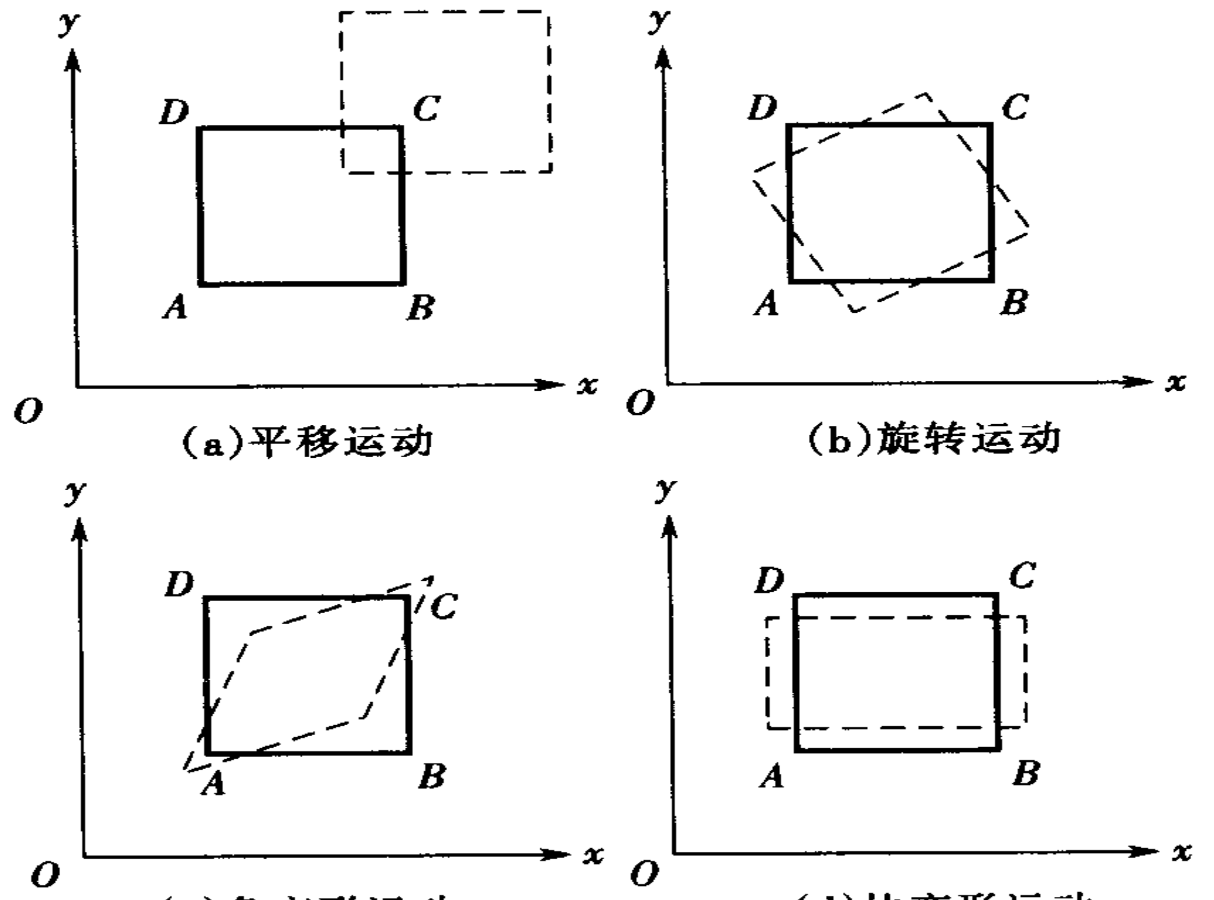
