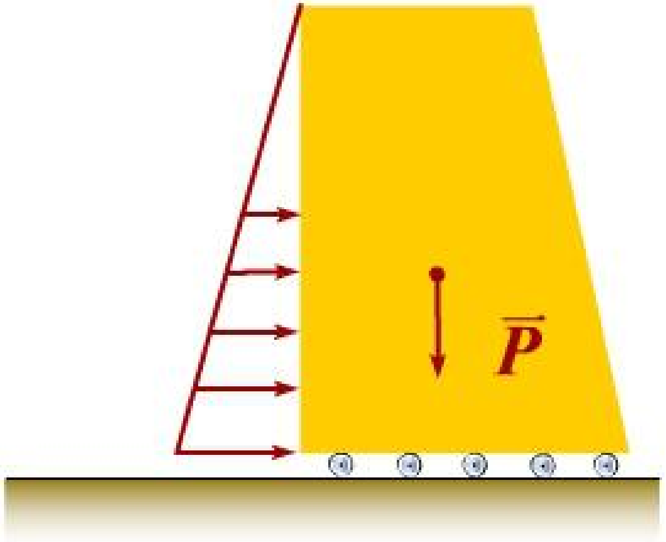
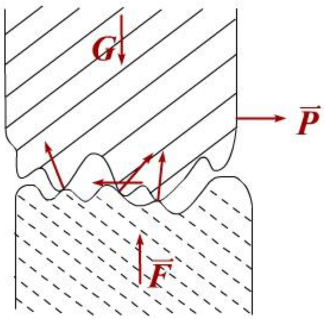
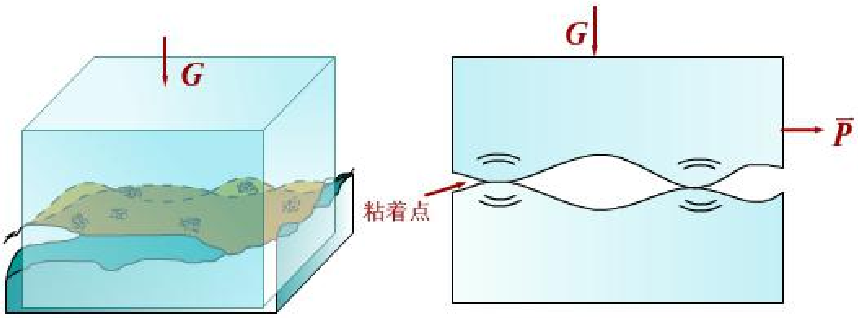
第四章　摩　擦

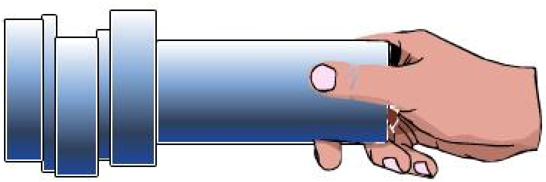


摩擦
- 滑动摩擦
  - 静滑动摩擦
  - 动滑动摩擦
- 滚动摩擦
  - 静滚动摩擦
  - 动滚动摩擦

摩擦
- 干摩擦
- 湿摩擦



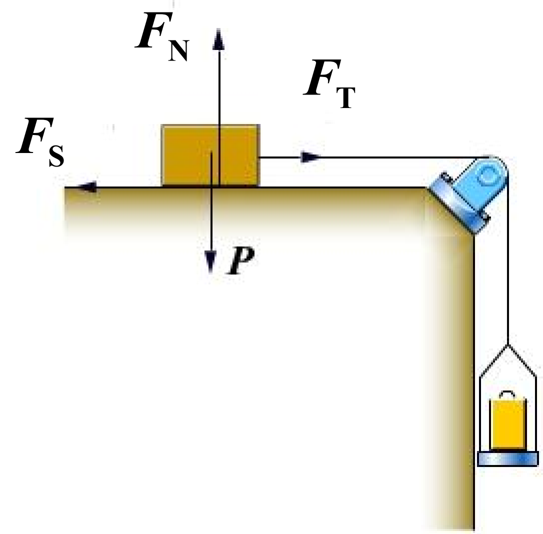




《摩擦学》

§4-1　滑动摩擦

$$\sum F_x = 0 \quad F_T - F_S = 0 \quad F_S = F_T$$



静滑动摩擦力的特点

方向：沿接触处的公切线，
与相对滑动趋势反向；

大小：　$0 \le F_s \le F_{max}$

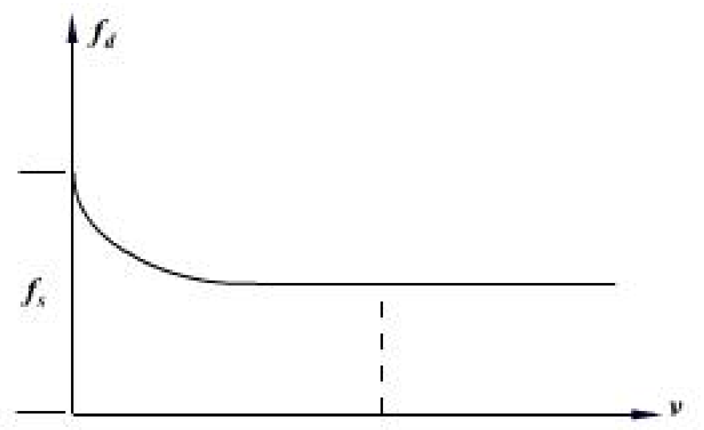
$$F_{max} = f_s F_N$$ （库仑摩擦定律）

动滑动摩擦力的特点

方向：沿接触处的公切线，与相对滑动趋势反向；

大小：$F_d = f_d F_N$

$f_d < f_s$ （对多数材料，通常情况下）



§4-2　摩擦角和自锁现象

一. 摩擦角

$\vec{F}_{RA}$ ----全约束力

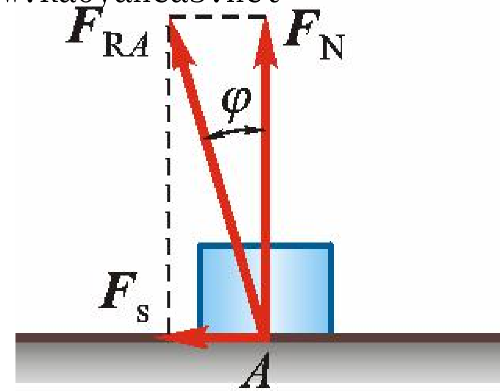
物体处于临界平衡状态时，全约束力和法线间的夹角----摩擦角

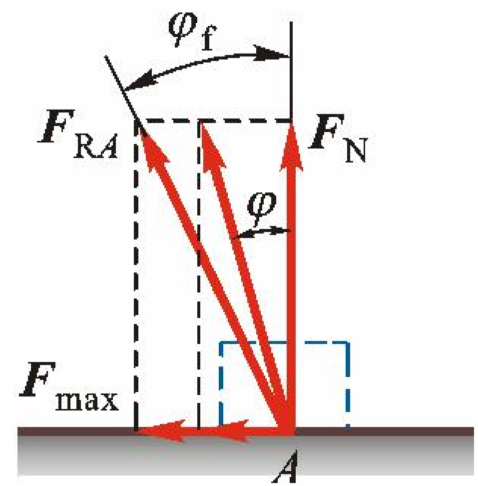
$$\tan\varphi_f = \frac{F_{max}}{F_N} = \frac{f_s F_N}{F_N} = f_s$$

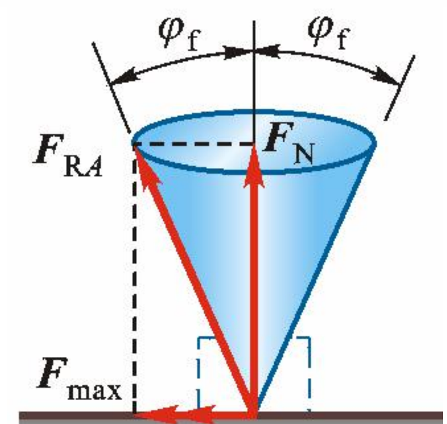
↓

全约束力和法线间的夹角的正切等于静滑动摩擦系数.

