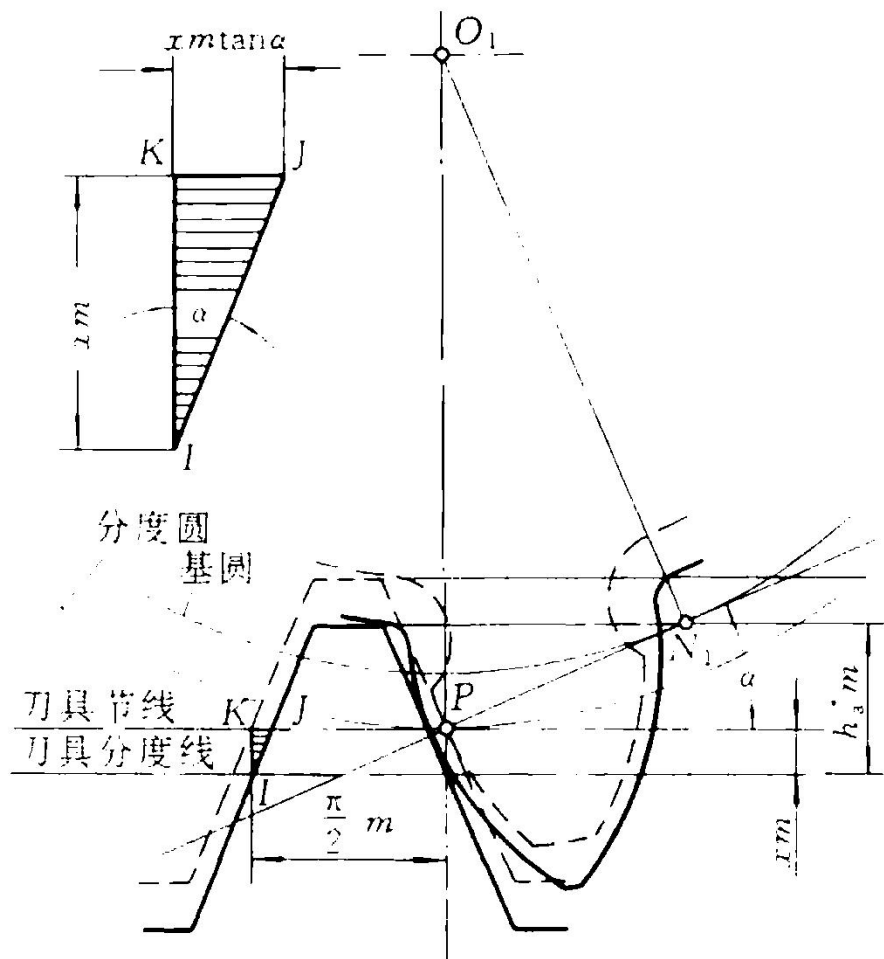
1、分度圆上的齿厚

见图：刀具节线的齿槽宽比中线齿槽宽 $2\overline{KJ}$ ， $\therefore$ 被切齿轮分度圆上的齿厚增加 $2\overline{KJ}$

在 $\triangle IJK$ 中: $\overline{KJ} = xmtg\alpha$

分度圆的齿厚:

$$S = \frac{\pi m}{2} + 2\overline{KJ} = \frac{\pi m}{2} + 2xmtg\alpha$$



2、齿顶高和齿根高

齿根高 h_f ：刀具加工节线到顶刃线之间的距离

$$h_f = (h_a^* m + c^* m) - xm = (h_a^* + c^* - x)m$$

对正变位： $x > 0$ ， h_f 比标准减小 xm

对负变位： $x < 0$ ， h_f 比标准增加 xm

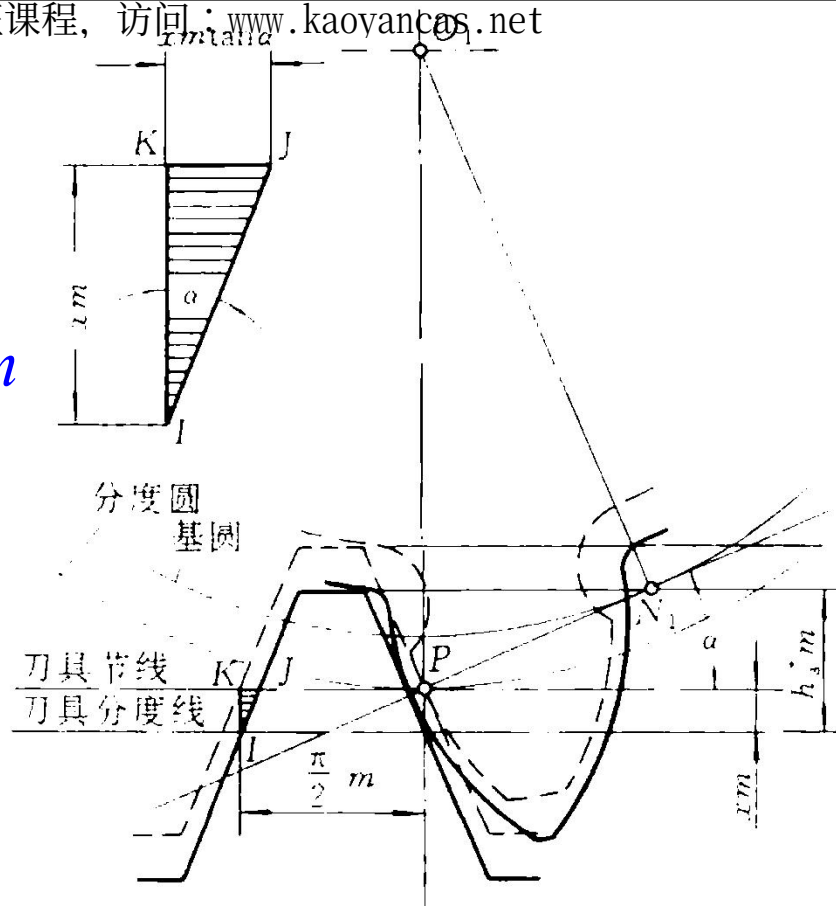
∴变位齿轮的齿根圆半径：

$$r_f = r - h_f = r - (h_a^* + c^* - x)m$$

齿顶高： ∵变位齿轮的分度圆与相应标准齿轮的分度圆一样，
∴变位齿轮的齿顶高仅决定于轮坯顶圆的大小。

为保证齿全高 $h = (2h_a^* + c^*)m$

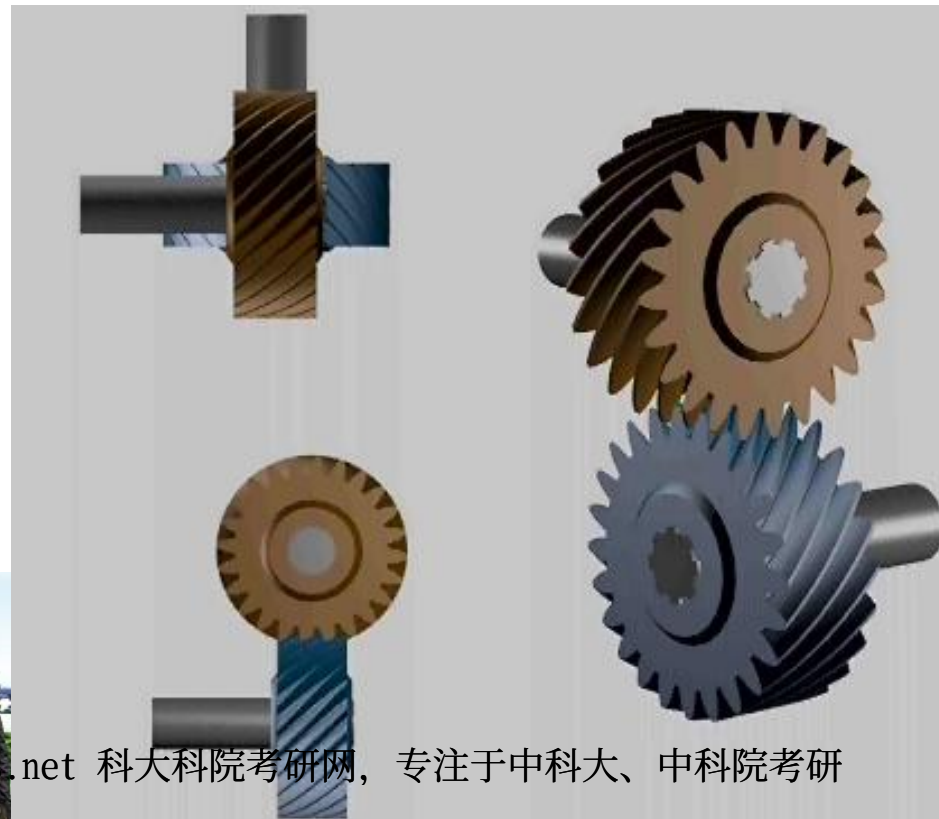
由于 $h_f = (h_a^* + c^* - x)m$ 所以 $h_a = (h_a^* + x)m$ $r_a = r + h_a = r + (h_a^* + x)m$



Helical Gears for Parallel Shafts (斜齿轮传动)



Crossed helical gears mechanism(螺旋齿轮机构)

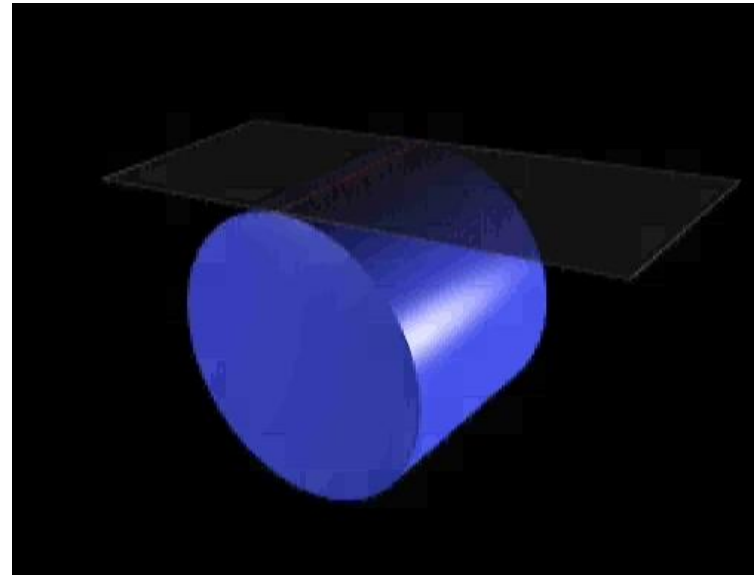
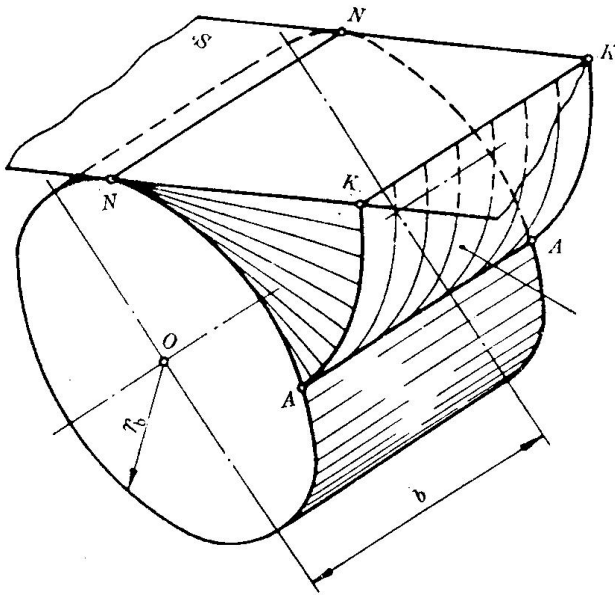
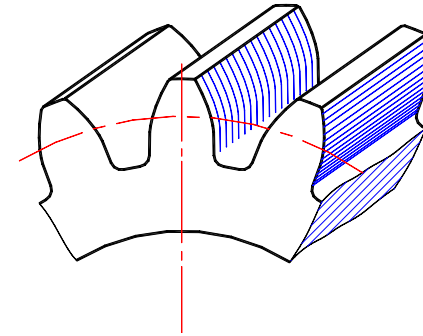


6.9 Helical Gears for Parallel Shafts

(平行轴斜齿轮传动)

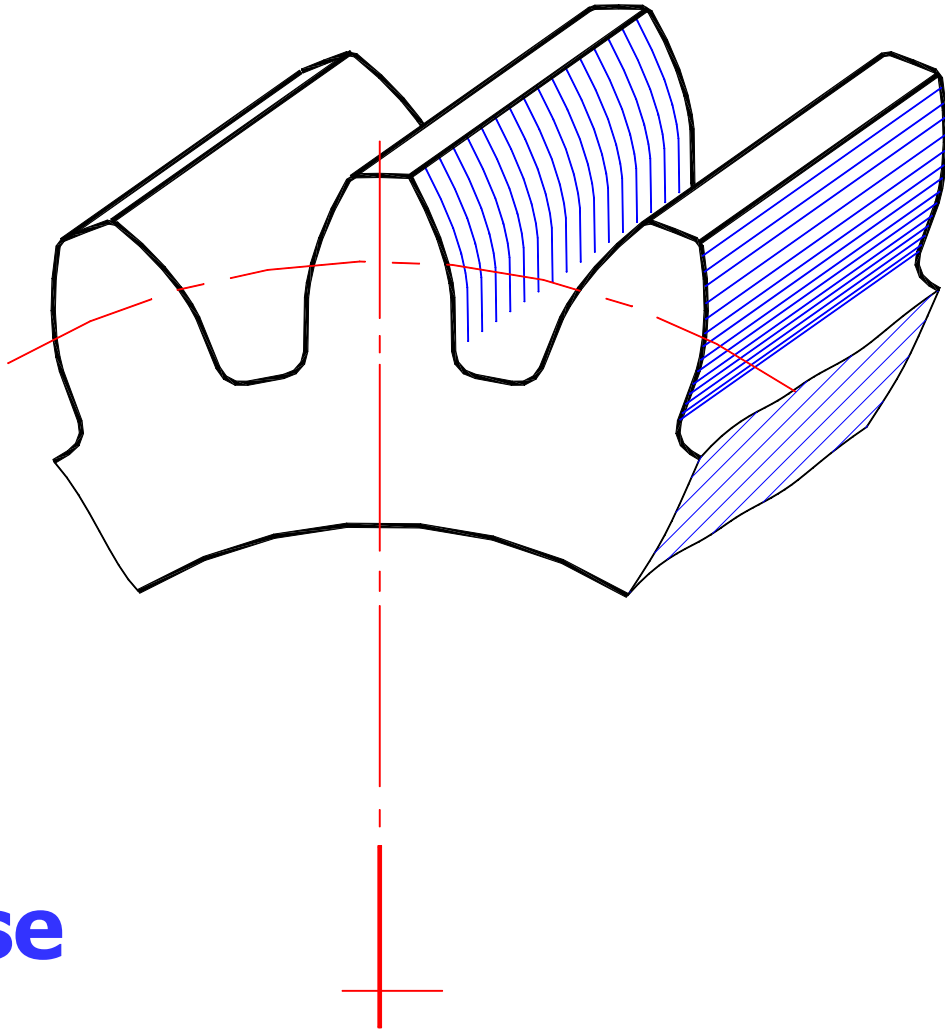
6.9.1 Generation and Characteristics of Helical Teeth

Spur Gear:



The tooth surface of the spur gear is therefore an involute cylinder(渐开线柱面).