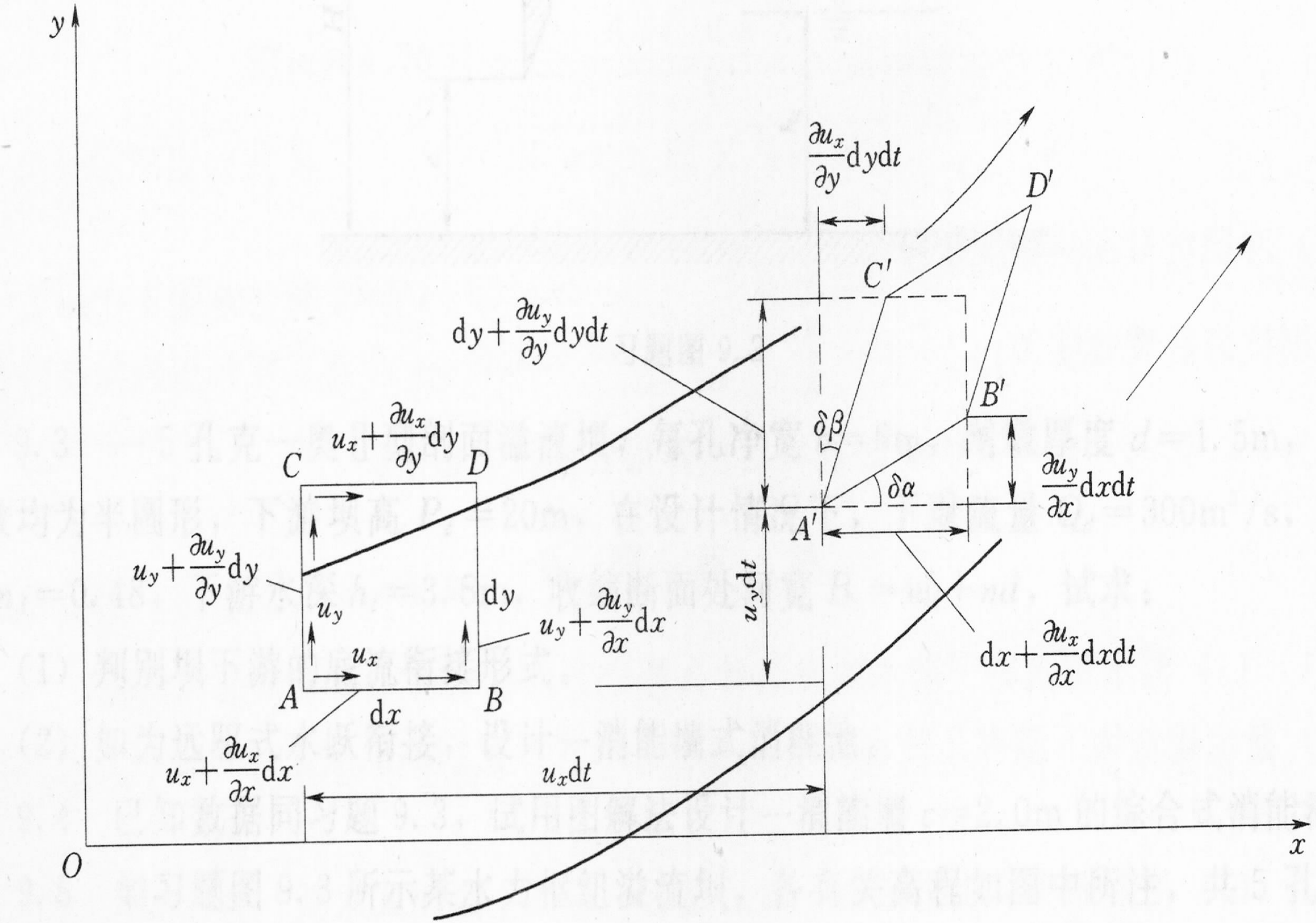
第 10 章　液体微团运动分析

