

5-6 解题过程：

$$\text{令 } e_1(t) = \frac{\pi}{\omega_c} \delta(t), \quad e_2(t) = \frac{\sin(\omega_c t)}{\omega_c t}$$

$$E_1(j\omega) = \mathcal{F}[e_1(t)] = \frac{\pi}{\omega_c}$$

$$E_2(j\omega) = \mathcal{F}[e_2(t)] = \frac{\pi}{\omega_c} [u(\omega + \omega_c) - u(\omega - \omega_c)] = \begin{cases} \frac{\pi}{\omega_c}, & |\omega| < \omega_c \\ 0, & \text{其他} \end{cases}$$

理想低通的系统函数的表达式 $H(j\omega) = |H(j\omega)|e^{j\varphi(\omega)}$

$$\text{其中} \quad |H(j\omega)| = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & |\omega| \geq \omega_c \end{cases} \quad \varphi(\omega) = -t_0\omega$$

因此有

$$R_1(j\omega) = H(j\omega)E_1(j\omega) = \begin{cases} \frac{\pi}{\omega_c} e^{-t_0\omega}, & |\omega| < \omega_c \\ 0, & \text{其他} \end{cases}$$

$$R_2(j\omega) = H(j\omega)E_2(j\omega) = \begin{cases} \frac{\pi}{\omega_c} e^{-t_0\omega}, & |\omega| < \omega_c \\ 0, & \text{其他} \end{cases}$$

$$R_1(j\omega) = R_2(j\omega)$$

$$\text{则 } \mathcal{F}^{-1}[R_1(j\omega)] = \mathcal{F}^{-1}[R_2(j\omega)]$$

5-8 解题过程：

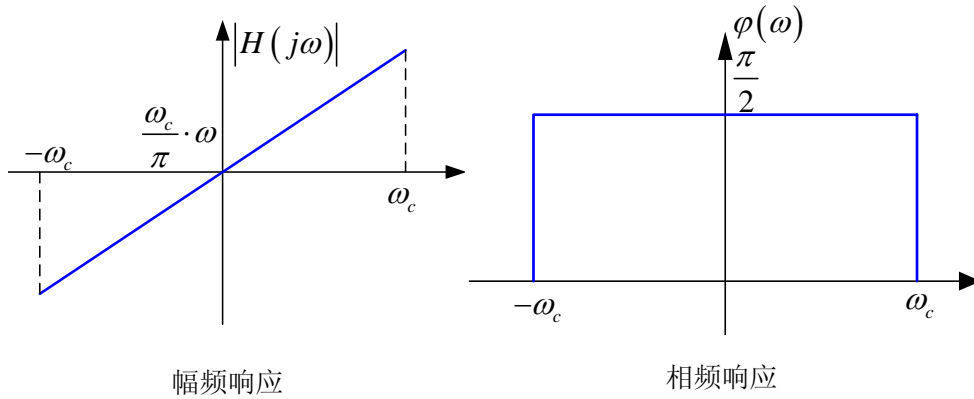
$$\text{记 } f(t) = \frac{\sin \omega_c t}{\pi t} = \frac{\sin \omega_c t}{\omega_c t} \cdot \frac{\omega_c}{\pi}$$

$$F(j\omega) = \mathcal{F}[f(t)] = \begin{cases} \frac{\pi}{\omega_c}, & |\omega| < \omega_c \\ 0, & |\omega| \geq \omega_c \end{cases}$$

$$\begin{aligned} H(j\omega) &= \mathcal{F}[h(t)] = \mathcal{F}\left\{\frac{d}{dt}\left[\frac{\sin(\omega_c t)}{\pi t}\right]\right\} \\ &= j\omega F(j\omega) = \begin{cases} \frac{\pi}{\omega_c} \cdot j\omega, & |\omega| < \omega_c \\ 0, & |\omega| \geq \omega_c \end{cases} \end{aligned}$$

$$\text{故 } H(j\omega) = \begin{cases} \frac{\omega_c}{\pi} \cdot \omega, & |\omega| < \omega_c \\ 0, & |\omega| \geq \omega_c \end{cases} \quad \varphi(\omega) = \begin{cases} \frac{\pi}{2}, & |\omega| < \omega_c \\ 0, & |\omega| \geq \omega_c \end{cases}$$