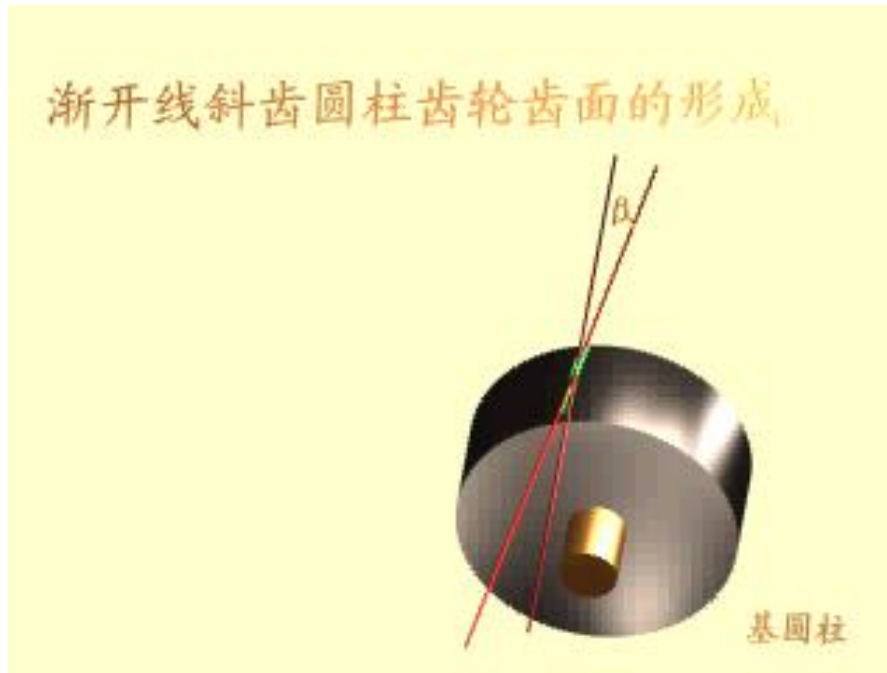
The tooth surfaces of spur gears contact on a straight line parallel to the axes of the gears.



→ **sudden loading and sudden unloading on teeth**

→ **Vibration and noise**

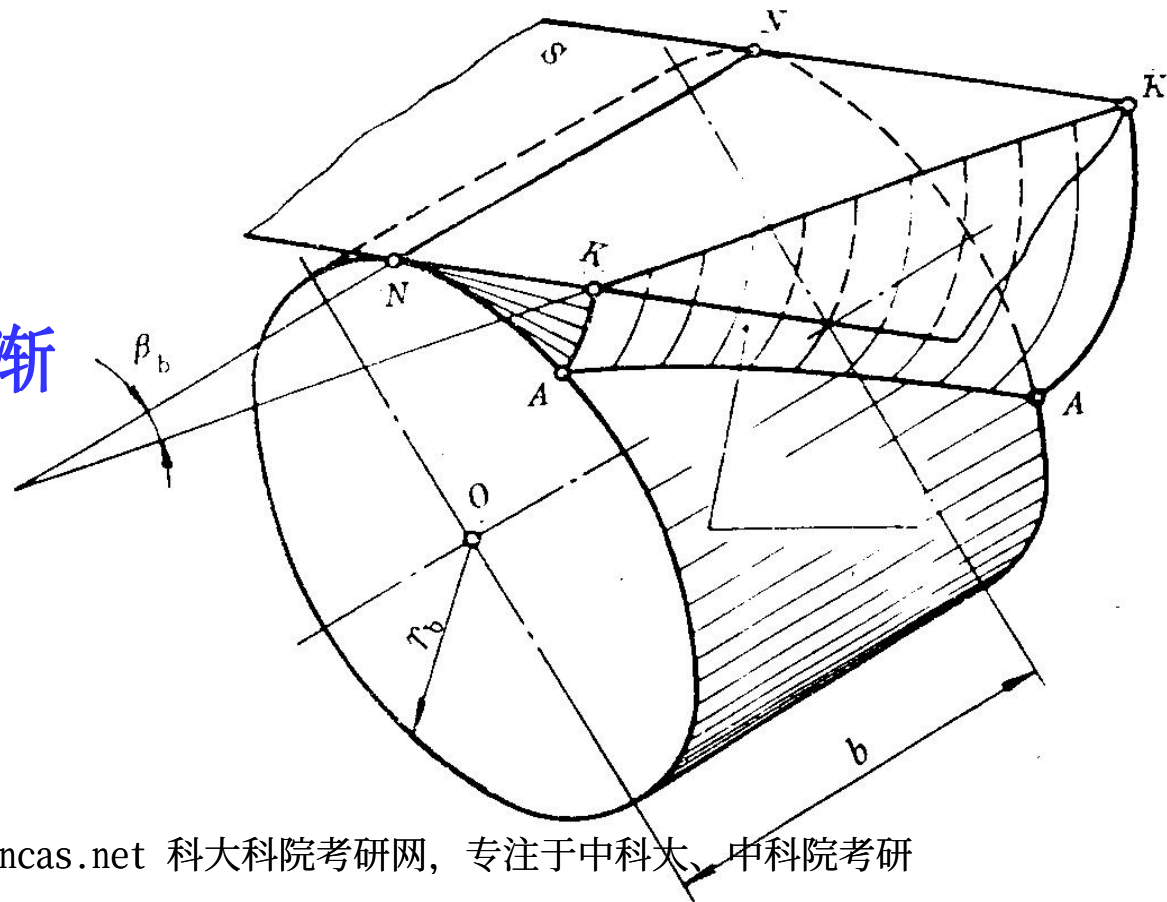
In generation of the tooth profile of a helical gear, the straight line KK on the generating plane is no longer parallel to the axis of the gear.



Every point on the straight line KK will produce an involute.

The curve connecting the starting points of the involutes on the base cylinder is a helix(螺旋) AA.

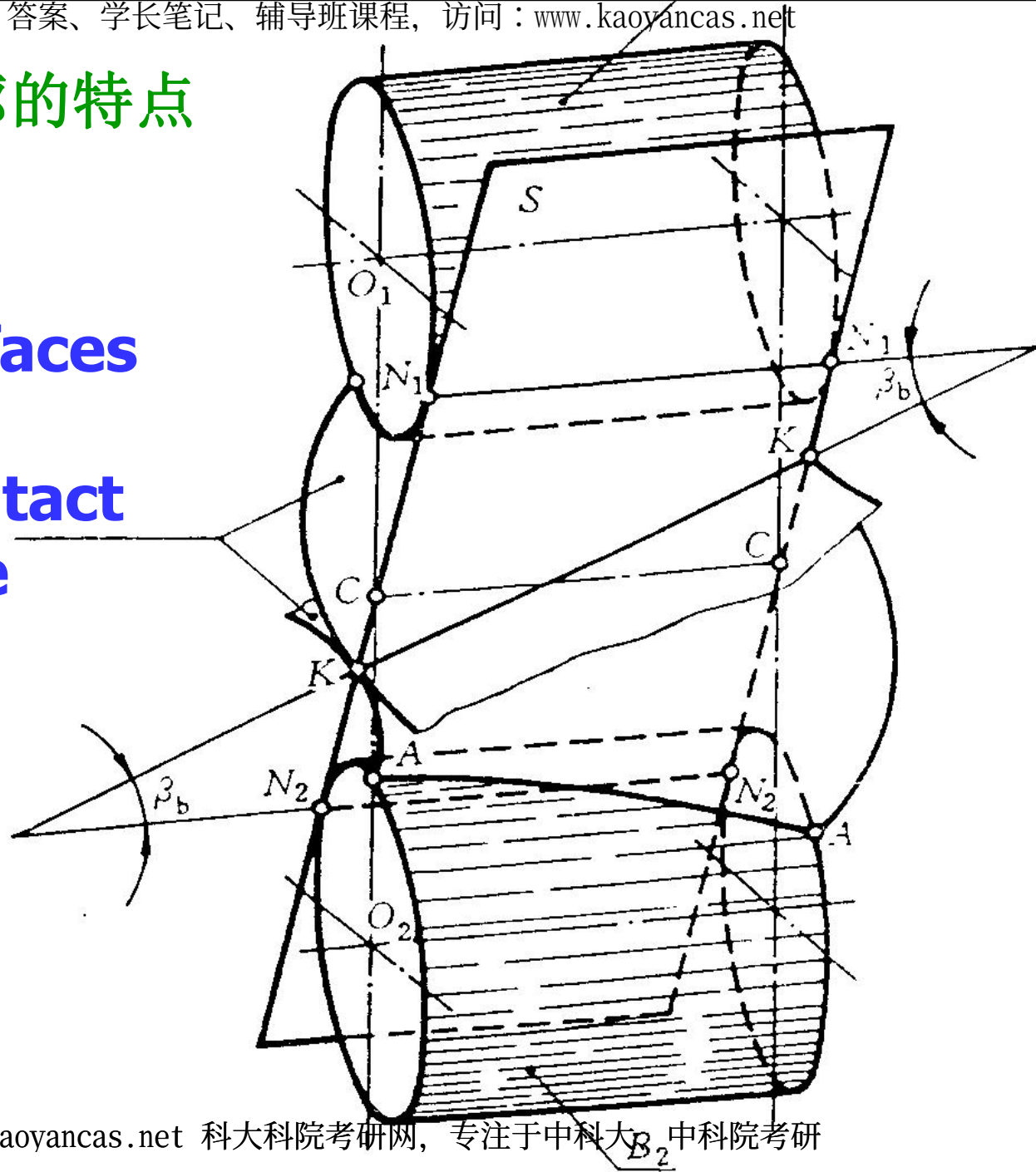
The surface profile of a helical gear is called an involute helicoid(渐开线螺旋面)