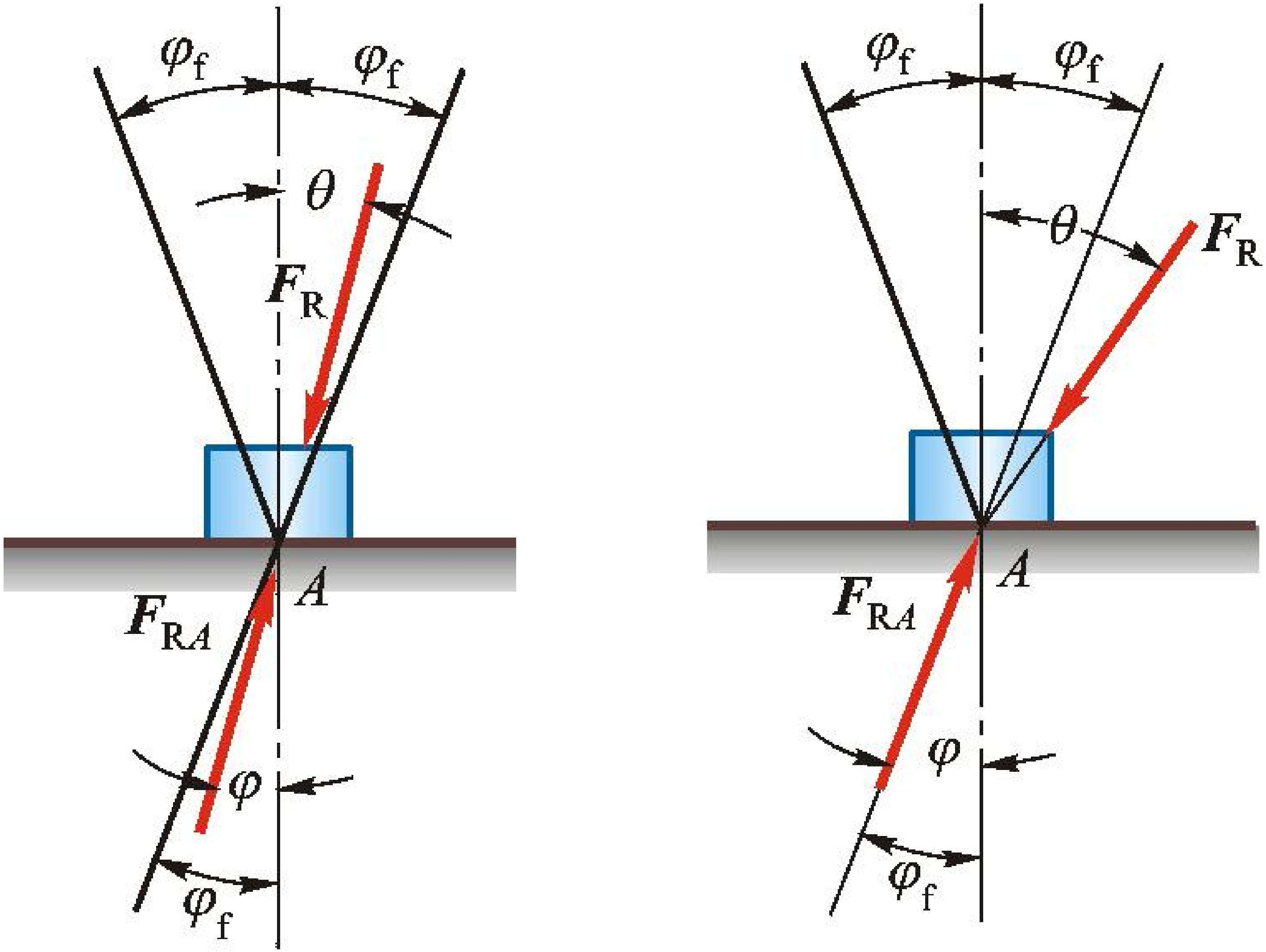
摩擦锥　　$0 \le \varphi \le \varphi_f$



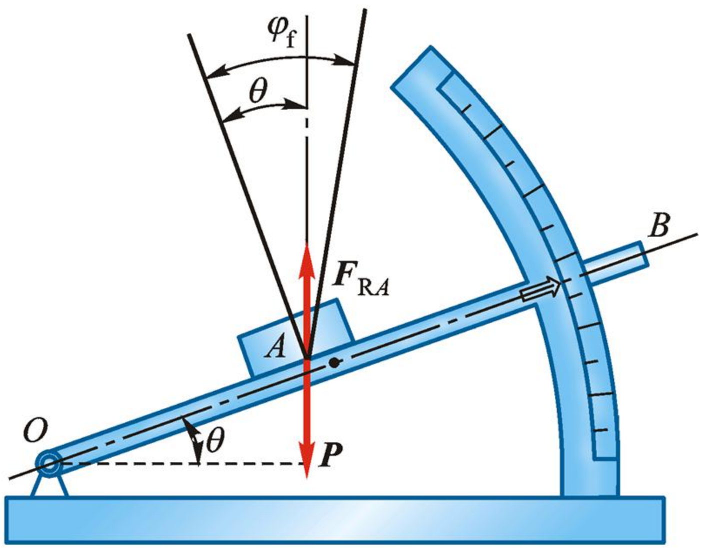




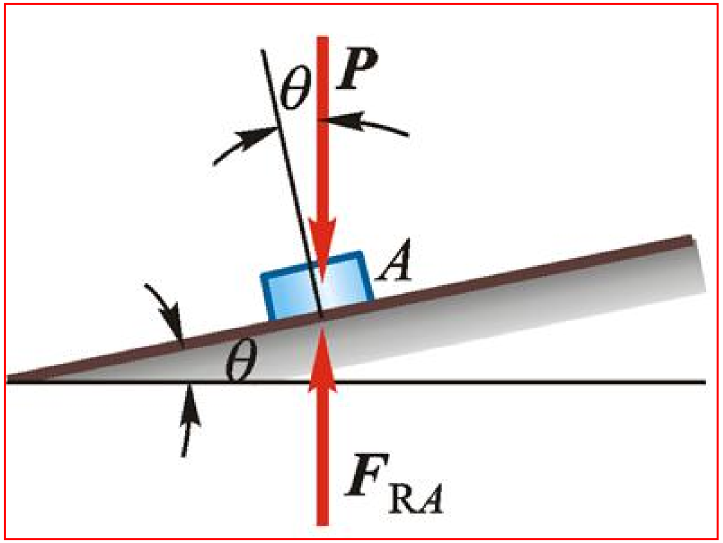
二. 自锁现象



三. 测定摩擦系数的一种简易方法，斜面与螺纹自锁条件



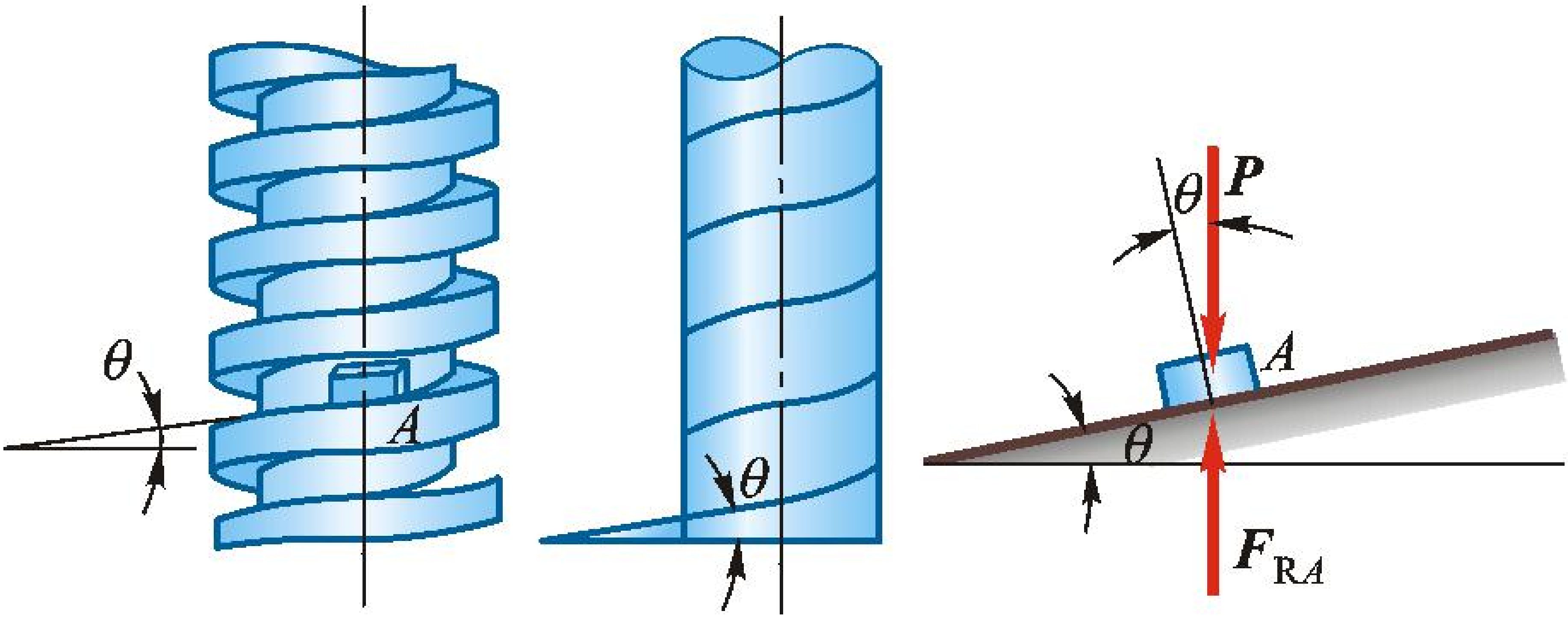
$$\tan\theta = \tan\varphi_f = f_s$$



斜面自锁条件 $\theta \le \varphi_f$

螺纹自锁条件

$$\theta \le \varphi_f$$



§4-3 考虑滑动摩擦时物体的平衡问题

仍为平衡问题，平衡方程照用，求解步骤与前面基本相同.

几个新特点

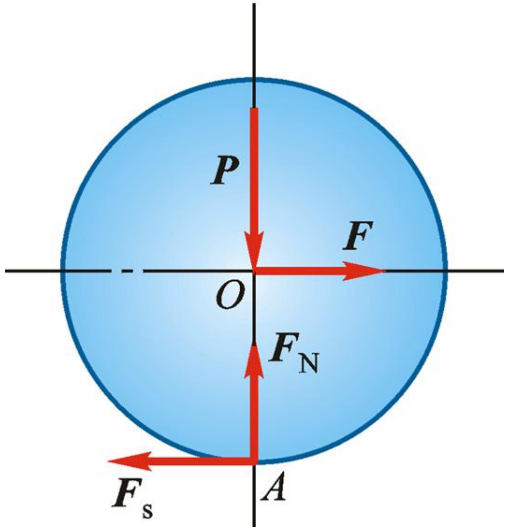
1 画受力图时，必须考虑摩擦力；

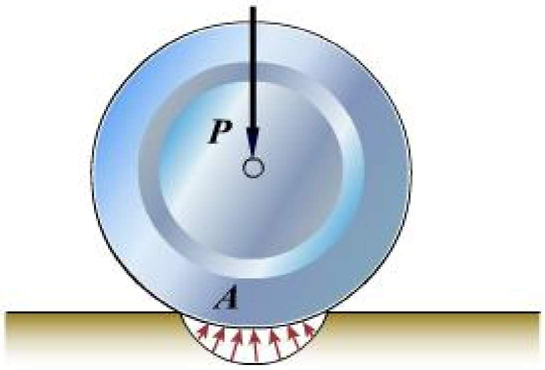
2 严格区分物体处于临界、非临界状态；

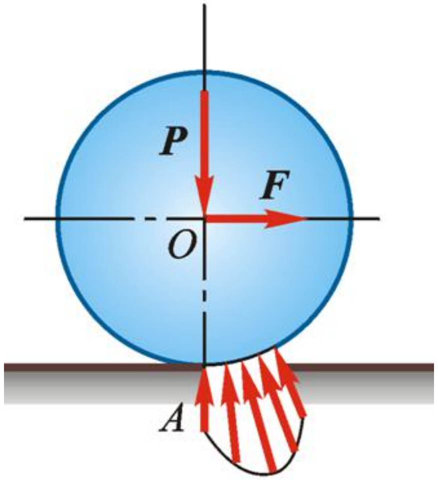
3 因$0 \le F_s \le F_{\max}$，问题的解有时在一个范围内.