10.1 基本形式：*平移、变形、旋转*

Translation, Transformation, Rotation



(a)平移运动　(b)旋转运动　(c)角变形运动　(d)块变形运动


y
x
O
C
D
A
B
dx
dy
u_x
u_y
$u_x+\frac{\partial u_x}{\partial y}dy$
$u_y+\frac{\partial u_y}{\partial y}dy$
$u_y+\frac{\partial u_y}{\partial x}dx$
$u_x+\frac{\partial u_x}{\partial x}dx$
$u_x dt$
$u_y dt$
$dy+\frac{\partial u_y}{\partial y}dydt$
$\frac{\partial u_x}{\partial y}dydt$
D'
C'
B'
A'
$\delta\beta$
$\delta\alpha$
$\frac{\partial u_y}{\partial x}dxdt$
$dx+\frac{\partial u_x}{\partial x}dxdt$

A点速度为 u_x, u_y

B点速度为

$$u_x+\frac{\partial u_x}{\partial x}dx,\qquad u_y+\frac{\partial u_y}{\partial x}dx$$

C点速度为

$$u_x+\frac{\partial u_x}{\partial y}dy,\qquad u_y+\frac{\partial u_y}{\partial y}dy$$

10.1.1 平移速度为 u_x, u_y

10.1.2 变形运动

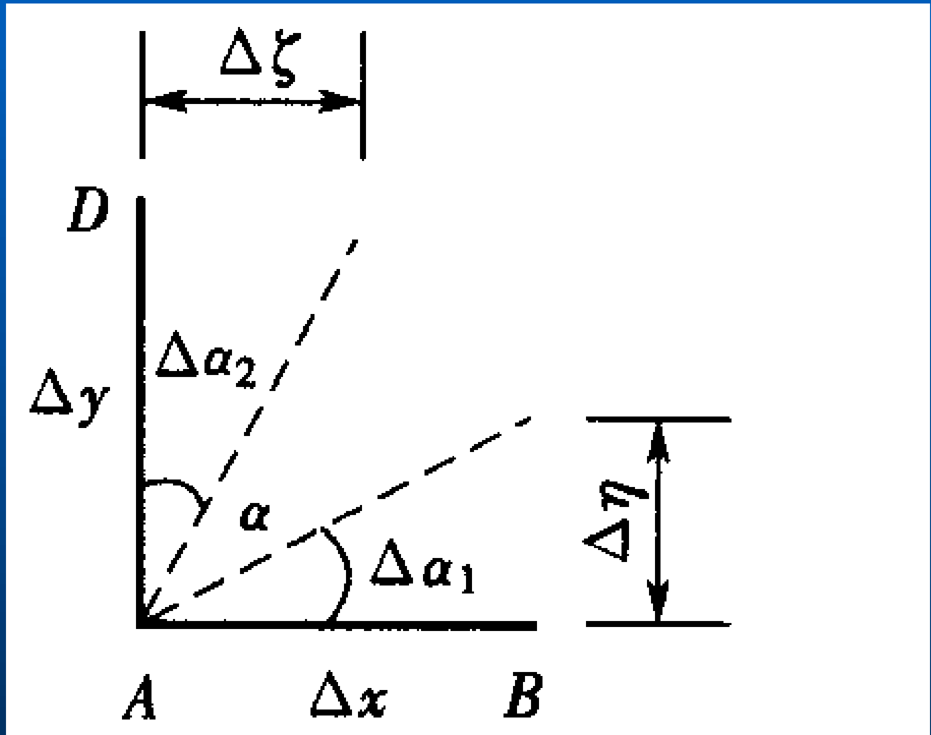
线变形速度(Linear strain rate)：
单位时间单位长度的线变形率。

$$e_{xx} = \frac{(u_B - u_A)dt}{dxdt} = \frac{\partial u_x}{\partial x} dxdt / dxdt = \frac{\partial u_x}{\partial x}, \quad e_{yy} = \frac{\partial u_y}{\partial y}, \quad e_{zz} = \frac{\partial u_z}{\partial z}$$