渐开线螺旋面齿廓的特点

1. The tooth surfaces of two engaging helical gears contact on a straight line inclined .

$$\beta_{b1} = -\beta_{b2}$$

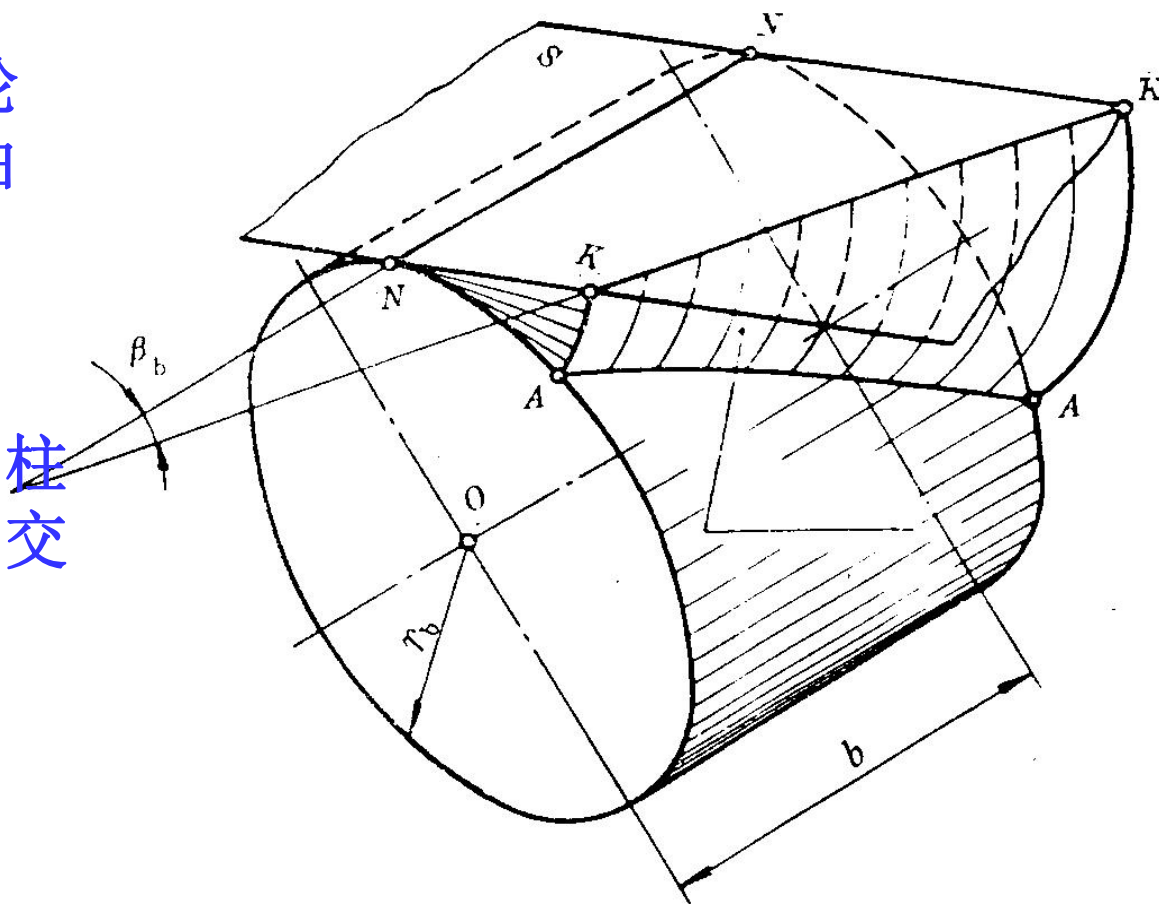




2. 端面（垂直于齿轮轴线的面）与齿廓曲面的交线为渐开线。

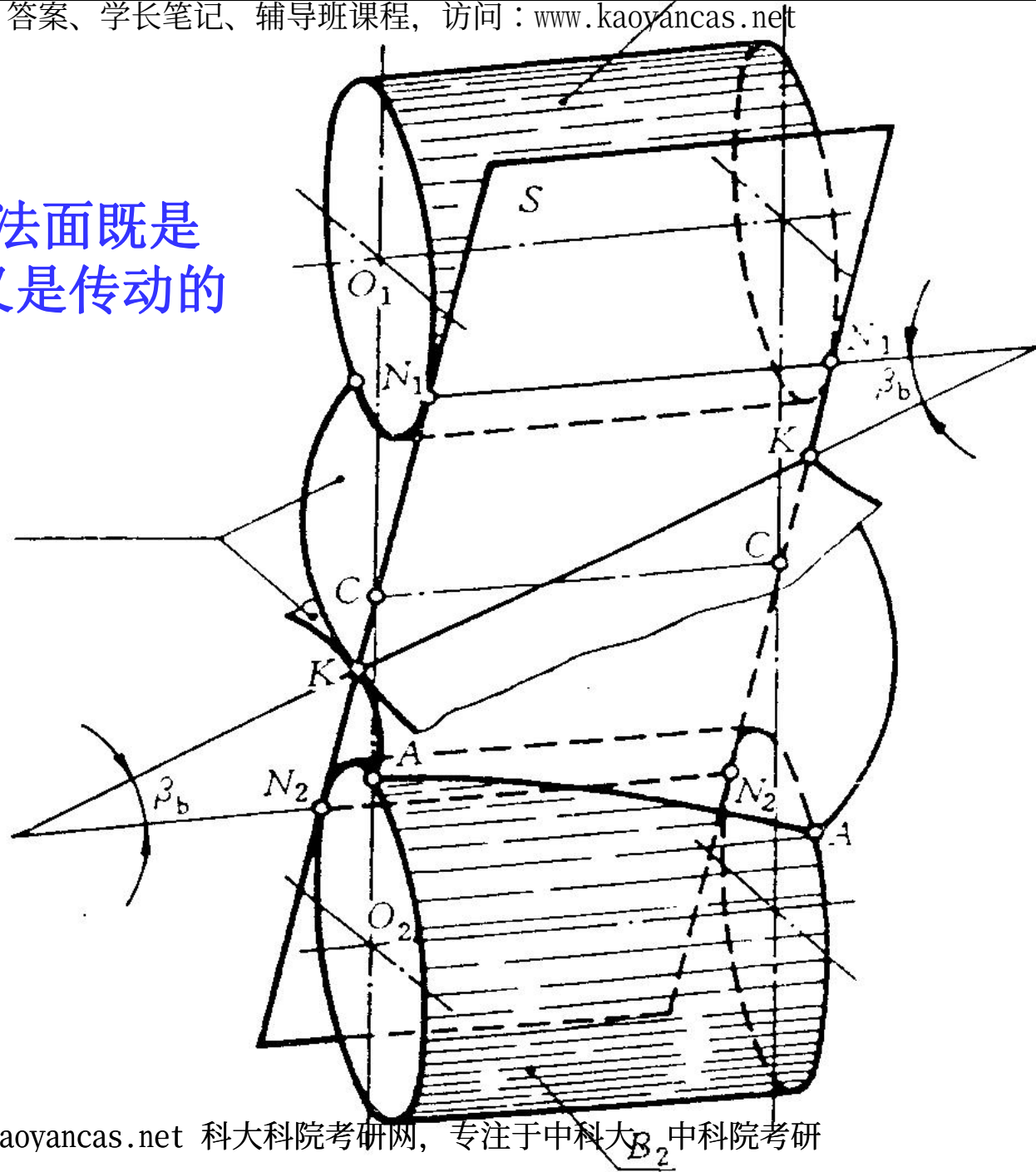
3. 与基圆柱同轴的圆柱面与渐开线螺旋面的交线为一螺旋线。

不同面→螺旋角不同



(1) 两斜齿齿廓的公法面既是两基圆柱的公切面，又是传动的啮合面

（2）两齿廓的接触 线与轴线夹角 β_p





The length of the contact line changes gradually from zero to maximum and then from maximum to zero.

The loading and unloading of the teeth become gradual and smooth. That is why helical gears can operate at a higher speed.





6.9.2 Parameters of Helical Gears

1、斜齿轮的切削加工：

①仿形法；②范成法：滚齿

（用仿形法加工斜齿轮时，铣刀是沿螺旋齿槽的方向进刀的）



Two sets of parameters :

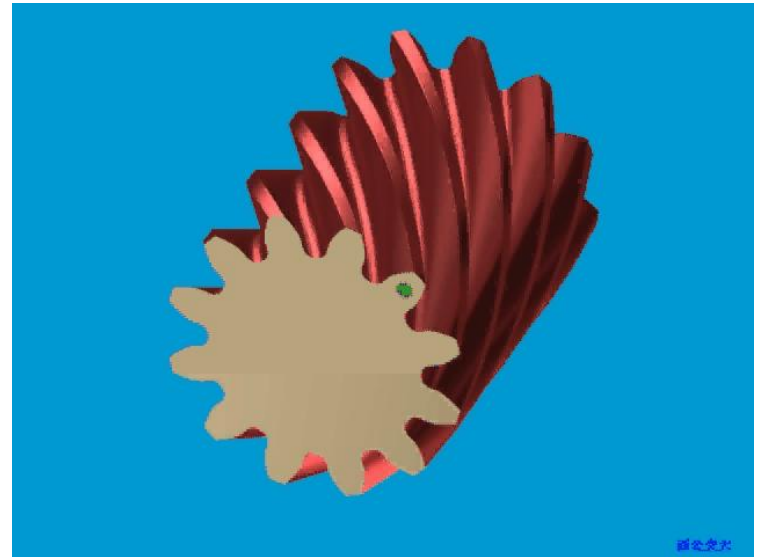
1. On the transverse plane(端面) (with subscript "t") $\perp$ 轴线的面

The transverse profile of a helical gear is an involute.

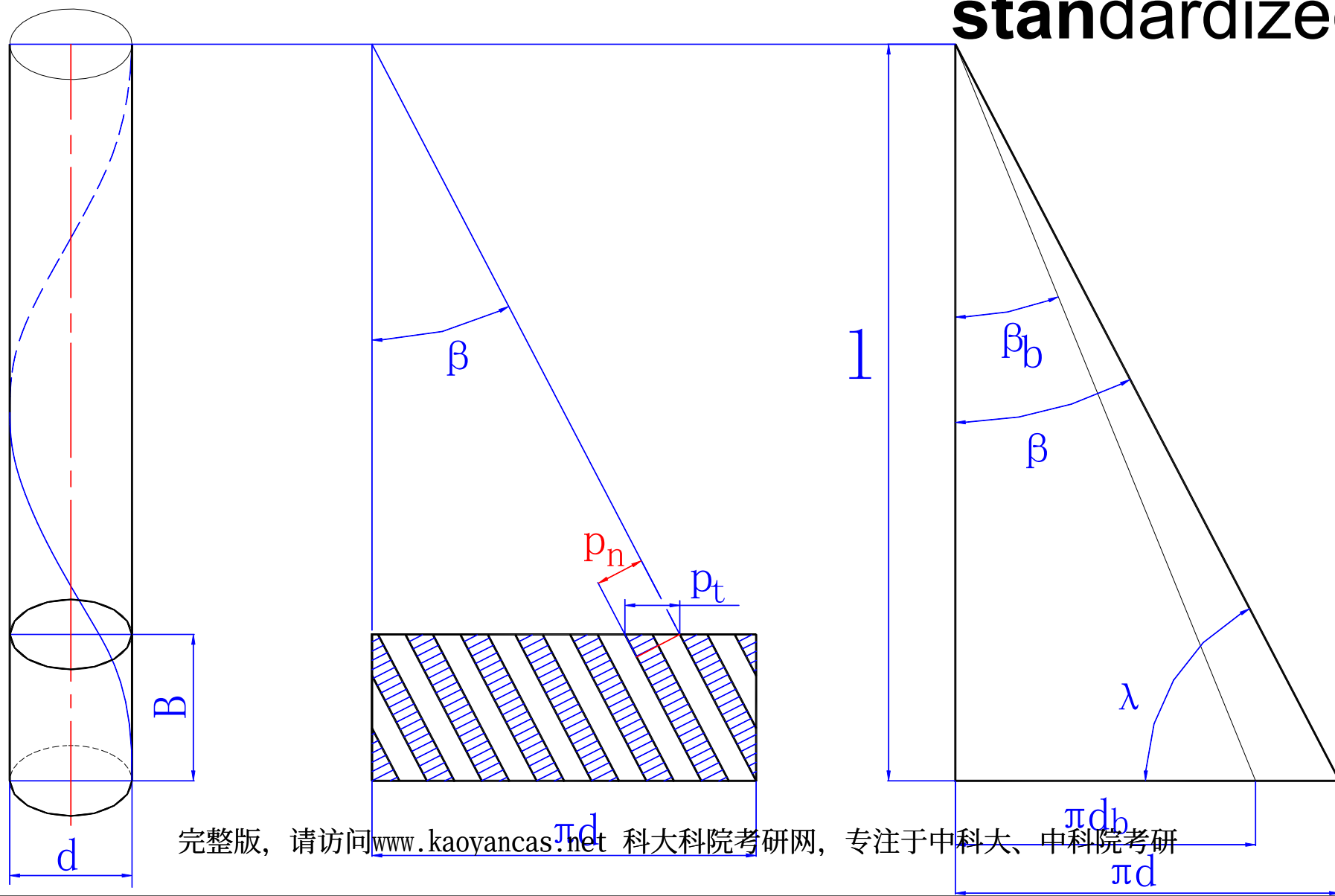
2. On the normal plane (法面) (with subscript "n")垂直于分度圆柱面螺旋线的切线的平面

Parameters on the normal plane are the standard values. They are determined by cutter.

刀具的齿形应于斜齿轮的法面齿形相同， $\rightarrow$ 法面上的模数和压力角为标准值。



$$p_n = p_t \cos \beta \Rightarrow m_n = m_t \cos \beta \quad m_n \text{ is standardized.}$$





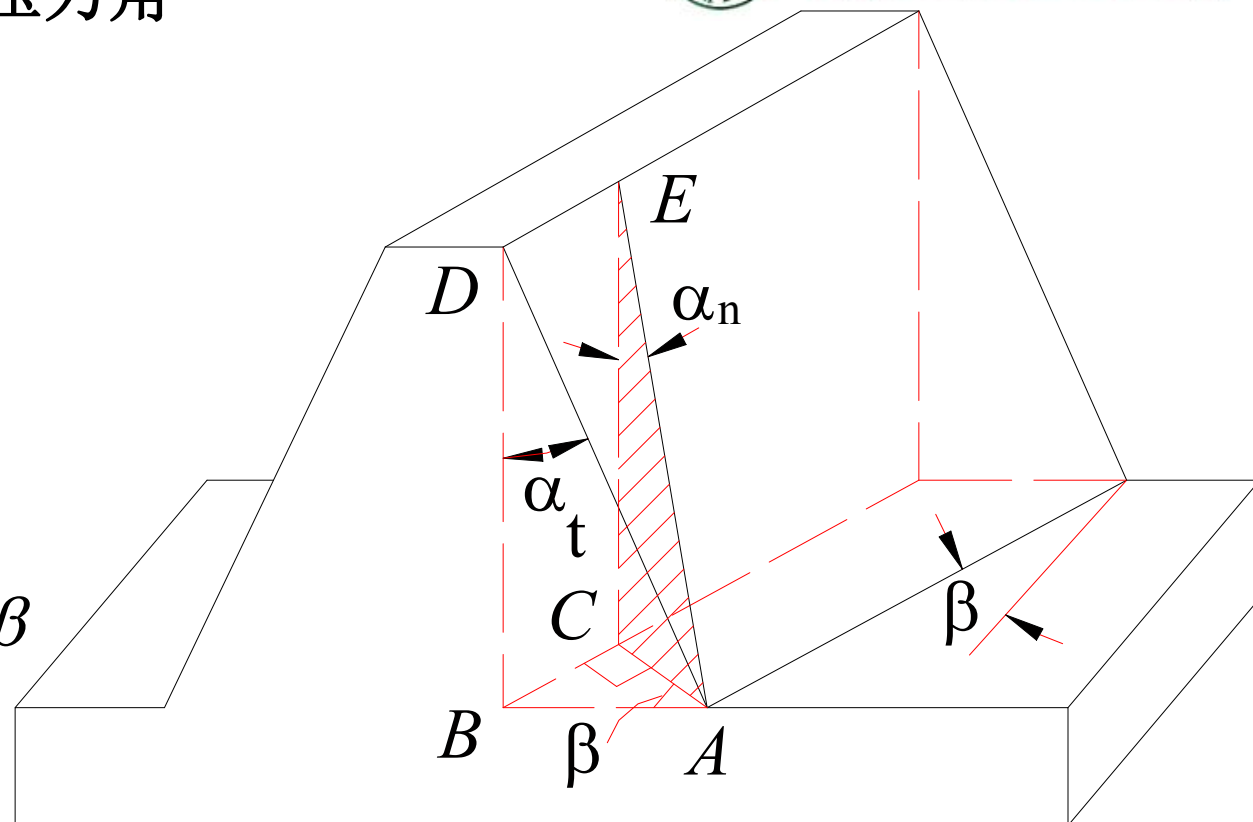
法面压力角与端面压力角

$$\operatorname{tg} \alpha_t = \frac{\overline{AB}}{\overline{BD}}$$

$$\operatorname{tg} \alpha_n = \frac{AC}{CE}$$

$$AC = AB \cos \beta$$

$$\because BD = CE$$



$$\operatorname{tg} \alpha_n = \operatorname{tg} \alpha_t \cos \beta$$

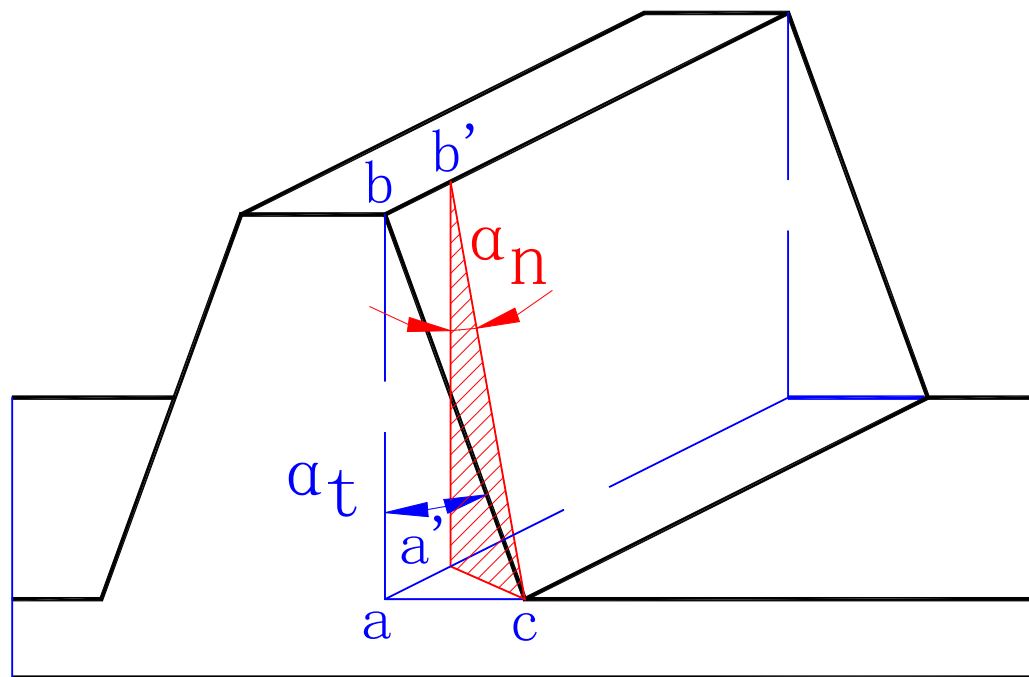


$$h_{an} = h_{at}$$

$$\therefore h_{an}^* m_n = h_{at}^* m_t, c_n^* m_n$$

$$m_n = m_t \cos \beta$$

$$\begin{cases} h_{at}^* = h_{an}^* \cos \beta \\ c_t^* = c_n^* \cos \beta \end{cases}$$



$$h_f = (h_{an}^* + c_n^*) m_n = 1.25 m_n \neq 1.25 m_t$$

螺旋角 β :

螺旋线的导程 P_z :