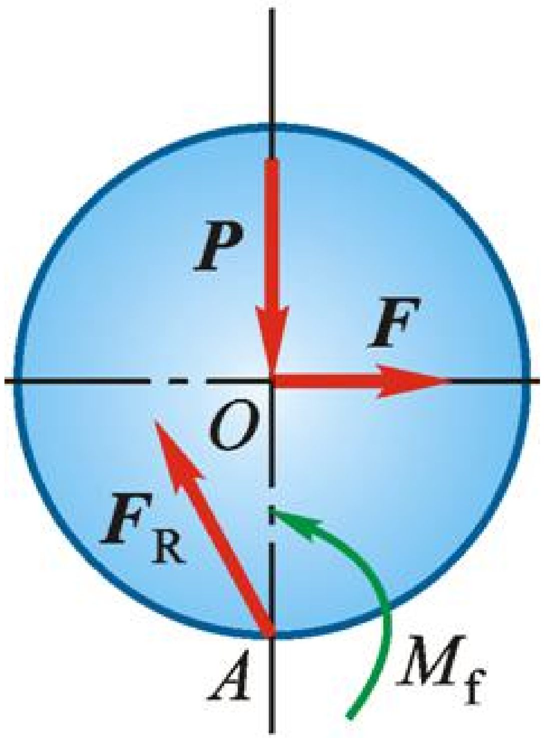
§4-4 滚动摩阻（擦）的概念

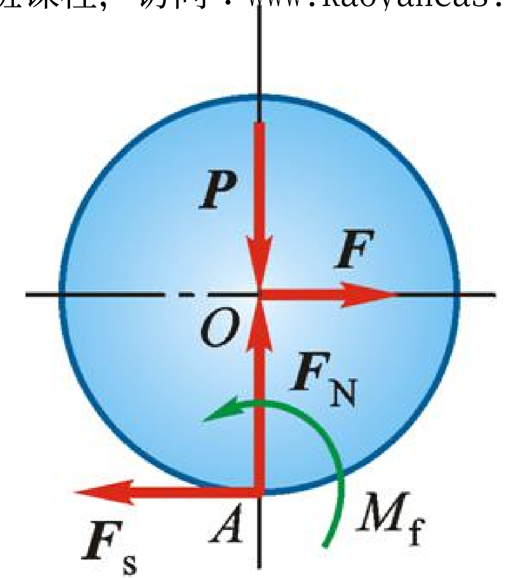
静滚动摩阻（擦）











$$\sum F_x = 0 \qquad F - F_s = 0$$

$$\sum M_A = 0 \qquad M - FR = 0$$

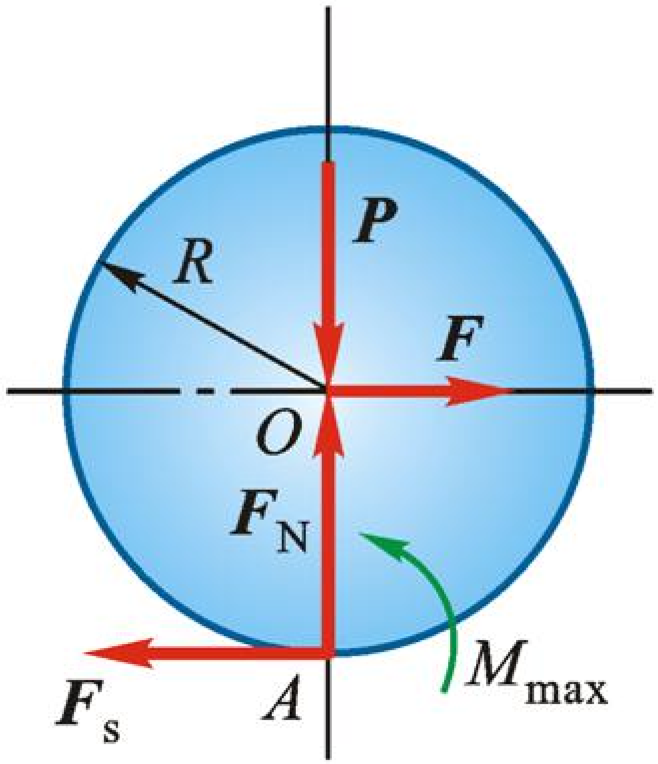
$$0 \le F_s \le F_{max}$$

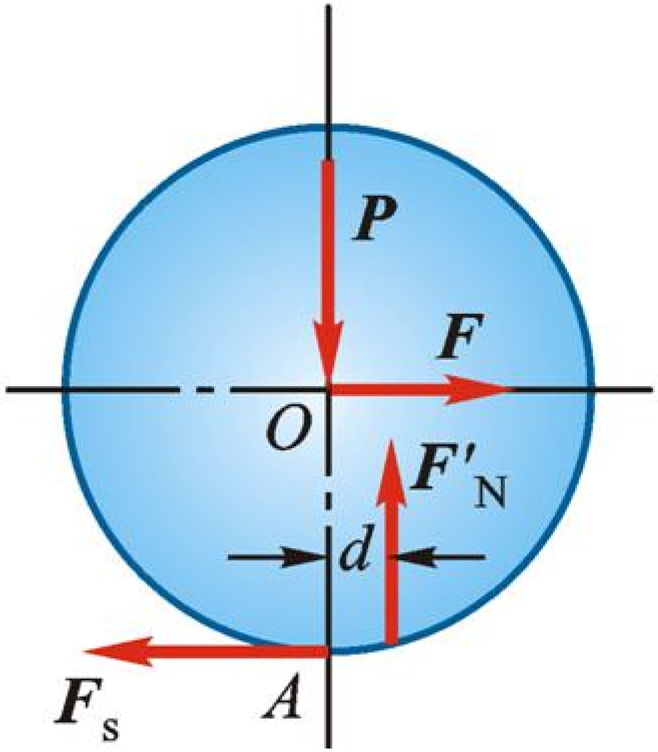
$$0 \le M \le M_{max}$$

$$F_{max} = f_s F_N \qquad M_{max} = \delta F_N$$ ——最大滚动摩阻（擦）力偶

δ 滚动摩阻（擦）系数，长度量纲

δ 的物理意义

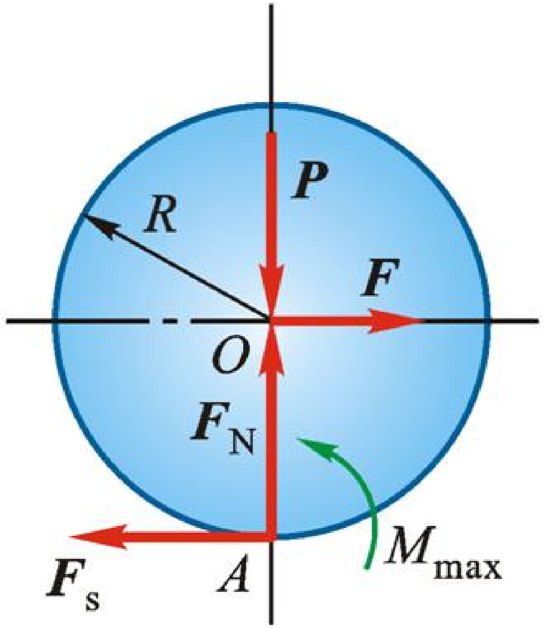




使圆轮滚动比滑动省力的原因

处于临界滚动状态

$$M_{max} = \delta F_N = F_1 R \qquad F_1 = \frac{\delta}{R} F_N$$



处于临界滑动状态

$$F_{max} = f_s F_N = F_2 \qquad F_2 = f_s F_N$$

一般情况下，$\frac{\delta}{R} < f_s$ 或 $\frac{\delta}{R} << f_s$

→ $F_1 < F_2$ 或 $F_1 << F_2$.

例：某型号车轮半径，$R = 450 \text{ mm}$

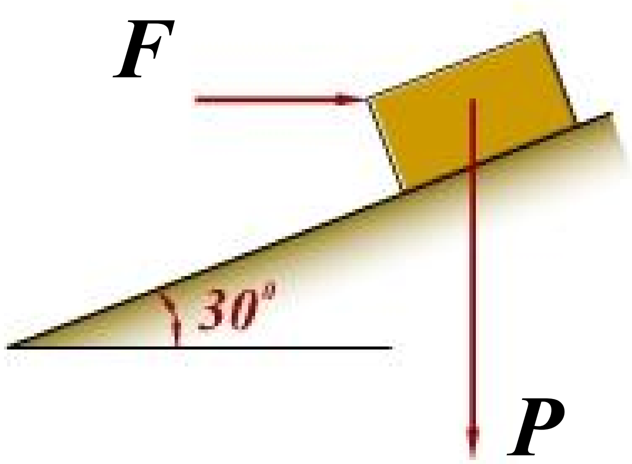
混凝土路面 $\delta = 3.15 \text{ mm}$ $f_s = 0.7$

$$\frac{F_2}{F_1} = \frac{f_s R}{\delta} = \frac{0.7 \times 350}{3.15} = 100$$