$|H(j\omega)|$ 和 $\varphi(\omega)$ 的图形如解图。



5-11 解题过程：

$$\text{由题图 5-11 有 } v_2(t) = [v_1(t-T) - v_1(t)] * h(t)$$

$$\text{据时域卷积定理有 } V_2(j\omega) = [V_1(j\omega)e^{-j\omega T} - V_1(j\omega)]H(j\omega)$$

$$(1) v_1(t) = u(t)$$

$$v_2(t) = [u(t-T) - u(t)] * h(t)$$

$$\text{由 } h(t) = \mathcal{F}^{-1}[H(j\omega)] = \frac{1}{\pi} Sa(t-t_0), \quad f(t) * u(t) = \int_{-\infty}^t f(\lambda) d\lambda, \quad \text{有}$$

$$\begin{aligned} v_2(t) &= \frac{1}{\pi} \int_{-\infty}^{t-T} Sa(\lambda - t_0) d\lambda - \frac{1}{\pi} \int_{-\infty}^t Sa(\lambda - t_0) d\lambda \\ &= \frac{1}{\pi} \int_{-\infty}^{t-t_0-T} Sa(\lambda') d\lambda' - \frac{1}{\pi} \int_{-\infty}^{t-t_0} Sa(\lambda'') d\lambda'' \end{aligned}$$

又知 $S_i(y) = \int_{-\infty}^y Sa(x) dx$ ，所有

$$v_2(t) = \frac{1}{\pi} [S_i(t-t_0-T) - S_i(t-t_0)]$$

$$(2) v_1(t) = \frac{2 \sin\left(\frac{t}{2}\right)}{t} = Sa\left(\frac{t}{2}\right)$$

$$V_1(j\omega) = F[v_1(t)] = \begin{cases} 2\pi & |\omega| < \frac{1}{2} \\ 0 & \text{其他} \end{cases}$$

$$\text{则 } V_2(j\omega) = V_1(j\omega)H(j\omega)(e^{-j\omega T} - 1) = \begin{cases} 2\pi e^{-j\omega t_0} (e^{-j\omega T} - 1) & |\omega| < \frac{1}{2} \\ 0 & \text{其他} \end{cases}$$

$$\text{所以 } v_2(t) = \mathcal{F}^{-1}[V_2(j\omega)] = Sa\left[\frac{1}{2}(t-t_0-T)\right] - Sa\left[\frac{1}{2}(t-t_0)\right]$$