螺旋线绕一周时它沿轴线方向前进的距离

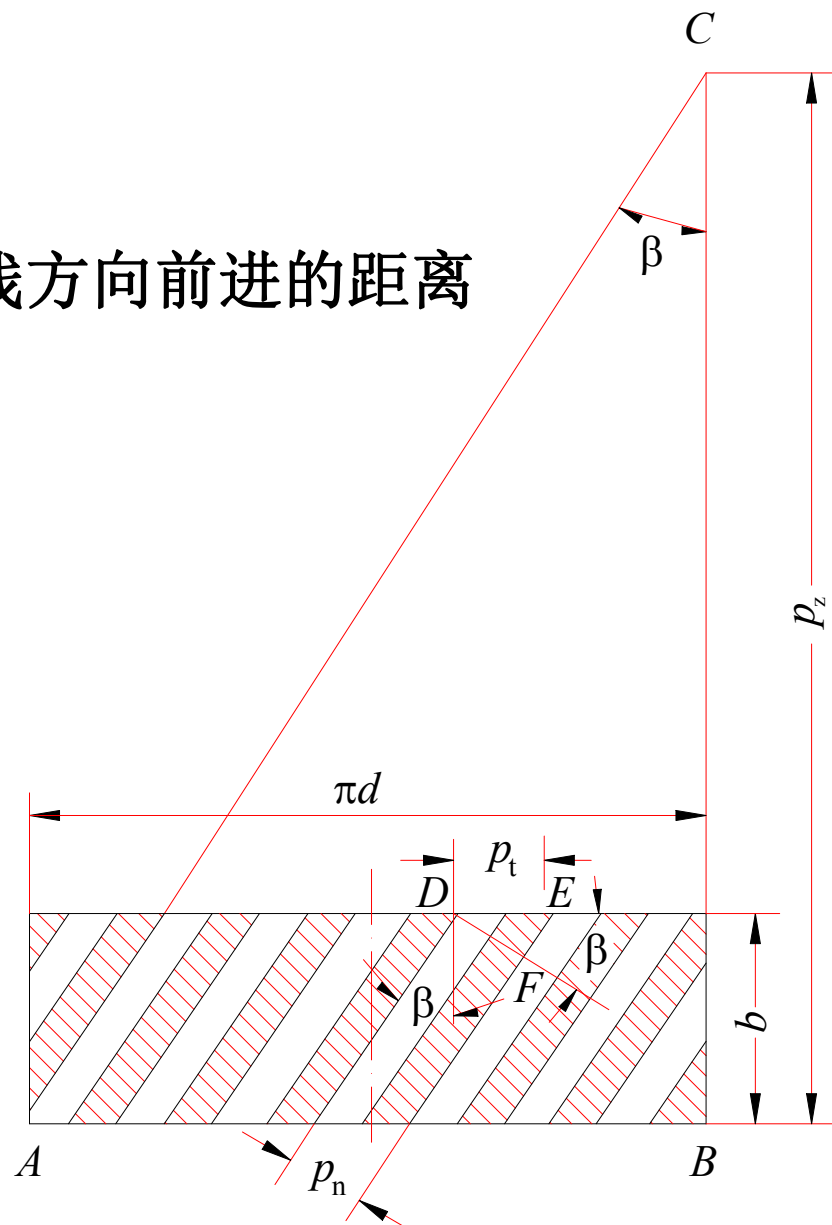
$$\operatorname{tg} \beta = \frac{\pi d}{P_z}$$

$$\operatorname{tg} \beta_b = \frac{\pi d_b}{P_z}$$

$$d_b = d \cos \alpha_t$$

$$\operatorname{tg} \beta_b = \frac{d_b}{d} \operatorname{tg} \beta = \operatorname{tg} \beta \cos \alpha_t$$

(不同圆柱面的螺旋角不等)



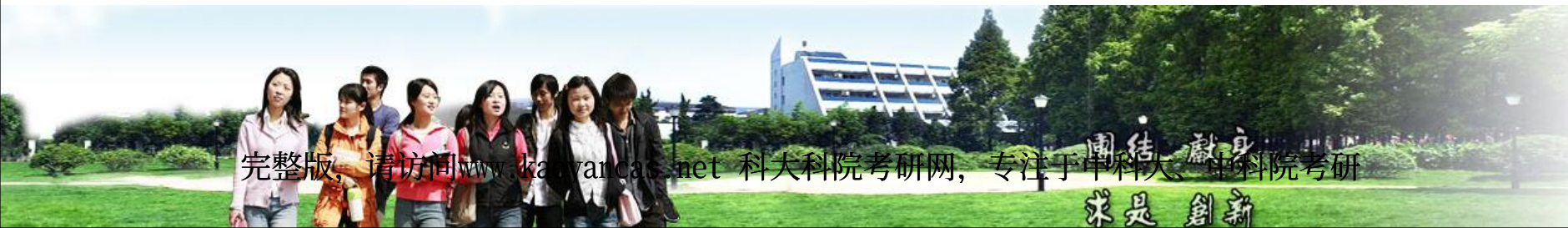


The reference diameter and centre distance should be calculated on the **transverse** plane as

$$d = Zm_t = \frac{Zm_n}{\cos \beta}$$

$$a = \frac{1}{2} (d_1 + d_2) = \frac{m_n (Z_1 + Z_2)}{2 \cos \beta}$$

Reference centre distance a can be **adjusted** by changing the **helix angle(螺旋角) β** . Standard helical gear drive can suit any centre distance.



6.9.3 Proper Meshing Conditions for Helical Gears

According to the proper meshing conditions of spur gear drive,

$$\left\{ \begin{array}{l} m_{t1} = m_{t2} \\ \alpha_{t1} = \alpha_{t2} \\ \beta_{b1} = \pm \beta_{b2} \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} m_{n1} = m_{n2} = \text{标准} \\ \alpha_{n1} = \alpha_{n2} = 20^\circ \\ \beta_1 = \pm \beta_2 \end{array} \right.$$

6.9.4 Contact Ratio for a Helical Gear Pair

transverse
contact ratio

overlap ratio

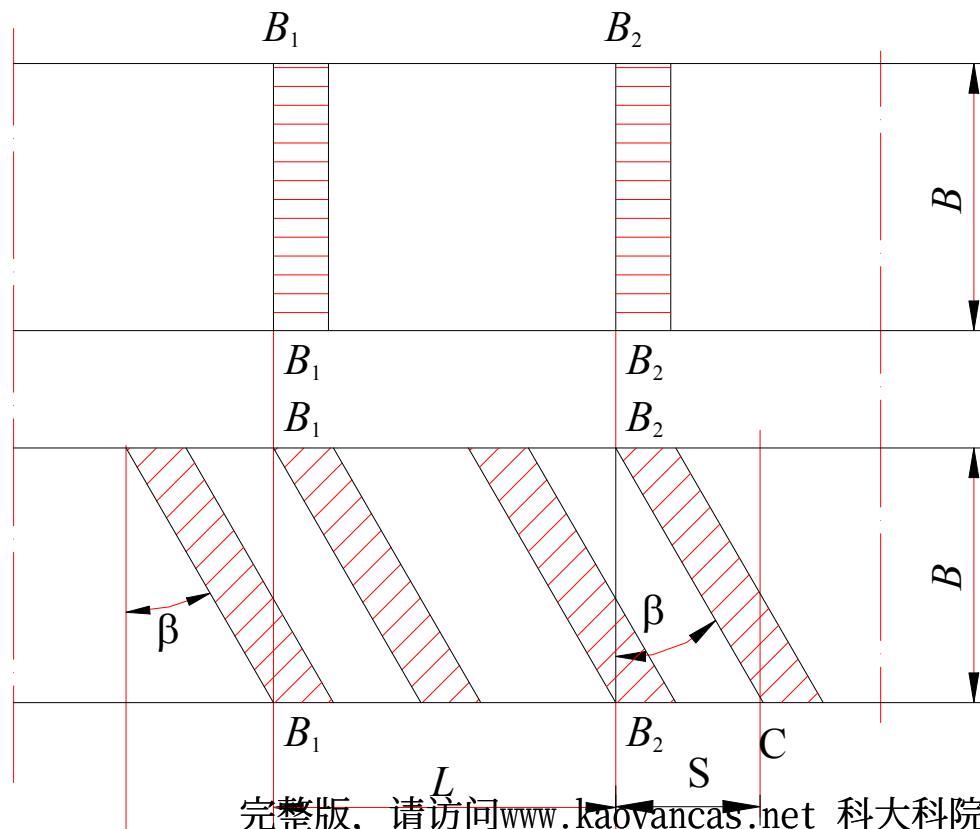
2、重合度

$$\varepsilon_{\gamma} = \varepsilon_{\alpha} + \varepsilon_{\beta}$$

$$\varepsilon_{\beta} = \frac{B \tan \beta}{\pi m_t} = \frac{B \sin \beta}{\pi m_n}$$

ε_{α} —— 端面重合度，与其端面齿形相同的直齿轮的重合度

ε_{β} —— 纵向重合度 $\beta \geq 8^{\circ}$





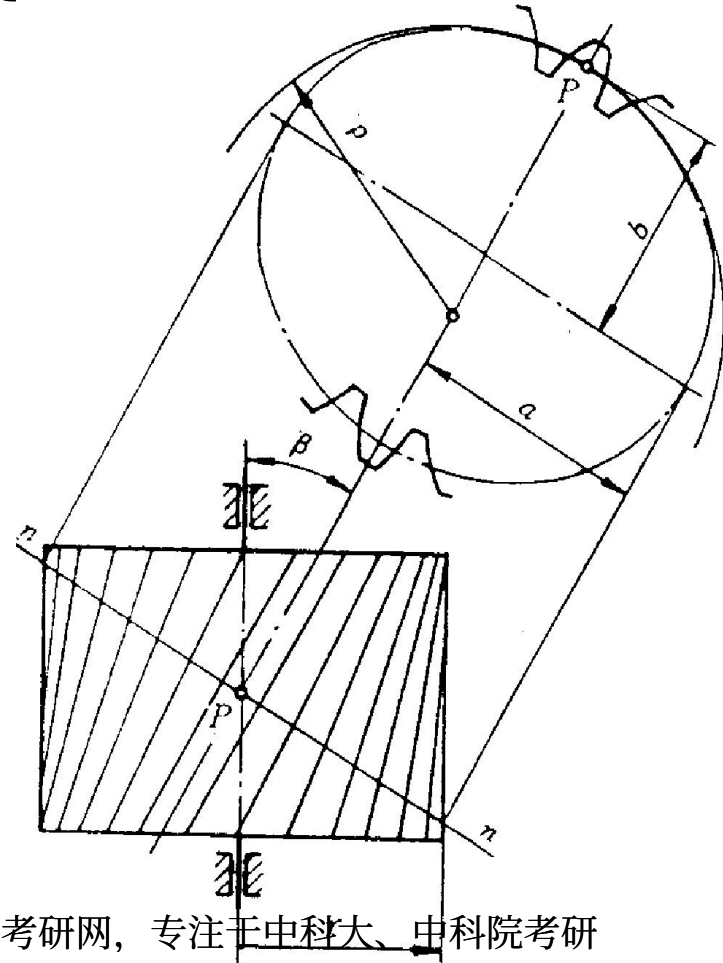
短齿制会降低重合度。短齿制斜齿轮传动即保证重合度，又提高安全性。



6.9.5 Virtual Number of Teeth (当量齿数) for Helical Gears

To calculate the strength of the tooth, the **virtual gear**(当量齿轮) of the helical gear is introduced.

The **virtual gear**(当量齿轮) of the helical gear is a spur gear, the tooth profile of which is equivalent to that of a helical gear on the normal plane.

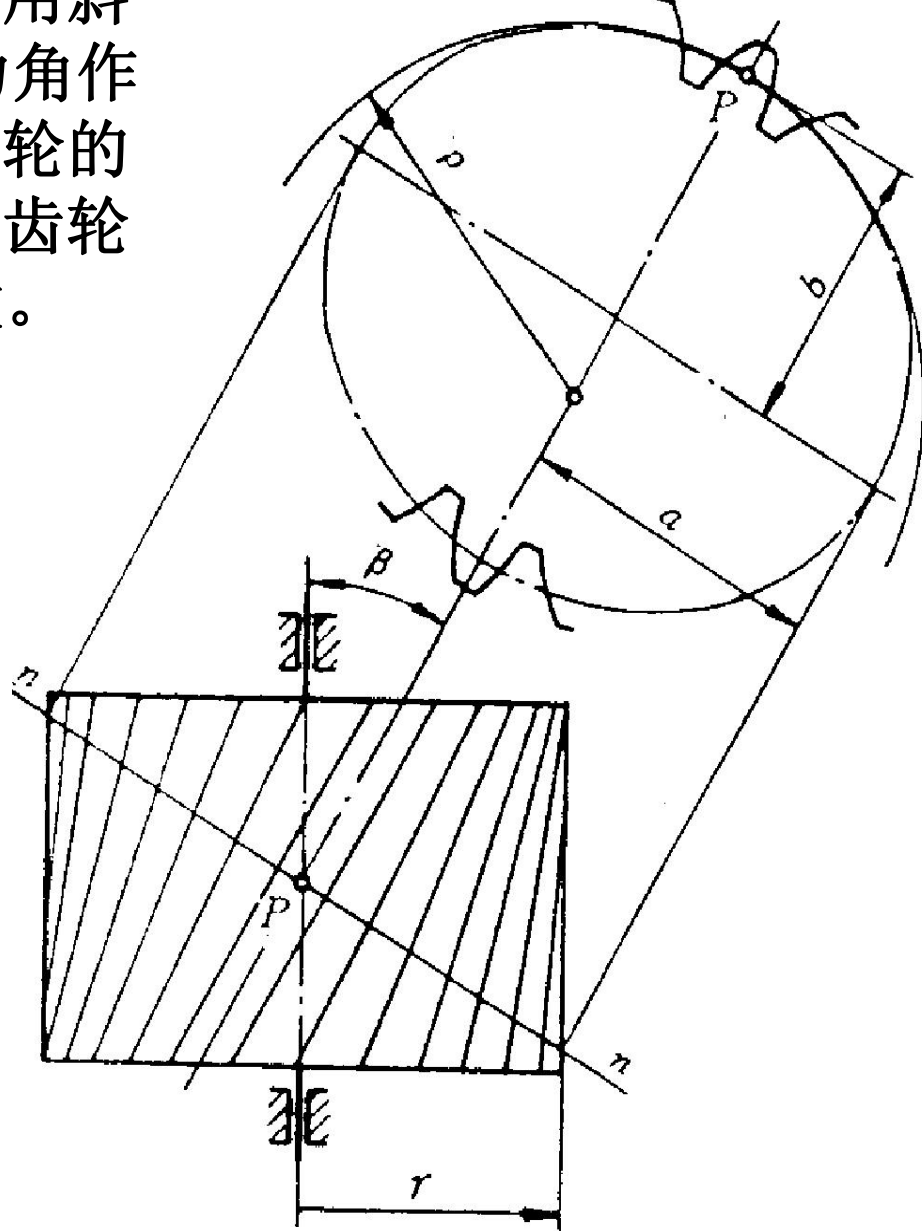


当量齿轮：以 ρ 为分度圆半径，用斜齿轮的 m_n 和 α_n 分别为模数和压力角作一虚拟的直齿轮，其齿形与斜齿轮的法面齿形最接近。这个齿轮称斜齿轮的当量齿轮，齿数 Z_v 称当量齿数。

$$a = \frac{r}{\cos \beta}, b = r$$

$$\rho = \frac{a^2}{b} = \frac{r}{\cos^2 \beta}$$

$$Z_v = \frac{2\rho}{m_n} = \frac{2r}{m_n \cos^2 \beta} = \frac{2\left(\frac{m_t Z}{2}\right)}{m_n \cos^2 \beta} = \frac{Z}{\cos^3 \beta}$$