例4-1

已知：$P = 1500\,\text{N}$，$f_s = 0.2$，$f_d = 0.18$，$F = 400\,\text{N}$ 。

求：物块是否静止，摩擦力的大小和方向.



解： 取物块，画受力图，设物块平衡

$$\sum F_x = 0 \qquad F\cos 30° - P\sin 30° - F_s = 0$$

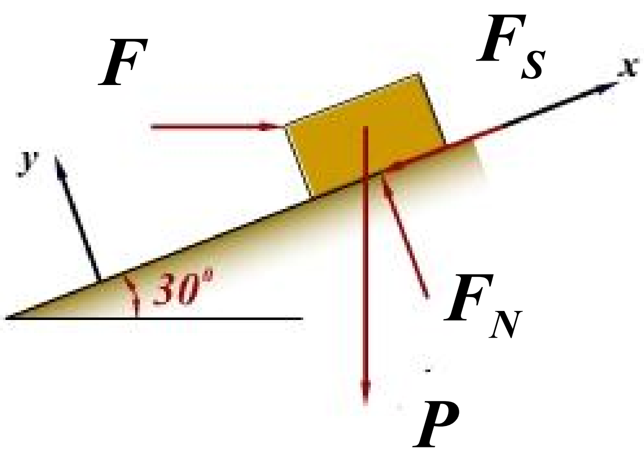
$$\sum F_y = 0 \qquad -F\sin 30° - P\cos 30° + F_N = 0$$

⟹ $F_s = -403.6\text{N}$（向上） $F_N = 1499\ \text{N}$

而 $F_{\max} = f_s F_N = 299.8\text{N}$

⟹ 物块处于非静止状态.

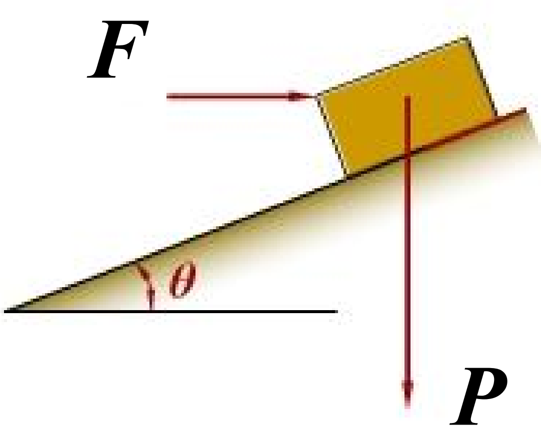
$F_d = f_d F_N = 269.8\text{N}$，向上.



例4-2

已知：P, θ, f_s .

求：使物块静止，水平推力$\boldsymbol{F}$ 的大小.



解： 使物块有上滑趋势时，推力为F_1

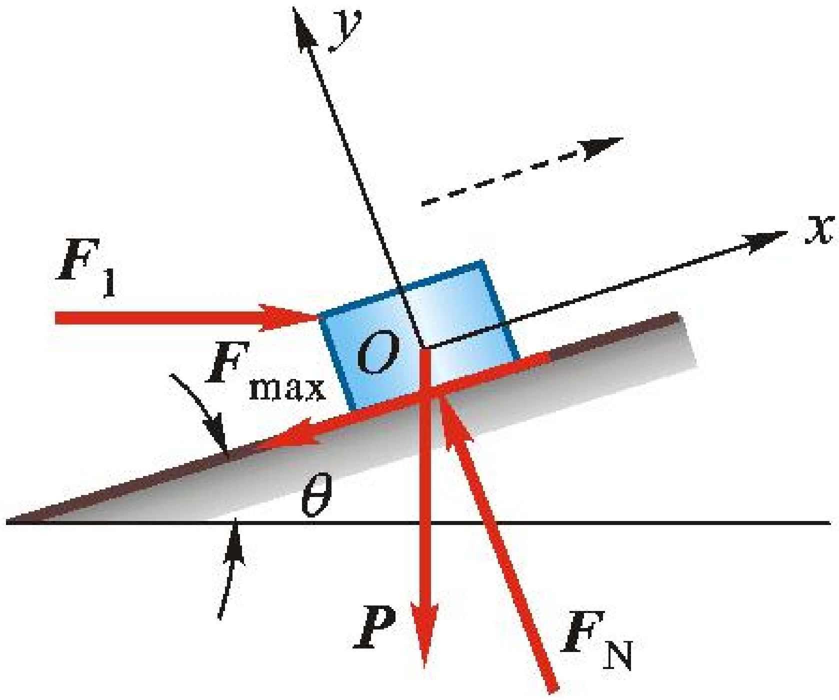
画物块受力图

$$\sum F_x = 0 \qquad F_1\cos\theta - P\sin\theta - F_{max} = 0$$

$$\sum F_y = 0 \qquad -F_1\sin\theta - P\cos\theta + F_N = 0$$

$$F_{max} = f_s F_N$$

$$\Rightarrow \quad F_1 = \frac{\sin\theta + f_s\cos\theta}{\cos\theta - f_s\sin\theta}P$$