5-18 解题过程：

信号 $g(t)$ 经过滤波器 $H(j\omega)$ 的频谱为

$$G_1(\omega) = G(\omega)H(j\omega) = -j \operatorname{sgn}(\omega)G(\omega)$$

信号 $g(t)$ 经过与 $\cos(\omega_0 t)$ 进行时域相乘后频谱为

$$G_2(\omega) = \frac{1}{2}[G(\omega + \omega_0) + G(\omega - \omega_0)]$$

信号 $g_1(t)$ 经过与 $-\sin(\omega_0 t)$ 进行时域相乘后频谱为

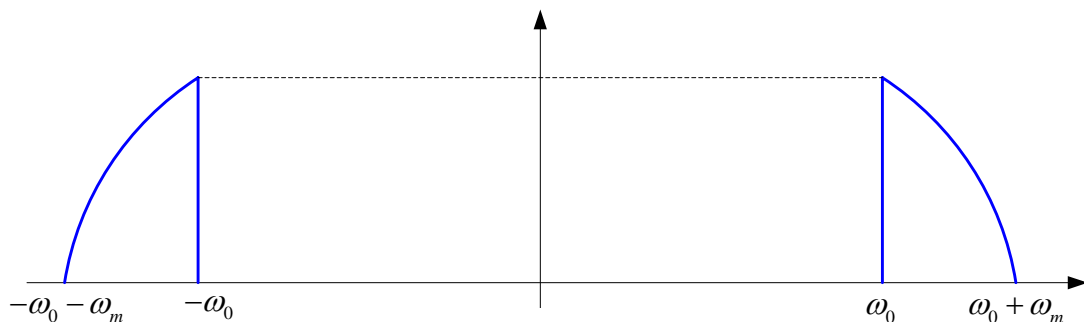
$$\begin{aligned} G_3(\omega) &= -\frac{j}{2}[G_1(\omega + \omega_0) - G_1(\omega - \omega_0)] \\ &= -\frac{1}{2}[G(\omega + \omega_0)\operatorname{sgn}(\omega + \omega_0) - G(\omega - \omega_0)\operatorname{sgn}(\omega - \omega_0)] \\ &= \frac{1}{2}[G(\omega - \omega_0)\operatorname{sgn}(\omega - \omega_0) + G(\omega + \omega_0)\operatorname{sgn}(\omega + \omega_0)] \end{aligned}$$

$$\begin{aligned} V(\omega) &= G_2(\omega) + G_3(\omega) \\ &= \frac{1}{2}[G(\omega + \omega_0) + G(\omega - \omega_0)] + \frac{1}{2}[G(\omega - \omega_0)\operatorname{sgn}(\omega - \omega_0) + G(\omega + \omega_0)\operatorname{sgn}(\omega + \omega_0)] \\ &= \frac{1}{2}\{G(\omega + \omega_0)[1 - \operatorname{sgn}(\omega + \omega_0)] + G(\omega - \omega_0)[1 + \operatorname{sgn}(\omega + \omega_0)]\} \end{aligned}$$

$$\text{又由于 } 1 + \operatorname{sgn}(\omega - \omega_0) = \begin{cases} 2 & (\omega > \omega_0) \\ 0 & (\omega < \omega_0) \end{cases}$$

$$\text{则 } V(\omega) = G(\omega - \omega_0)U(\omega - \omega_0) + G(\omega + \omega_0)U(\omega + \omega_0)$$

其图形如图所示



5-20 解题过程：

(1) 系统输入信号为 $\delta(t)$ 时， $\delta(t)\cos(\omega_0 t) = \delta(t)$

所以虚框所示系统的冲激响应 $h(t)$ 就是 $h_i(t)$

$$\text{即 } h(t) = \mathcal{F}^{-1}[H_i(j\omega)] = \frac{\sin[2\Omega(t-t_0)]}{\pi(t-t_0)}$$

(2) 输入信号与 $\cos(\omega_0 t)$ 在时域相乘之后

$$e(t)\cos\omega_0 t = \left[\frac{\sin(\Omega t)}{\Omega t}\right]^2 \cos^2(\omega_0 t) = \left[\frac{\sin(\Omega t)}{\Omega t}\right]^2 \frac{1+\cos(2\omega_0 t)}{2}$$

又由 $H_i(j\omega)$ 的表达式可知 $\omega_0 \gg \Omega$ 时，载波为 $2\omega_0$ 的频率成分被滤除

而且 $\varphi(\omega) = -\omega t_0$

$$\text{故 } r(t) = \frac{1}{2} \left[\frac{\sin\Omega(t-t_0)}{\Omega(t-t_0)}\right]^2$$

(3) 输入信号 $e(t)$ 与 $\cos\omega_0 t$ 在时域相乘之后

$$e(t)\cos\omega_0 t = \left[\frac{\sin\Omega(t)}{\Omega t}\right]^2 \sin\omega_0 t \cos\omega_0 t = \left[\frac{\sin\Omega(t)}{\Omega t}\right]^2 \cdot \frac{1}{2} \sin(2\omega_0 t)$$

$\omega_0 \gg \Omega$ 时，载波为 $2\omega_0$ 的频率成分被滤除

故 $r(t) = 0$

(4) 由于理想低通滤波器能够无失真的传输信号，只是时间上的搬移，故理想低通滤波器是线性时变系统；又 $h(t) = h_i(t)$ 所以该系统是线性时变的。