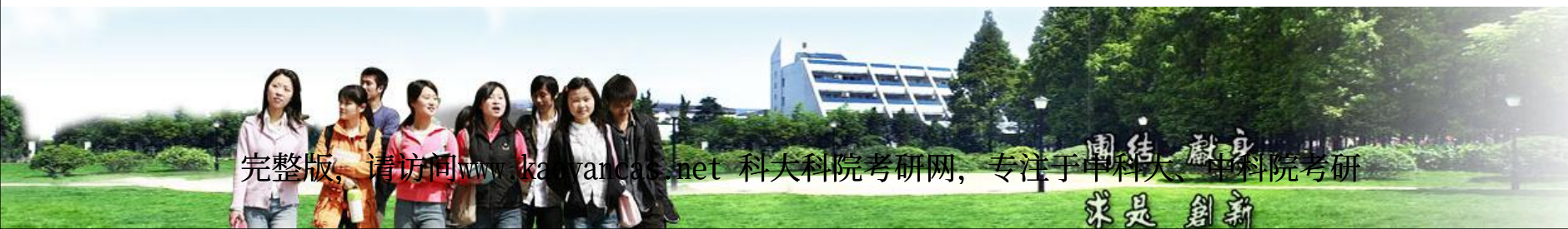
$$Z_{\min} = Z \cos^3 \beta < \text{直齿轮的最少齿数}$$



Advantage of helical gears:

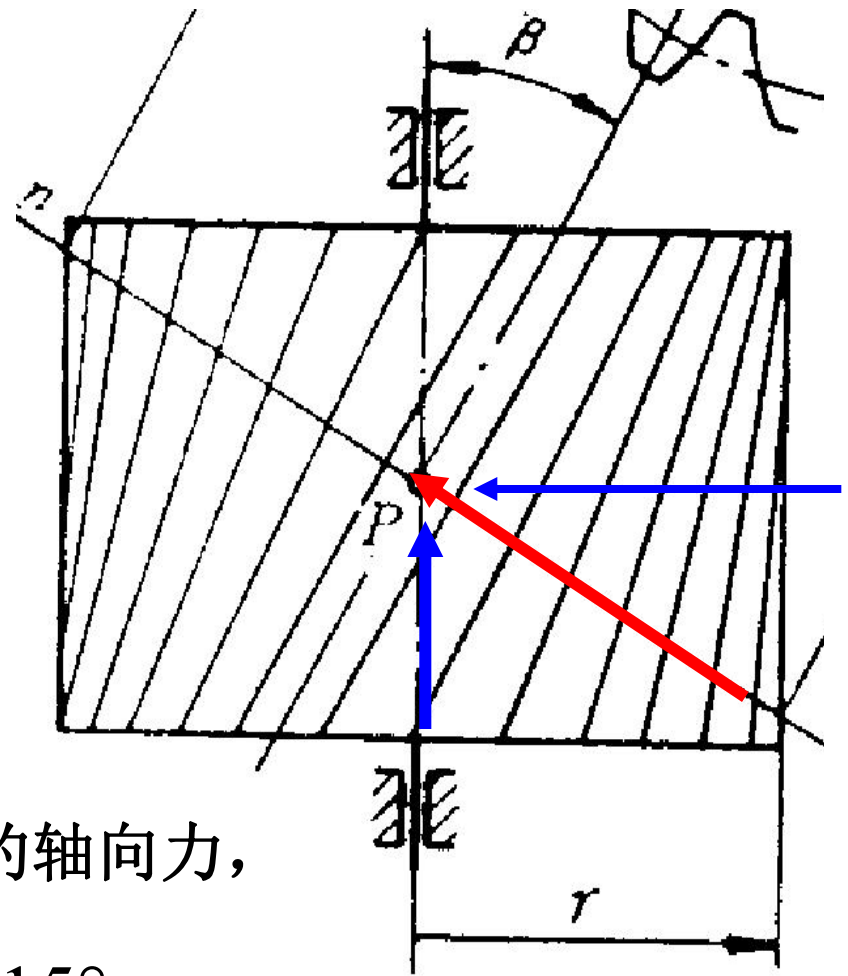
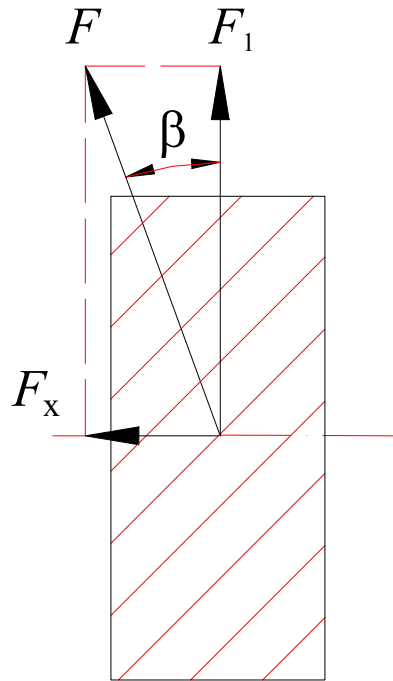
- ((1)) Helical gears operate more smoothly.
- ((2)) Have higher strength.
- ((3)) The working centre distance of a helical gear pair can be adjusted by the helix angle β .
- ((4)) 比直齿轮小，机构更紧凑
- ((5)) 制造成本与直齿轮相同

Helical gears are therefore widely used for high speed or heavy load transmission.



The main disadvantage of helical gears is:

They produce an **axial thrust**(轴向推力).



为了保证有足够的重合度和较小的轴向力，
一般对螺旋角有限制

$$\beta = 8^\circ \sim 15^\circ$$

6.10 Worm Gearing(蜗杆传动)

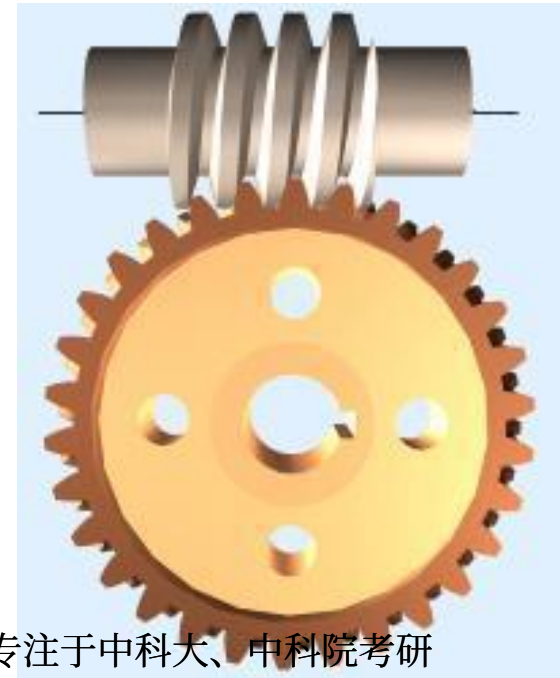
6.10.1 Worm Gearing and its Characteristics

蜗杆传动由交错轴斜齿轮机构传动演变而来，主要传递交错轴之间的运动和动力。

Non-intersecting and non-parallel shafts

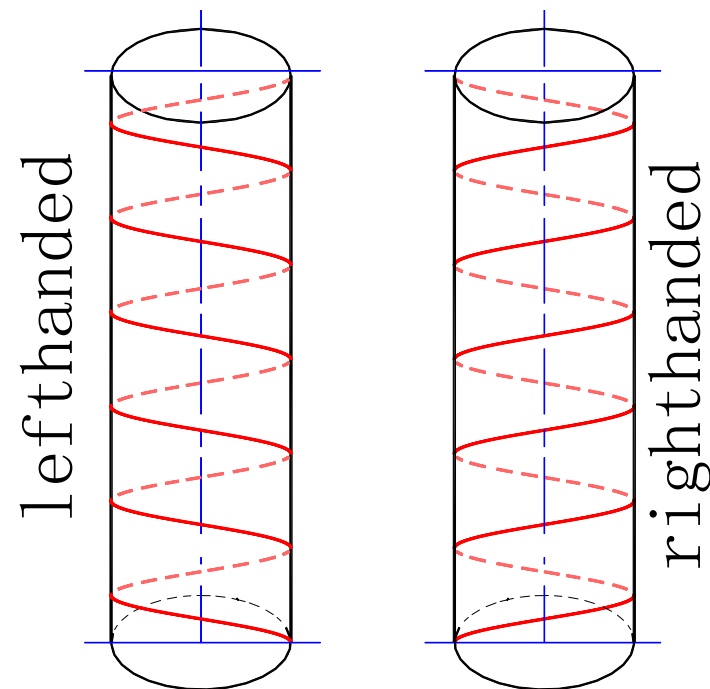
Main Characteristics:

- 1、 Smooth silent operation.*
- 2、 Greater speed reduction*
- 3、 Self-locking (usually)*
- 4、 Lower efficiency*



蜗轮蜗杆旋向的判断

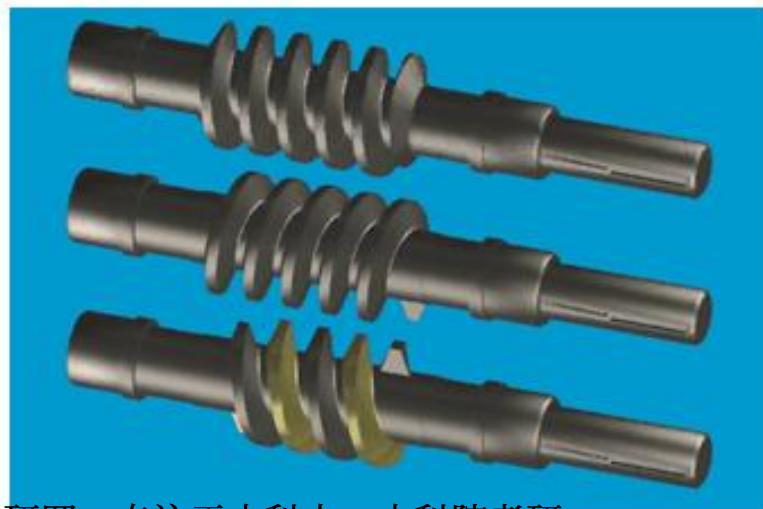
Worms are a kind of screw, usually **right handed** (右旋) for convenience of cutting, or **lefthanded** (左旋) if necessary. Worms are usually drivers to reduce the speed.



左右旋、单双头蜗杆



蜗轮
蜗杆
旋向
相同

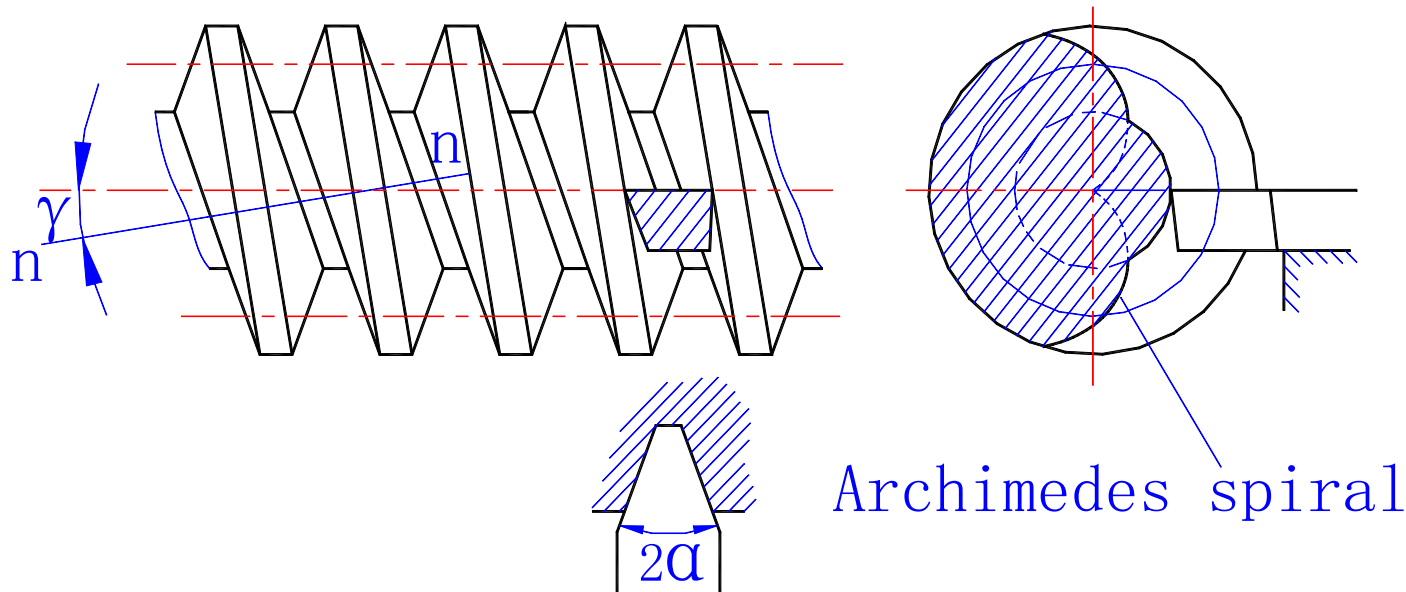


6.10.2 Types of Worms

(1) Straight sided axial worms

-----**ZA-worm**阿基米德蜗杆

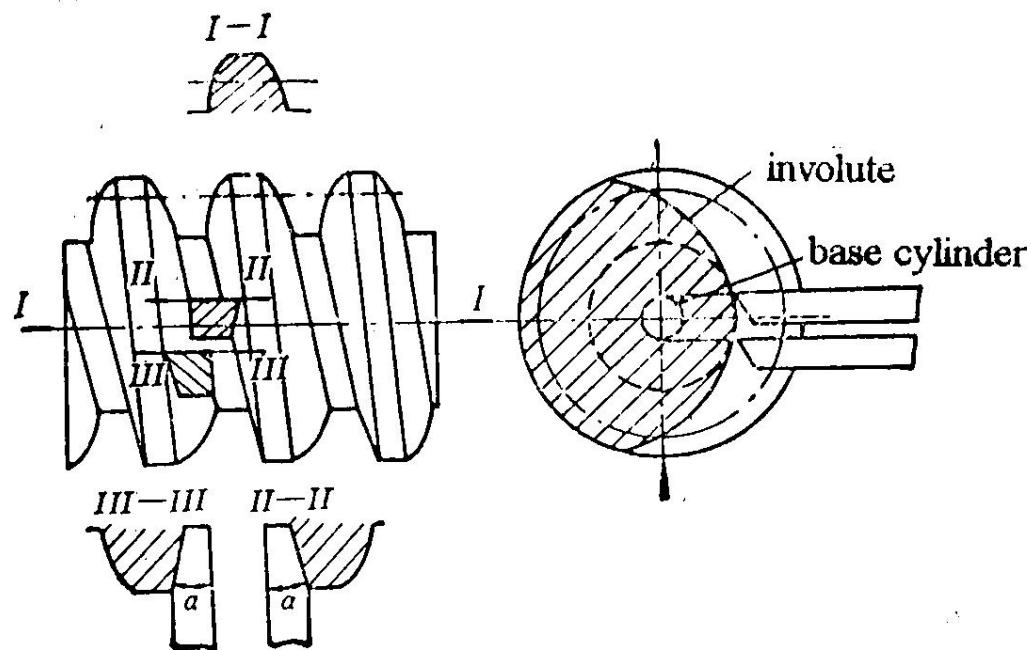
The **transverse section**(横切面) is Archimedes spiral(螺旋线). The axial section is involute rack.