设物块有下滑趋势时，推力为 F_1

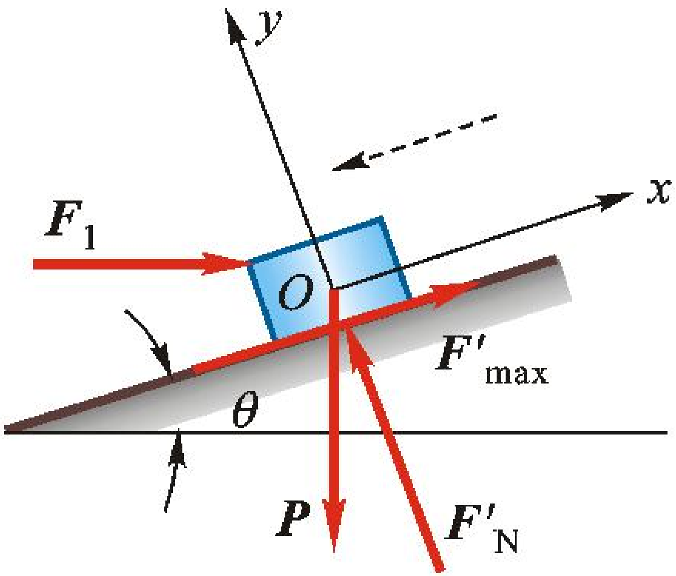
画物块受力图

$$\Sigma F_x = 0 \qquad F_1\cos\theta - P\sin\theta + F_{max}' = 0$$

$$\Sigma F_y = 0 \qquad -F_1\sin\theta - P\cos\theta + F_N' = 0$$

$$F_{max}' = f_s F_N'$$

$$\Rightarrow \quad F_1 = \frac{\sin\theta - f_s\cos\theta}{\cos\theta + f_s\sin\theta}P$$

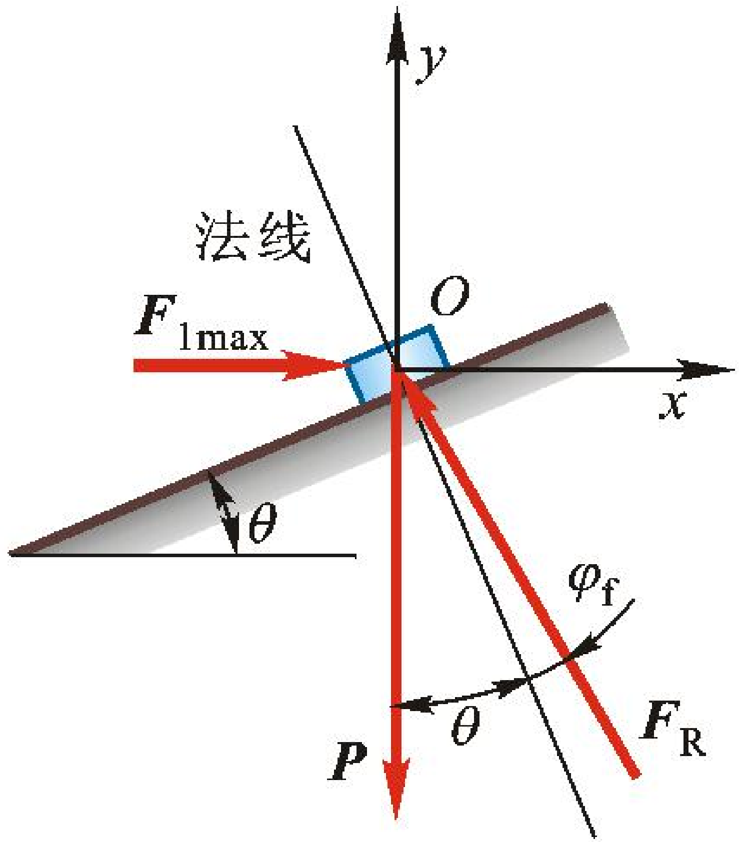


$$\Rightarrow$$

$$\frac{\sin\theta - f_s\cos\theta}{\cos\theta + f_s\sin\theta}P \le F \le \frac{\sin\theta + f_s\cos\theta}{\cos\theta - f_s\sin\theta}P$$

