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中国科学院 2004 年高等代数真题解析

1.(15 分) $\begin{cases} x_{n+1} = x_n + 4y_n \\ y_{n+1} = 2x_n + y_n \end{cases}$，已知 $x_0 = 1$， $y_0 = 0$，求 x_{100}, y_{100}。

【解答】

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix}，因此，\begin{pmatrix} x_{100} \\ y_{100} \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}^{100} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}^{100} \begin{pmatrix} 1 \\ 0 \end{pmatrix}。$$

令 $A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}$，容易得到 A 的特征值为 $\lambda_1 = 1 + 2\sqrt{2}$， $\lambda_2 = 1 - 2\sqrt{2}$，令

$\varphi_1(\lambda) = \lambda - (1 - 2\sqrt{2})$， $\varphi_2(\lambda) = \lambda - (1 + 2\sqrt{2})$，

$$G_1 = \frac{\varphi_1(A)}{\varphi_1(\lambda_1)} = \frac{\begin{pmatrix} 2\sqrt{2} & 4 \\ 2 & 2\sqrt{2} \end{pmatrix}}{4\sqrt{2}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix}$$

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