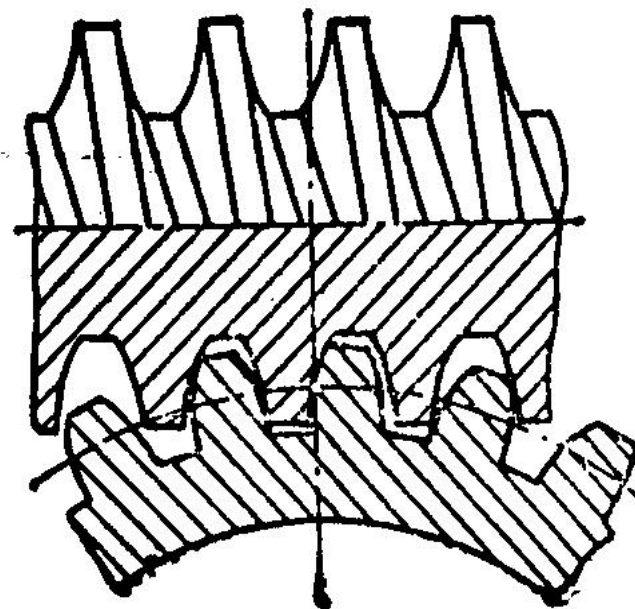
(2) Involute helicoid worms

----ZI-worm 渐开线蜗杆



((3)) Arc-contact worms

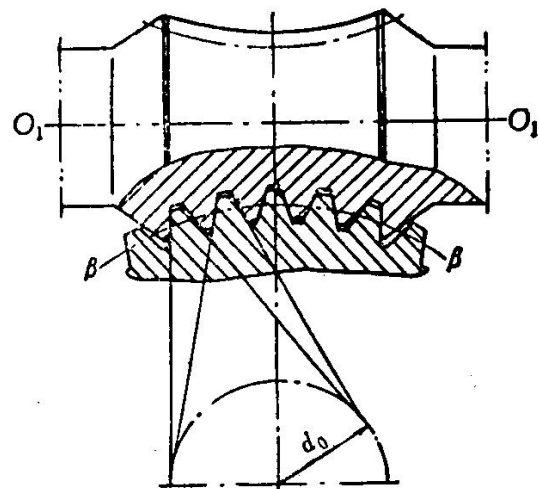
-----ZC-worm 圆弧齿蜗杆



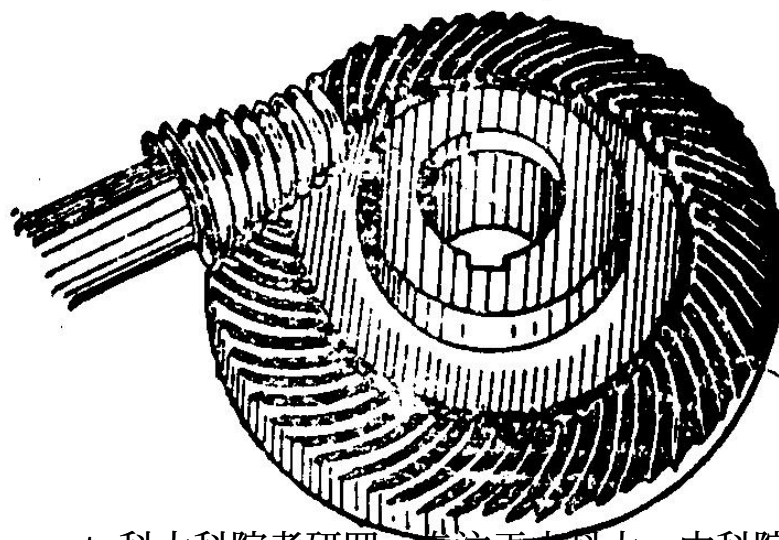


(4) Enveloping worm(包络蜗杆)

-----more teeth engaged.



(5) Spiroid 锥蜗杆



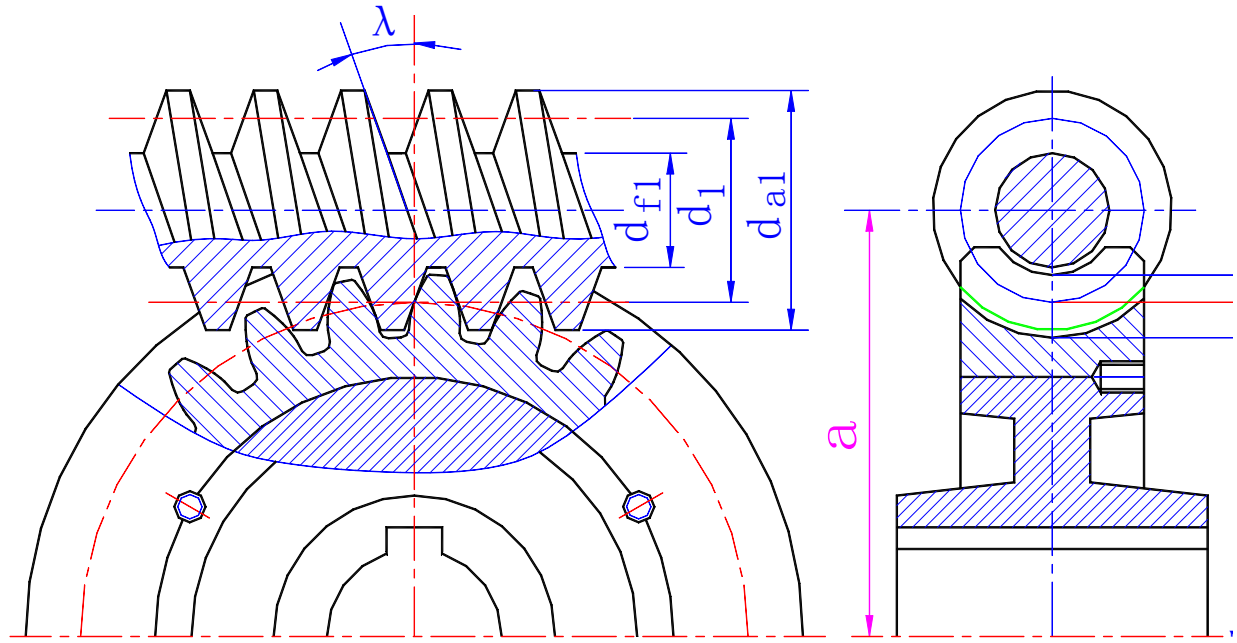
6.10.3 Proper Meshing Conditions for Worm Drives

The mid-plane---- The transverse plane of a worm wheel passing through the axis of the worm .

The **engagement**(啮合) between a Archimedes worm and a worm wheel on the mid-plane corresponds to that of an **involute rack and pinion**.

Proper Meshing Conditions for Worm Drives:

$$\begin{cases} m_{t2} = m_{x1} \\ \alpha_{t2} = \alpha_{x1} \\ \lambda_1 = \beta_2 \end{cases}$$





6.10.4 Main Parameters and Dimensions for Worm Drives

1) The axial pressure angle of worm α_{x1} is 20° . This can be increased to 25° in power transmission or decreased to 15° or 12° in **indexing devices**(分度装置).

2) The module m ---- standard values

The series of modules for worms is somehow different from those for gears. Table6-2

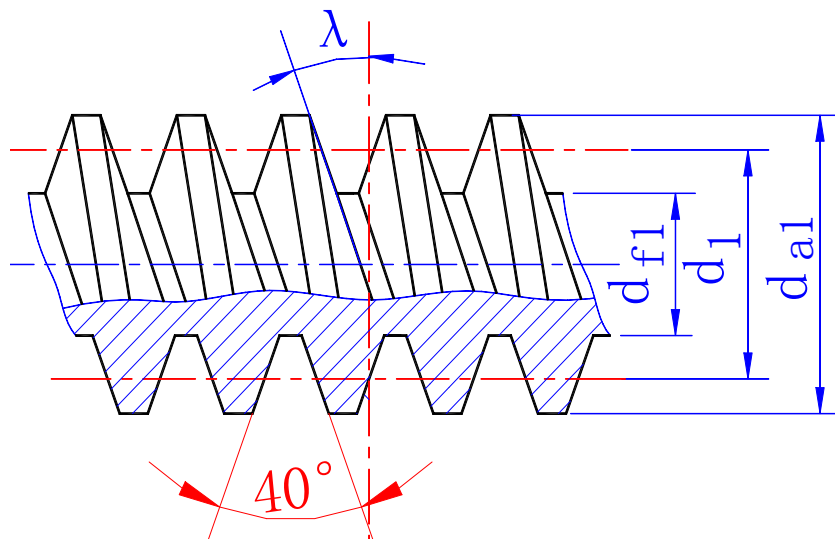




Table 6.2

Modules and Reference Diameters of Worms(mm) (GB10085-88)

m	1	1.25	1.6	2	2.5	3.15
Mid-diameter d1	18	20 22.4	20 28	(18) 22.4 (28) 35.5	(22.4) 28 (35.5) 45	(28) 35.5 (45) 56
m	4	5	6.3	8	10	12.5
Mid-diameter d1	(31.5) 40 (50) 71	(40) 50 (63) 90	(50) 63 (80) 112	(63) 80 (100) 140	(71) 90 (112) 160	(90) 112 (140) 200

3) The **number of threads(头数)** of the worm Z_1

左右旋、单双头蜗杆

$Z_1 = 1, 2, 4, 6$ are preferable.

$Z_1 \uparrow$, then efficiency $\eta \uparrow$

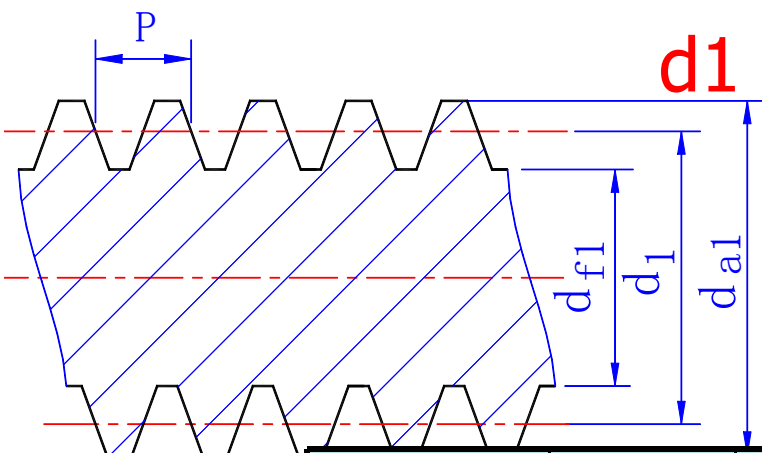


The number of teeth of a worm wheel Z_2 .

$$Z_2 = i_{12} Z_1$$

$$Z_2 = 27 \sim 80$$

4) The mid-diameter d_1 of worm(蜗杆中径)



d_1 — The mid-diameter of worm
蜗杆中径，蜗杆分度圆直径

d_1 is standardized to reduce the number of cutters and should be

m	1	1.25	1.6	2	2.5	3.15
Mid-diameter d_1	18	20 22.4	20 28	(18) 22.4 (28) 35.5	(22.4) 28 (35.5) 45	(28) 35.5 (45) 56
m	4	5	6.3	8	10	12.5
Mid-diameter d_1	(31.5) 40 (50) 71	(40) 50 (63) 90	(50) 63 (80) 112	(63) 80 (100) 140	(71) 90 (112) 160	(90) 112 (140) 200

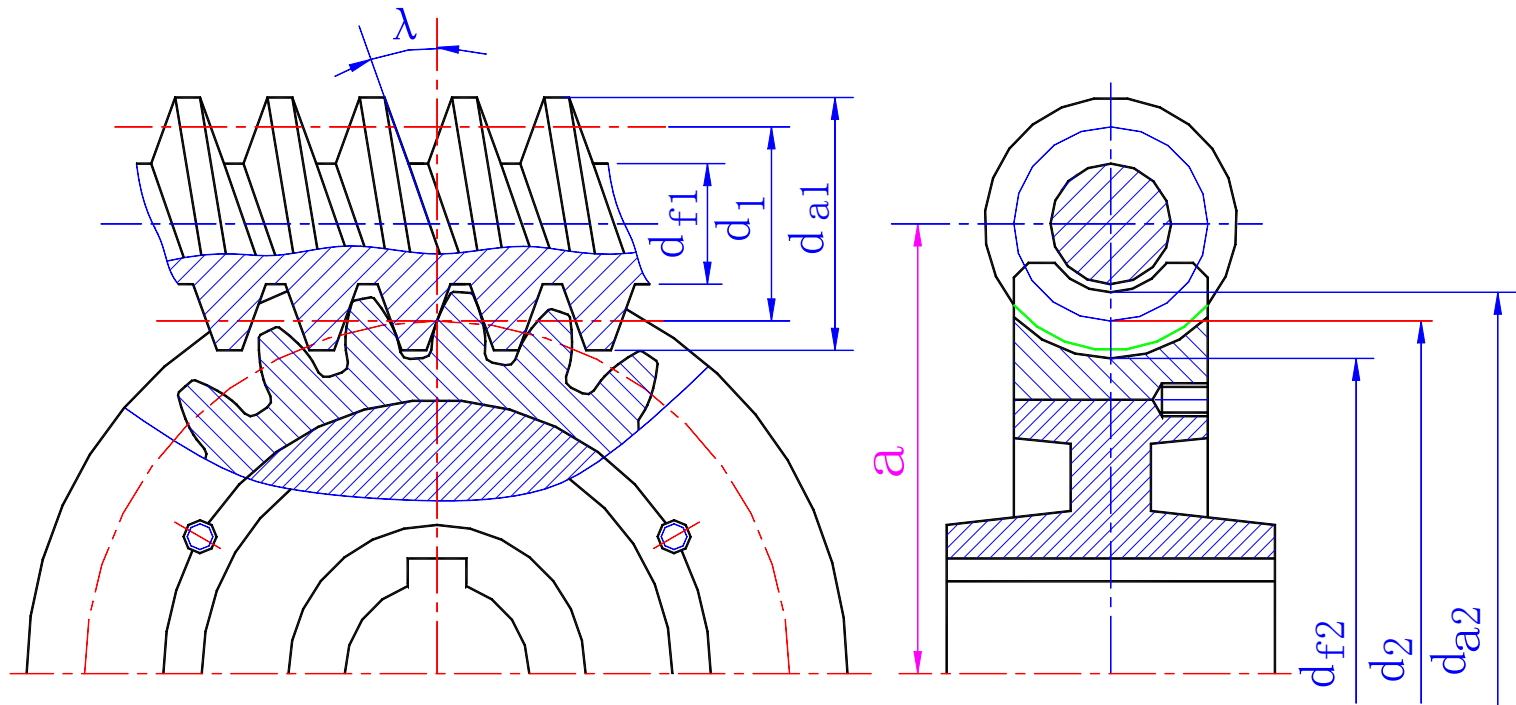
为减少刀具，将 d_1 与 m 相搭配，见表6-2



d_2 —— The reference diameter of worm wheel.