用几何法求解

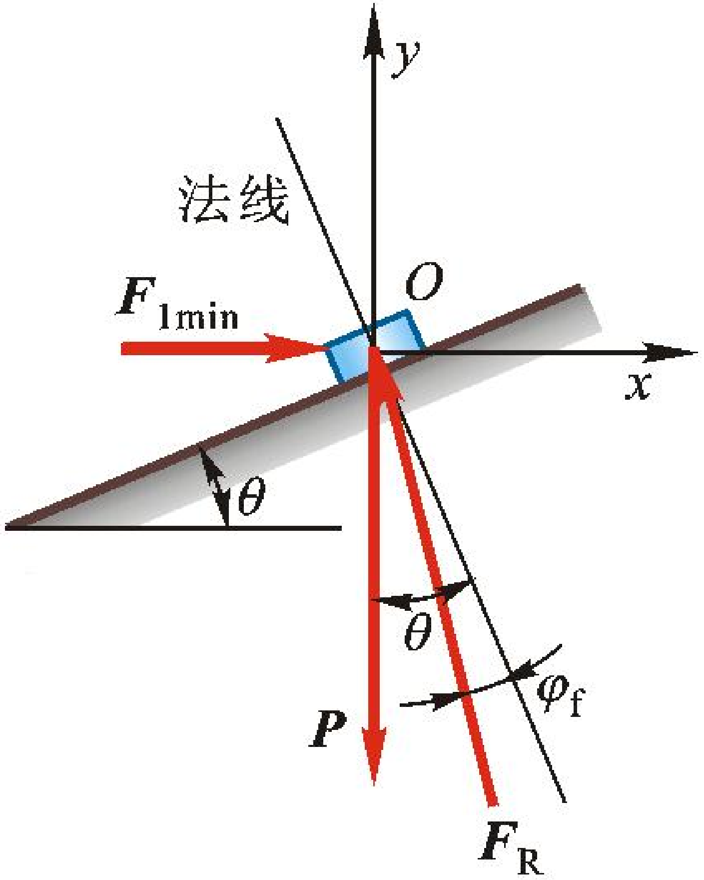
解： 物块有向上滑动趋势时



$$F_{1max} = P\tan(\theta + \varphi)$$

物块有向下滑动趋势时

$$F_{1\min} = P\tan(\theta - \varphi)$$



$$P\tan(\theta - \varphi) \le F \le P\tan(\theta + \varphi)$$

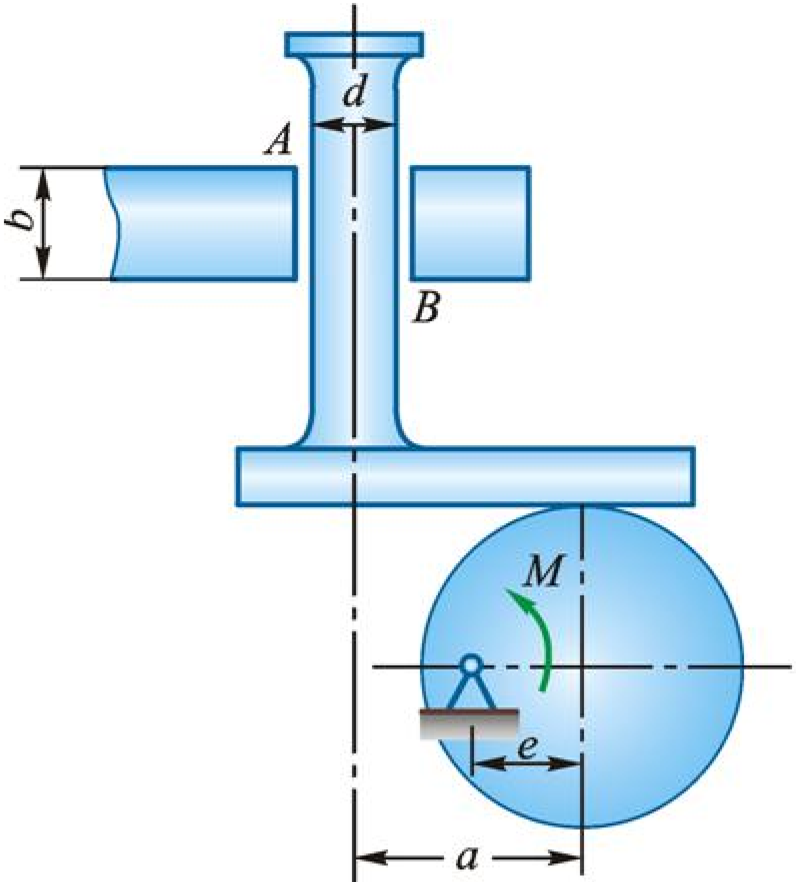
利用三角公式与 $\tan\varphi = f_s$，

$$P\frac{\sin\theta - f_s\cos\theta}{\cos\theta + f_s\sin\theta} \le F \le P\frac{\sin\theta + f_s\cos\theta}{\cos\theta - f_s\sin\theta}$$

例4-3

已知：b , d , f_s , 不计凸轮与挺杆处摩擦，不计挺杆质量；

求：挺杆不被卡住之 a 值.



解： 取挺杆，设挺杆处于刚好卡住位置.

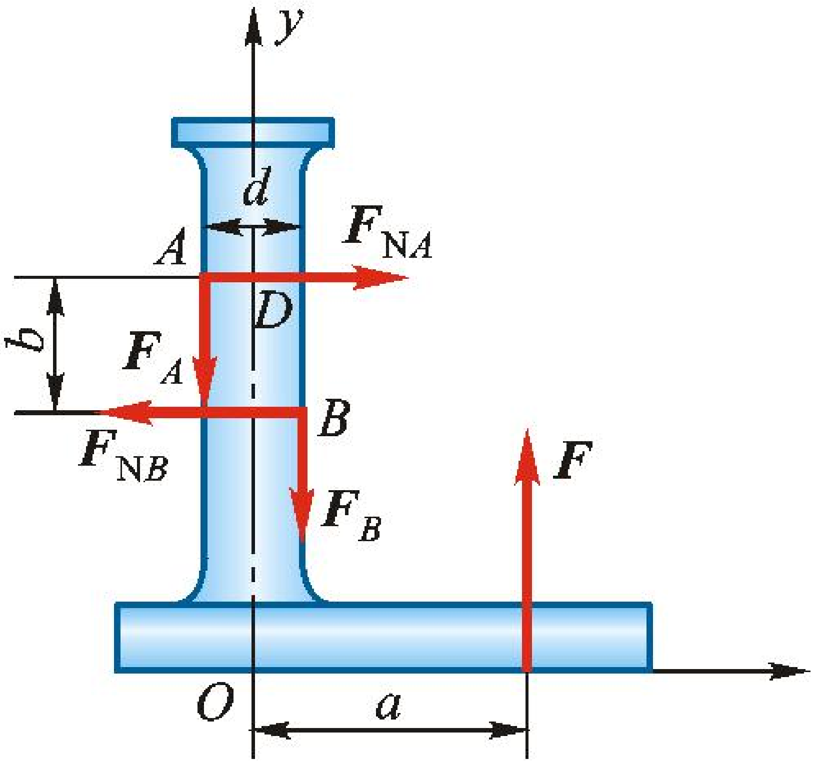
$$\Sigma F_x = 0 \qquad F_{NA} - F_{NB} = 0$$

$$\Sigma F_y = 0 \qquad -F_A - F_B + F = 0$$

$$\Sigma M_A = 0$$

$$F(a + \frac{d}{2}) - F_B d - F_{NB} b = 0$$

$$F_A = f_s F_{NA} \qquad F_B = f_s F_{NB}$$



⇒ $$a = \frac{b}{2f_s}$$

⇒ 挺杆不被卡住时 $a < \dfrac{b}{2f_s}$

用几何法求解

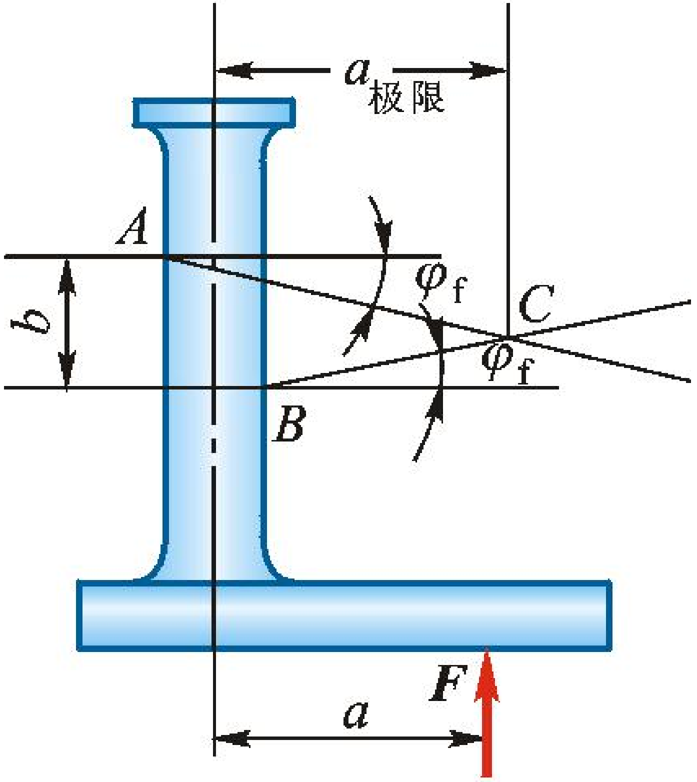
解：

$$b=(a_{极限}+\frac{d}{2})\tan\varphi+(a_{极限}-\frac{d}{2})\tan\varphi$$

$$=2a_{极限}\tan\varphi=2a_{极限}f_s$$

$$\Rightarrow\quad a_{极限}=\frac{b}{2f_s}$$

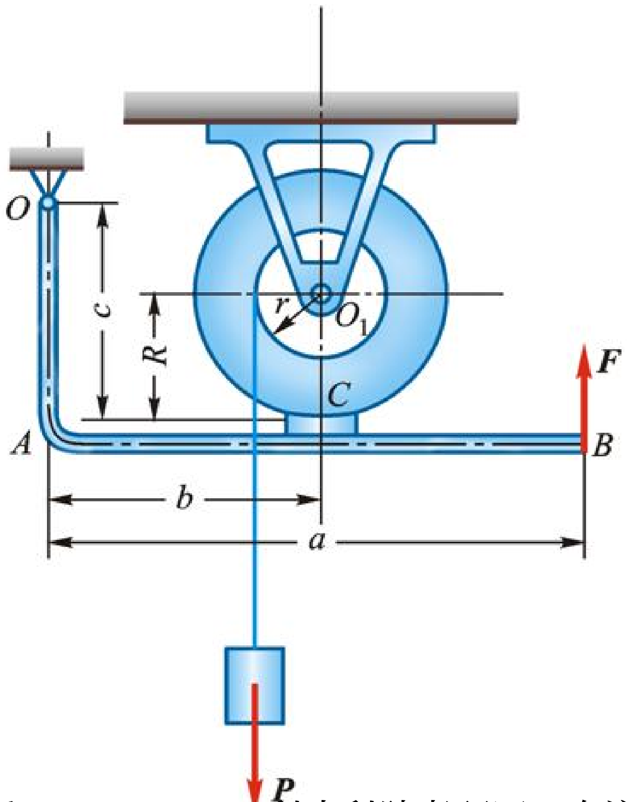
$$a<\frac{b}{2f_s}$$



例4-4

已知：物块重 P, 鼓轮重心位于 O_1处，闸杆重量不计，f_s ，各尺寸如图所示.

求：制动鼓轮所需铅直力F.