剪切变形速度 (Shear strain rate)：

平面上原相互正交线相互靠近角变形速度的一半/

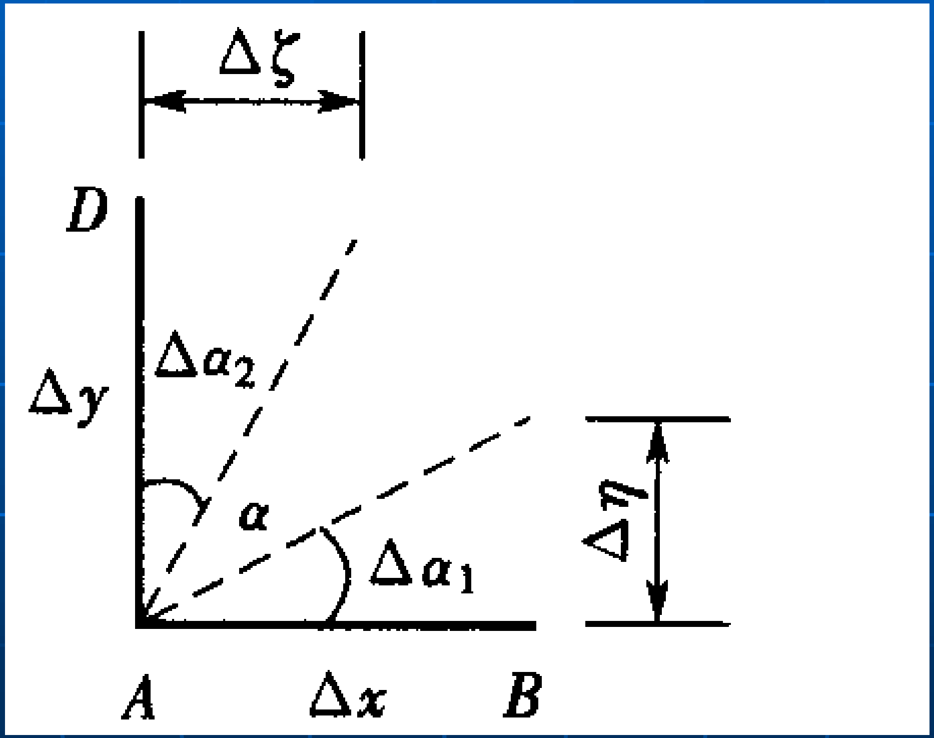
the rate of angle closure of the initially perpendicular lines。



$$e_{xy}=e_{yx}=\frac{1}{2}\frac{\delta\alpha+\delta\beta}{dt}=\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right),\quad e_{yz}=e_{zy}=\frac{1}{2}\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right),\quad e_{xz}=e_{zx}=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right)$$

10.1.3 旋转角速度 (Anglular velocity)

液体微团上两相邻原来相互正交边逆时针旋转角度的平均值/the average rate of counter-clockwise turning of the two originally perpendicular lines



$$\omega_z=\frac{1}{2}\frac{\delta\beta-\delta\alpha}{dt}=\frac{1}{2}\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right),\quad \omega_x=\frac{1}{2}\left(\frac{\partial u_z}{\partial y}-\frac{\partial u_y}{\partial z}\right),\quad \omega_y=\frac{1}{2}\left(\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}\right)$$

10.1.3 旋转角速度 (Anglular velocity)

$$\omega_z = \frac{1}{2}\frac{\delta\beta - \delta\alpha}{dt} = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right), \quad \omega_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right), \quad \omega_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)$$

$$\vec{\omega} = \omega_x i + \omega_y j + \omega_z k = \frac{1}{2}\nabla \times \vec{u} = \frac{1}{2} rot\,\vec{u}$$

10.2 速度分解定理

设在时刻t，液体微团p点的速度为**u**(u_x,u_y,u_z)，邻近q点的速度为**u**+d**u**

$$du_x=\frac{\partial u_x}{\partial x}dx+\frac{\partial u_x}{\partial y}dy+\frac{\partial u_x}{\partial z}dz$$

$$=\frac{\partial u_x}{\partial x}dx+\frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)dy+\frac{1}{2}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right)dz+$$

$$\frac{1}{2}\left(\frac{\partial u_x}{\partial y}-\frac{\partial u_z}{\partial x}\right)dz-\frac{1}{2}\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right)dy$$

$$=e_{xx}dx+(e_{xy}-\omega_z)dy+(e_{xz}+\omega_y)dz$$

$$u_{qx}=u_x+e_{xx}dx+(e_{xy}-\omega_z)dy+(e_{xz}+\omega_y)dz$$

$$u_{qy}=u_y+e_{yy}dy+(e_{xy}+\omega_z)dx+(e_{yz}-\omega_x)dz$$

$$u_{qz}=u_z+e_{zz}dz+(e_{zx}-\omega_y)dx+(e_{yz}+\omega_x)dy$$