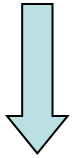
蜗轮分度圆直径

$$d_2 = mz_2$$

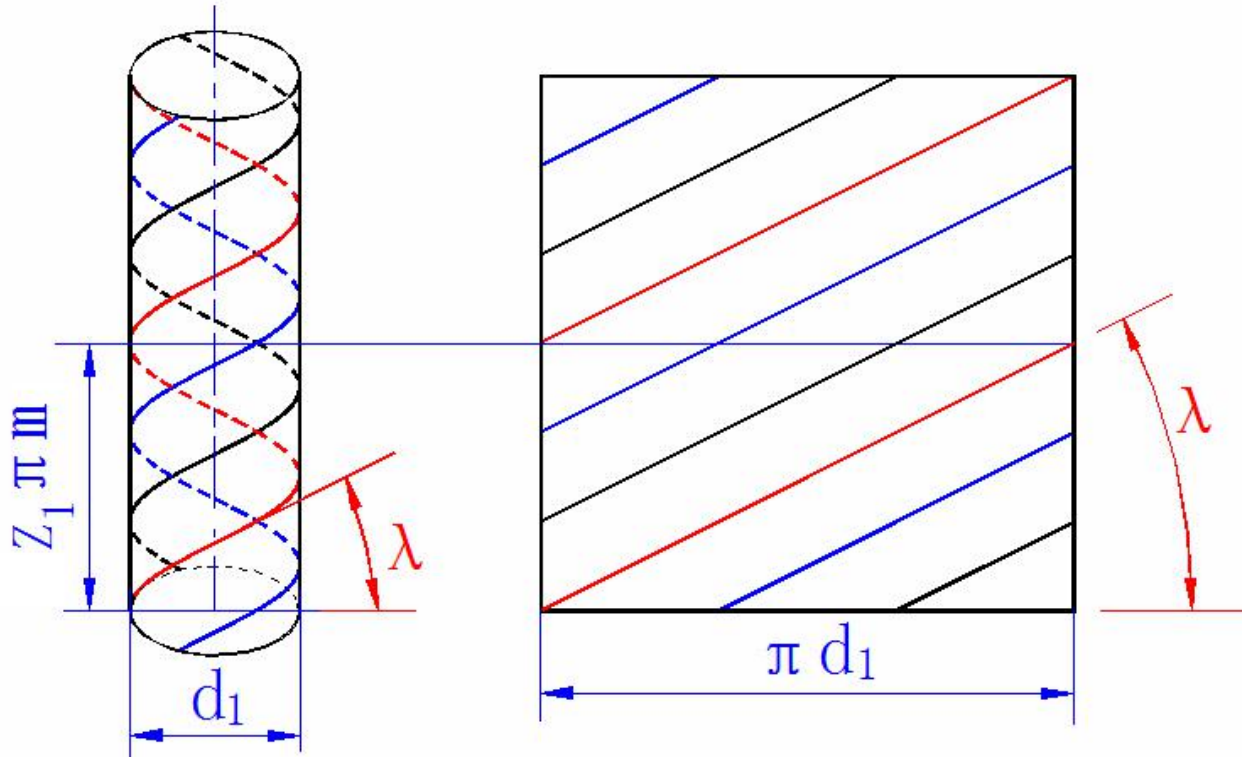


5) The lead angle(导程角) λ_1 of the worm.

$$\tan \lambda_1 = \frac{Z_1 \pi m}{\pi d_1} = \frac{m Z_1}{d_1}$$



$$d_1 = m \cdot \frac{Z_1}{\tan \gamma}$$



$$\frac{d_1}{m} = q = \frac{Z_1}{\tan \gamma}$$

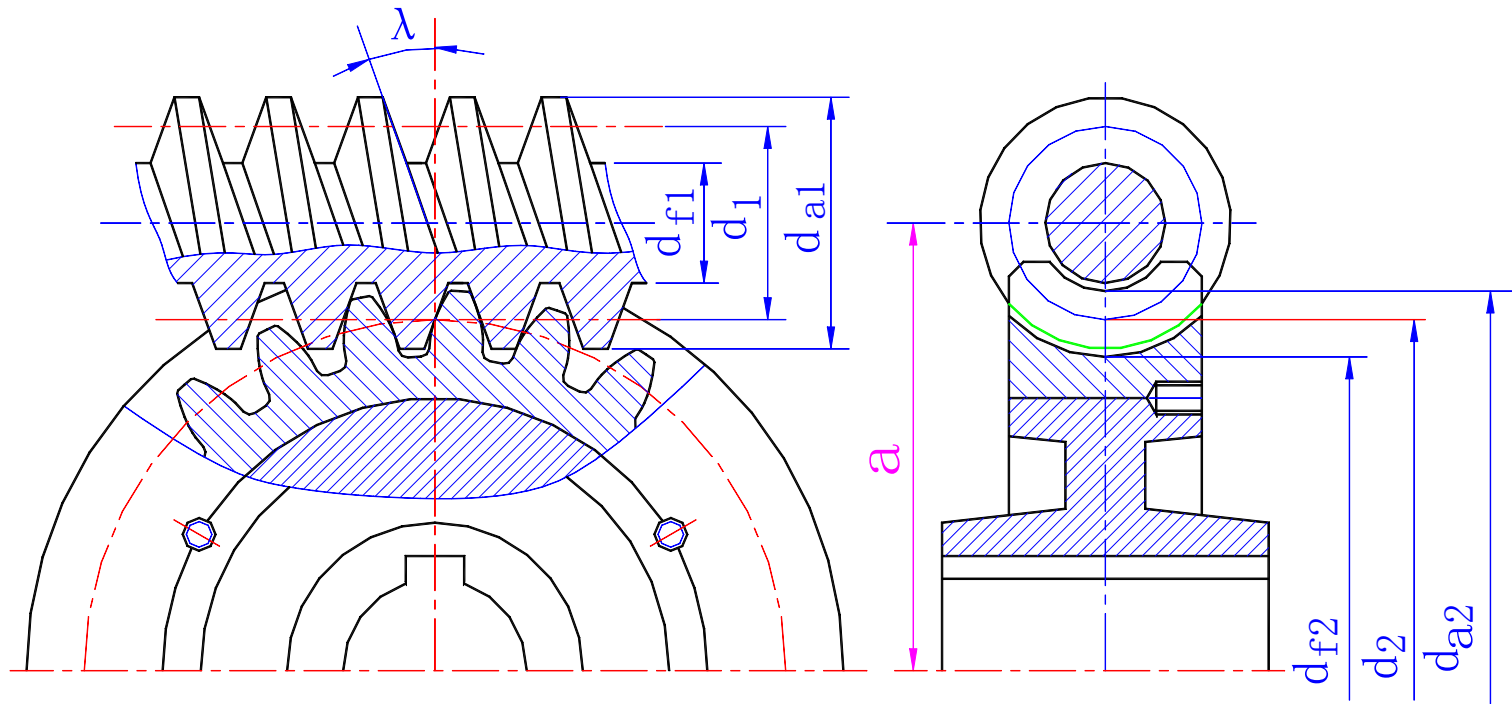
q——直径系数

$$d_1 = m q$$

$$\neq m Z_1$$

6) The centre distance a' of the worm gear pair.

$$a' = r_1 + r_2 + x_2 m = \frac{1}{2}(d_1 + mZ_2 + 2x_2)$$

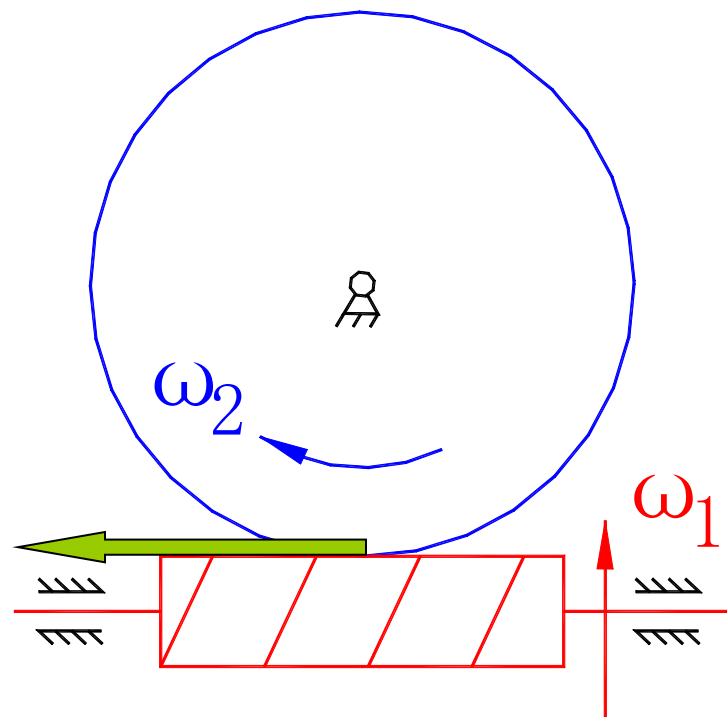


7) The transmission ratio and the rotation direction of wormwheel

传动比的大小

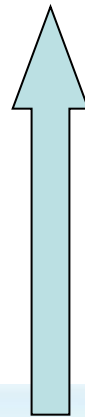
$$\pi m z_2 n_2 = \pi m z_1 n_1$$

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1}$$



判断蜗轮的转向：

蜗杆左（右）旋，伸左（右）手，四指握住蜗杆转动方向，拇指的反向为蜗轮的啮合点圆周速度的方向。



6.11 Bevel Gears(锥齿轮)

6.11.1 Types and Applications of Bevel Gears

Bevel gears are used to transmit motion and power between intersecting shafts.

The teeth of a bevel gear are distributed on the **frustum**(截锥体) of a cone(圆锥体).

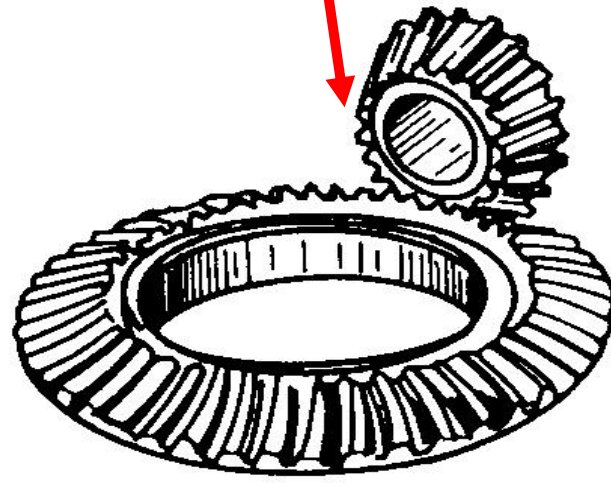
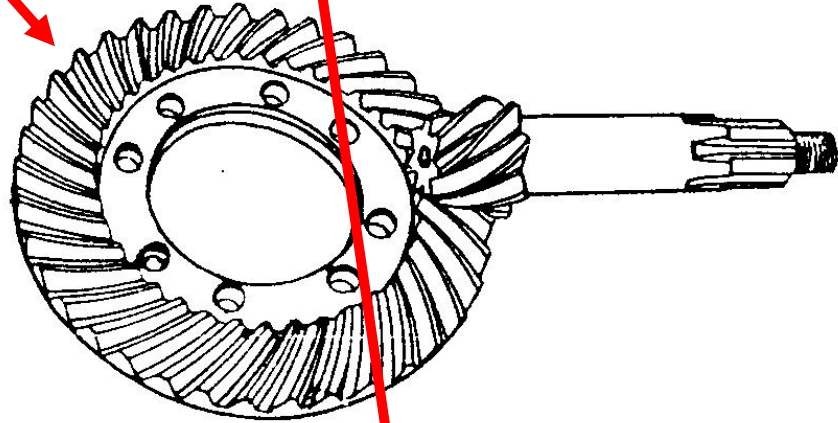
Parameters and dimensions at the large end are taken to be standard values.

The shaft angle of a bevel gear pair can be any required value. In most cases, the two shafts intersect at a right angle.





There are straight, helical and spiral (直齿、斜齿、曲齿) bevel gears.

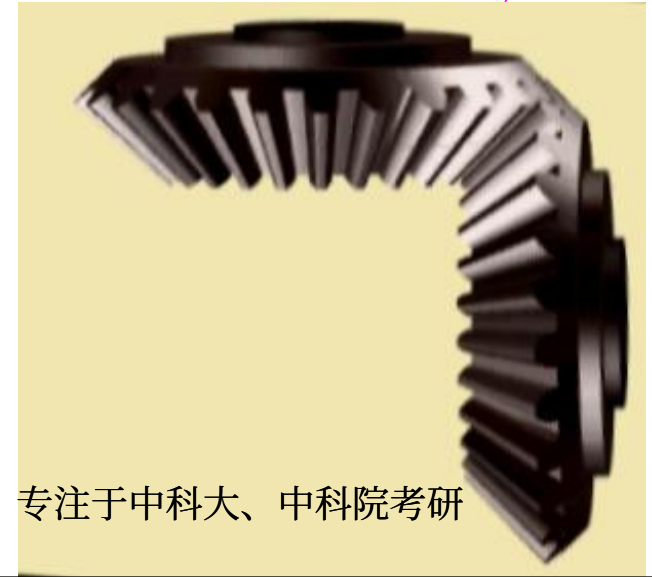
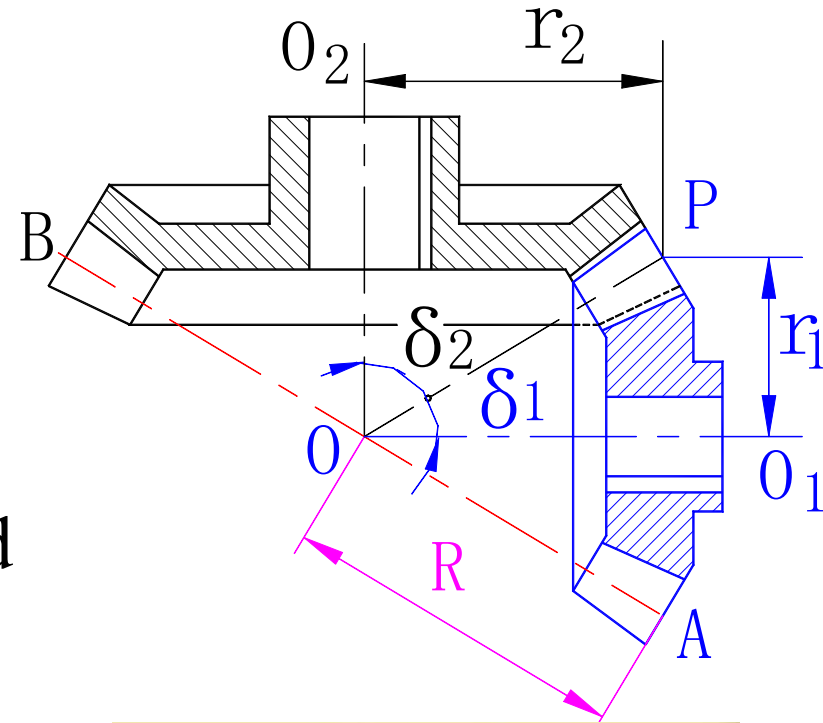


6.11.2 Back Cone(背锥) and Virtual Gear(当量齿轮) of a Bevel Gear

The bottom of the **reference cone(分度圆锥)** is called the reference circle.