解： 分别取闸杆与鼓轮

设鼓轮被制动处于平衡状态

对鼓轮， $\sum M_{O_1} = 0 \qquad rF_{\mathrm{T}} - RF_{\mathrm{s}} = 0$

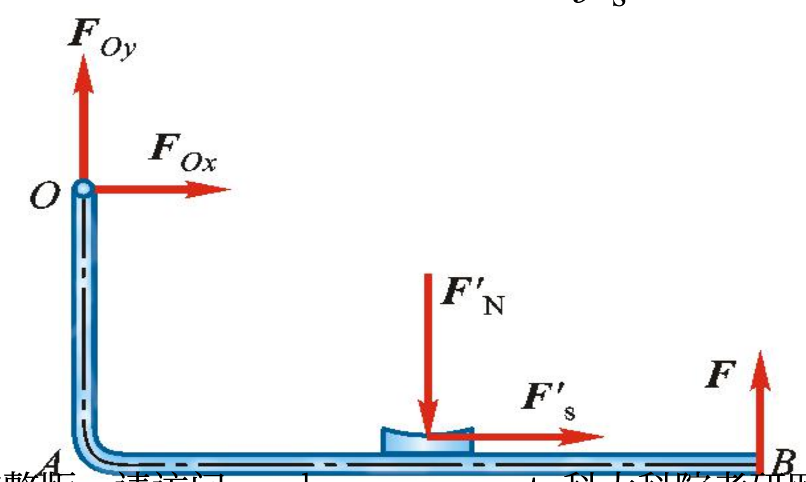
对闸杆， $\sum M_{O} = 0 \qquad Fa - F_{\mathrm{N}}'b - F_{\mathrm{s}}'c = 0$

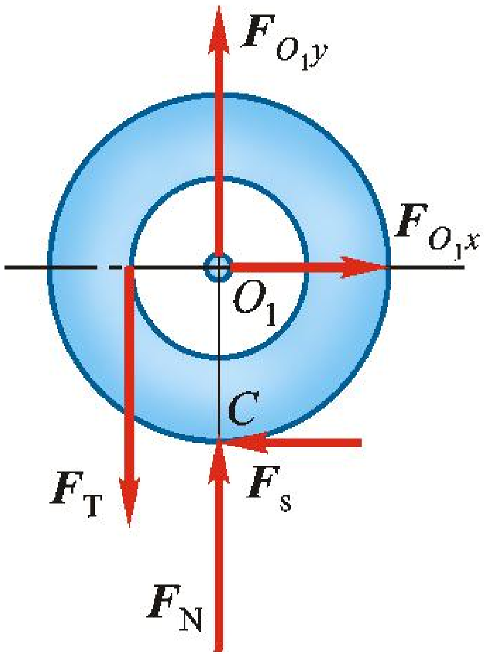
且 $F_{\mathrm{s}}' \leq f_{\mathrm{s}} F_{\mathrm{N}}'$

而 $F_{\mathrm{T}} = P, \qquad F_{\mathrm{s}}' = F_{\mathrm{s}}$

解得

$$F \geq \frac{rP(b - f_{\mathrm{s}}c)}{f_{\mathrm{s}}Ra}$$



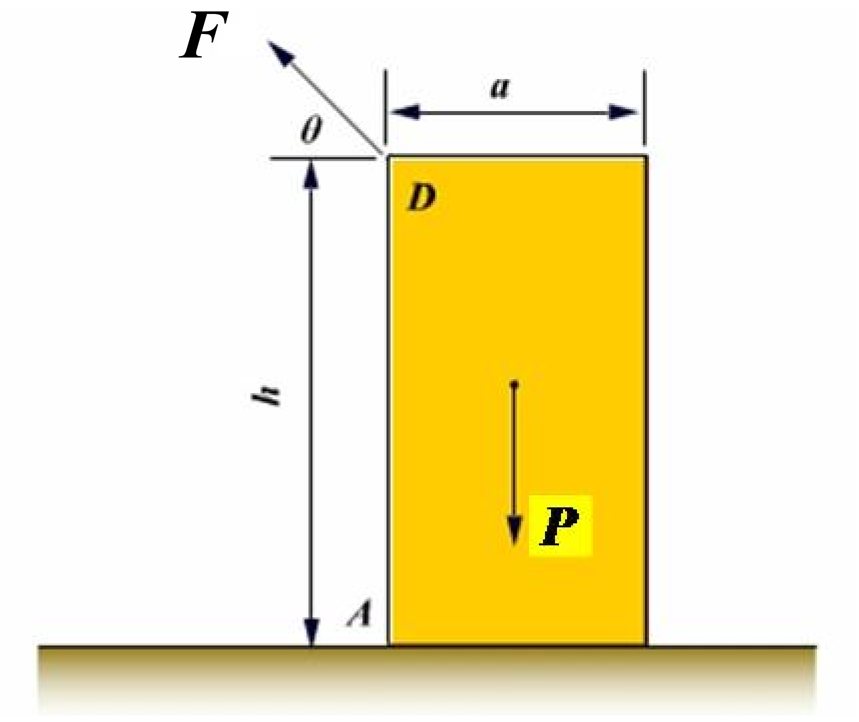


例4-5

已知：均质木箱重 $P=5\text{kN}$，$f_s=0.4$，$h=2a=2\text{m}$，$\theta=30^o$

求：（1）当D处拉力 $F=1\text{kN}$ 时，木箱是否平衡?

（2）能保持木箱平衡的最大拉力.



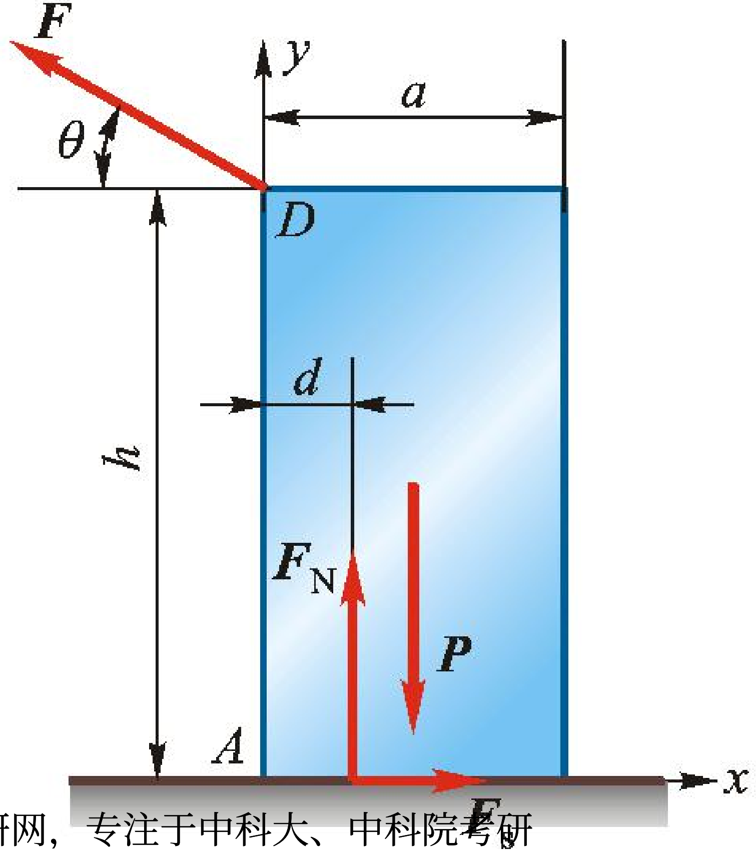
解：（1）取木箱，设其处于平衡状态.

$\Sigma F_x = 0 \quad F_s - F\cos\theta = 0$

$\Sigma F_y = 0 \quad F_N - P + F\sin\theta = 0$

$\Sigma M_A = 0 \quad hF\cos\theta - P\cdot\frac{a}{2} + F_N d = 0$

⟹ $F_s = 866\text{N}$, $F_N = 4500\text{N}$, $d = 0.171\text{m}$



而 $F_{max} = f_s F_N = 1800\text{N}$

因 $F_s < F_{max}$， 木箱不会滑动；

又 $d > 0$，木箱无翻倒趋势.