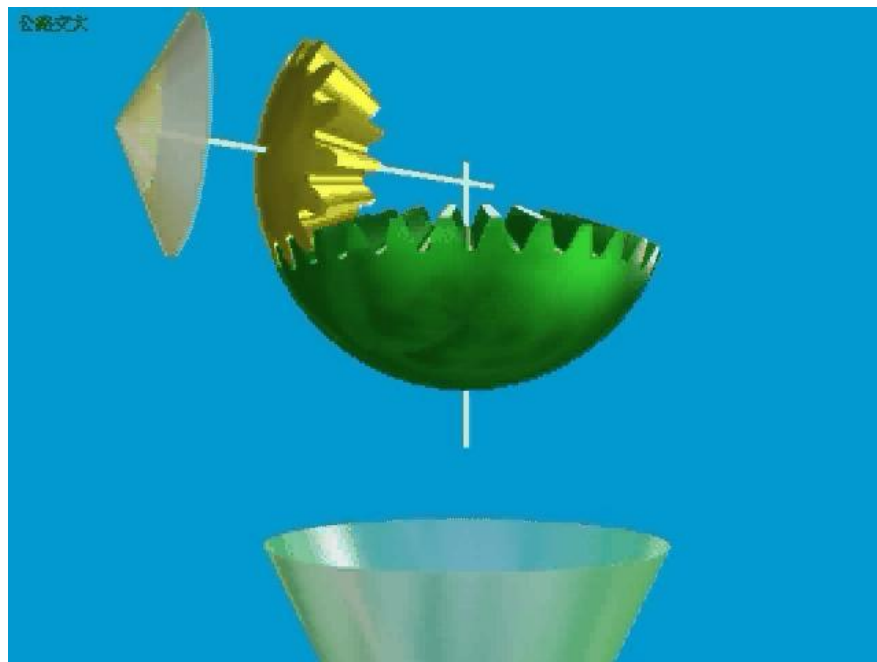
$$r = \frac{mz}{2}$$

Its theoretic tooth profile should be spherical(球面的) involute.

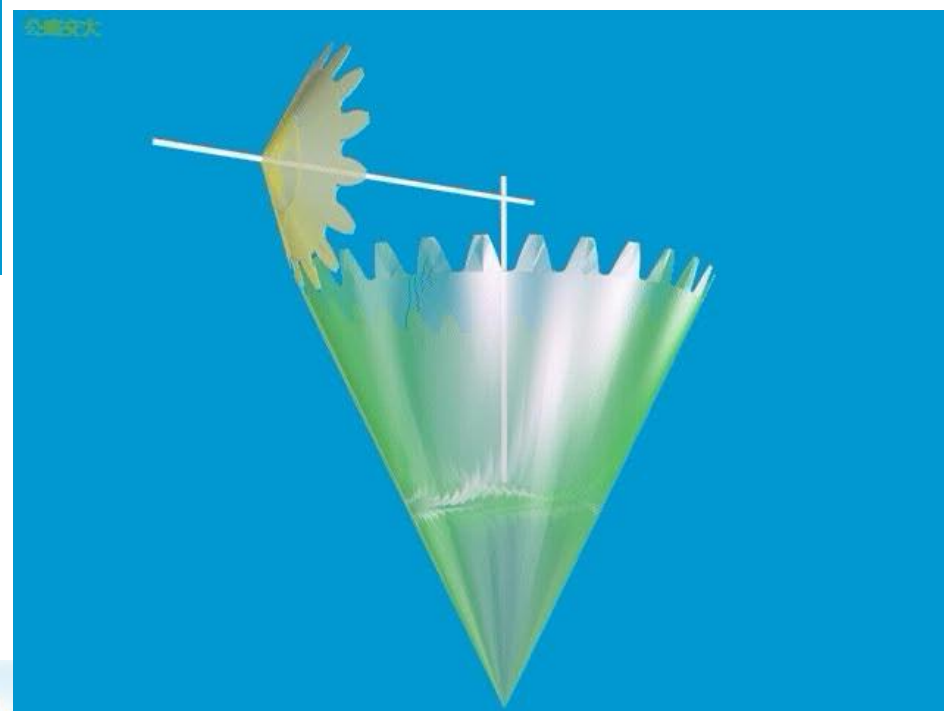




球面渐开线用一个锥体渐开线来近似



一对背锥齿轮的啮合相当于一对锥齿轮大端的啮合



分度圆上背锥圆锥面上的渐开线代替球面渐开线



The back cone can be unfolded(展开) imaginarily(假想地) into a sector gear(扇形齿轮) with Z teeth and further filled up to a full gear with Z_v teeth.

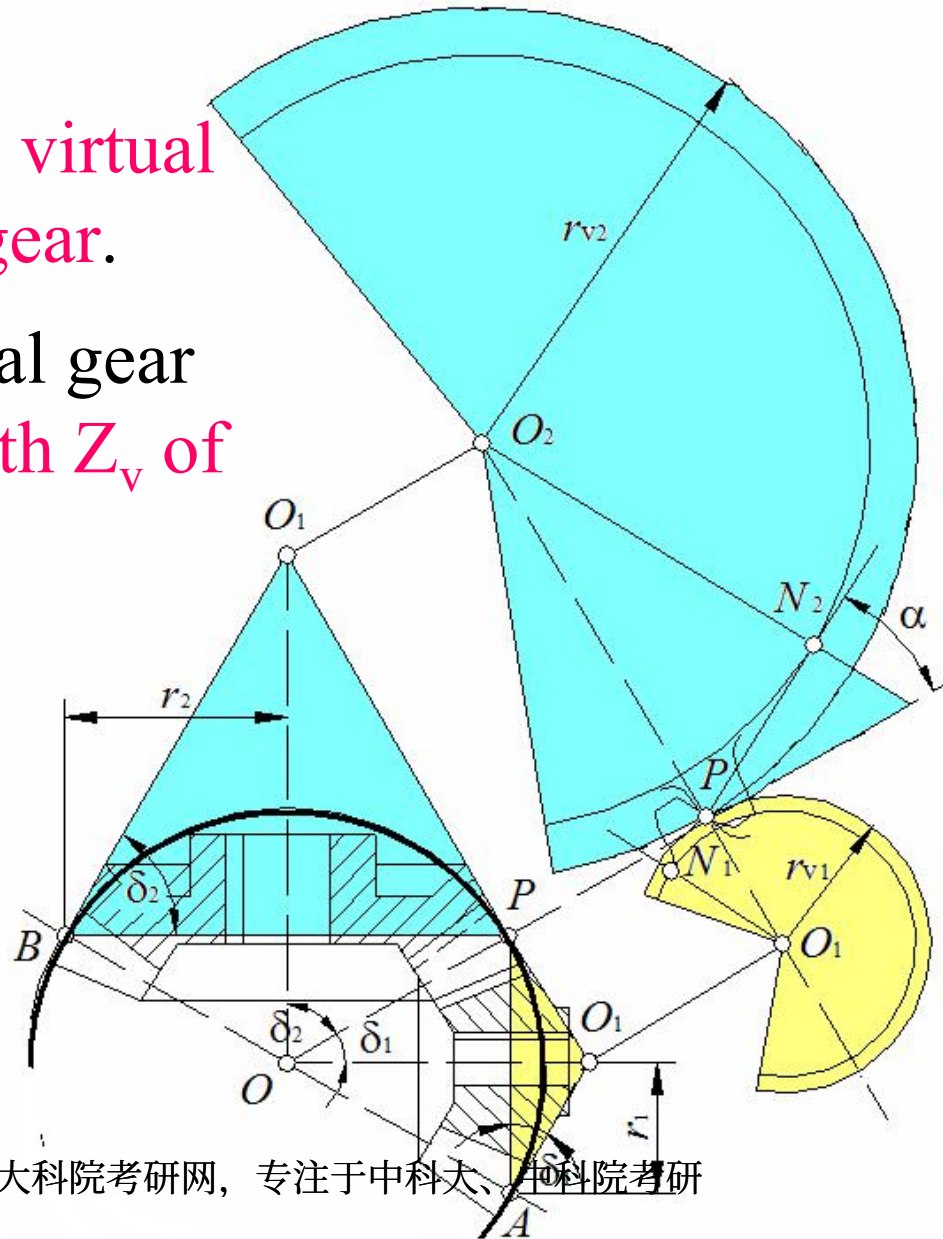
This imaginary gear —— the virtual gear(当量齿轮) of the bevel gear.

The tooth number of the virtual gear —— the virtual number of teeth Z_v of the bevel gear. 当量齿数

$$r_{v2} = \overline{O_2P} = \frac{r_2}{\cos \delta_2} = \frac{Z_2 m}{2 \cos \delta_2}$$

$$Z_{v2} = \frac{2r_{v1}}{m} = \frac{Z_2}{\cos \delta_2}$$

$$Z_{v1} = \frac{Z_1}{\cos \delta_1} \quad Z_v \text{ should not be less than } 17$$



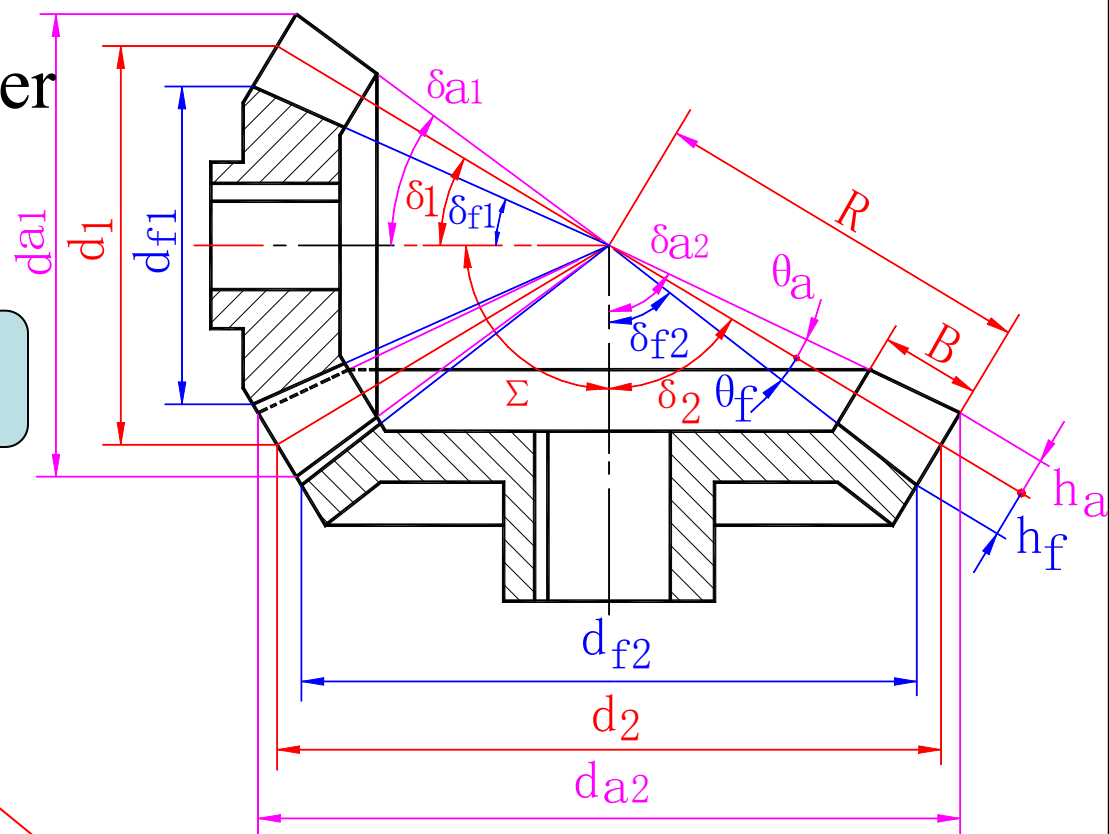
6.11.3 Parameters and Dimensions of Bevel Gears

1、the reference diameter

$$d = mZ = 2R \sin \delta$$

The outer
cone distance

The reference
cone angle



2. 正确啮合条件:

$$m_1 = m_2$$

$$\alpha_1 = \alpha_2$$

轴角

$$\Sigma = \delta_1 + \delta_2$$

Modules of bevel gears must
be chosen from Table 6-3



3、The transmission ratio

$$i_{12} = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{r_2}{r_1} = \tan \delta_2 = \cot \delta_1$$

$\uparrow$
 $\Sigma = 90^\circ$

4、The addendum h_a and dedendum h_f are measured along the **generatrix**(**母线**) of the back cone.

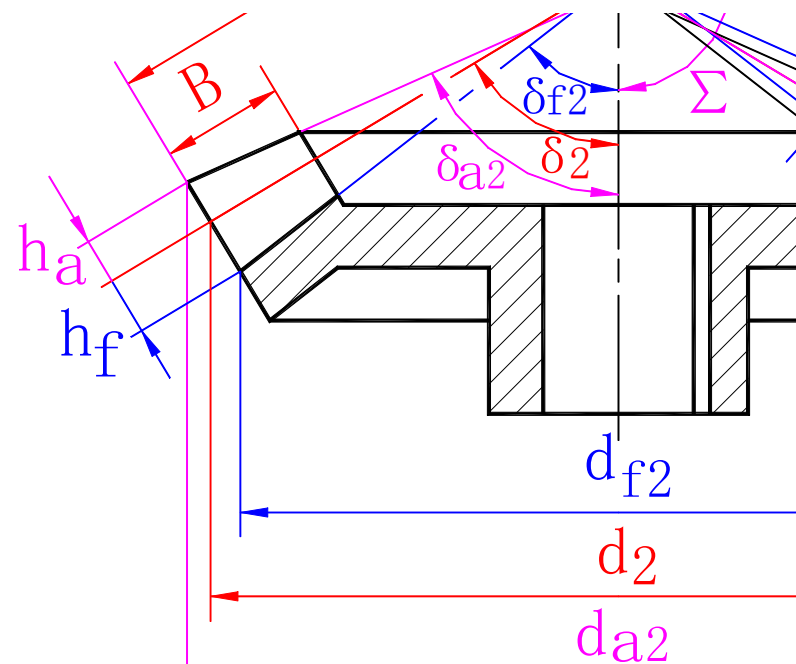
$$d_a = d + 2h_a \cos \delta$$

$$d_f = d - 2h_f \cos \delta$$

$$h_a = h_a^* m = 1m$$

$$h_f = (h_a^* + c^*) m = 1.2m$$

Here $c^* = 0.2$, **not 0.25**.





Chapter 6

Attentions

1. Parameters of standard Spur involute gear
2. Calculate the working pressure angle and the radius of pitch circles with actual centre distance
3. Calculate the contact ratio
4. Parameters of standard helical involute gear
5. Virtual gear of helical involute gear
6. Parameters of worm and worm wheel
7. Parameters of bevel gear
8. Virtual gear of bevel gear



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