⟹ 木箱平衡

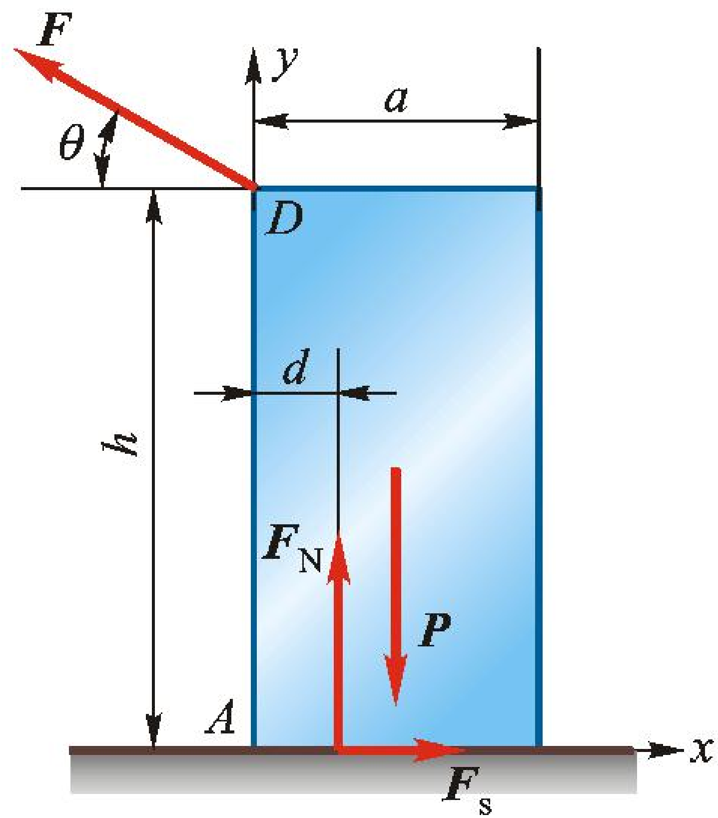
（2）设木箱将要滑动时拉力为 F_1

$$\Sigma F_x = 0 \qquad F_s - F_1\cos\theta = 0$$

$$\Sigma F_y = 0 \qquad F_N - P - F_1\sin\theta = 0$$

又 $F_s = F_{max} = f_s F_N$

$$\Rightarrow F_1 = \frac{f_s P}{\cos\theta + f_s\sin\theta} = 1876\text{N}$$



设木箱有翻动趋势时拉力为 F_2

$$\Sigma M_A = 0 \quad F_2\cos\theta\cdot h - P\cdot\frac{a}{2} = 0$$

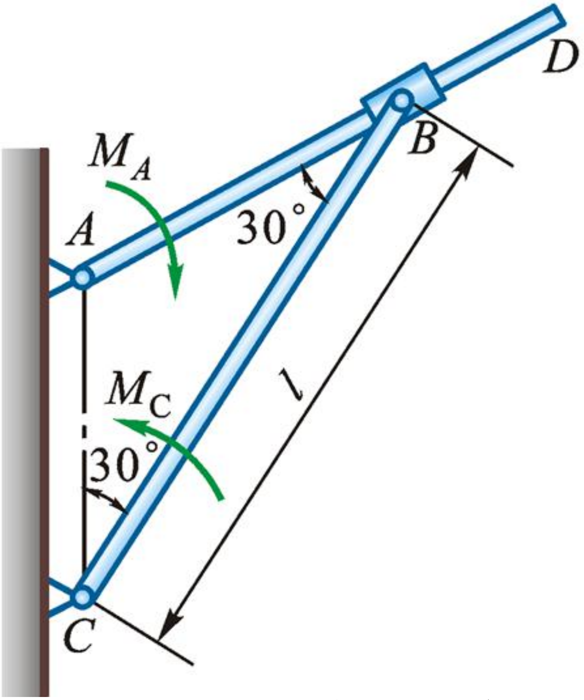
$$\Rightarrow F_2 = \frac{Pa}{2h\cos\theta} = 1443\text{N} \Rightarrow$$ 最大拉力为 1443N

例4-6

已知：$M_A = 40\text{N}\cdot\text{m}$，$f_s = 0.3$，各构件自重不计，

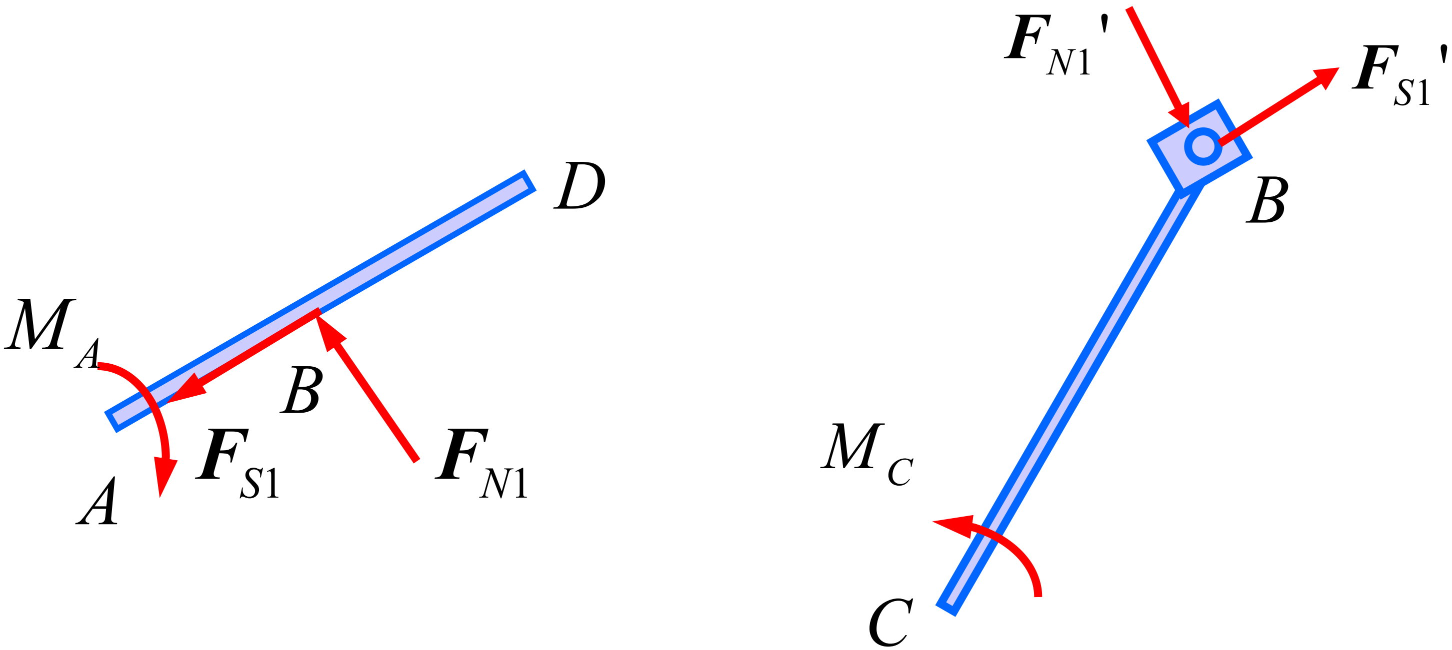
尺寸如图；

求：保持系统平衡的力偶矩 M_C.



解： 设 $M_C = M_{C1}$ 时，系统即将逆时针方向转动

画两杆受力图.



$$\Sigma M_A = 0$$

$$F_{N1} \cdot AB - M_A = 0$$

$$\Sigma M_C = 0$$

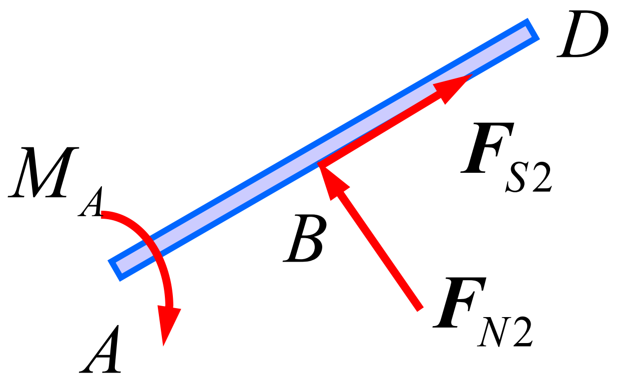
$$M_{C1} - F'_{N1} \cdot l \sin 60^\circ - F'_{s1} \cdot l \cos 60^\circ = 0$$

又 $F'_{s1} = F_{s1} = f_s F_{N1} = f_s F'_{N1}$

$M_{C1} = 70.39\text{N}\cdot\text{m}$

设 $M_C = M_{C2}$ 时，系统有顺时针方向转动趋势

画两杆受力图.



$\Sigma M_A = 0$

$F_{N2} \cdot AB - M_A = 0$

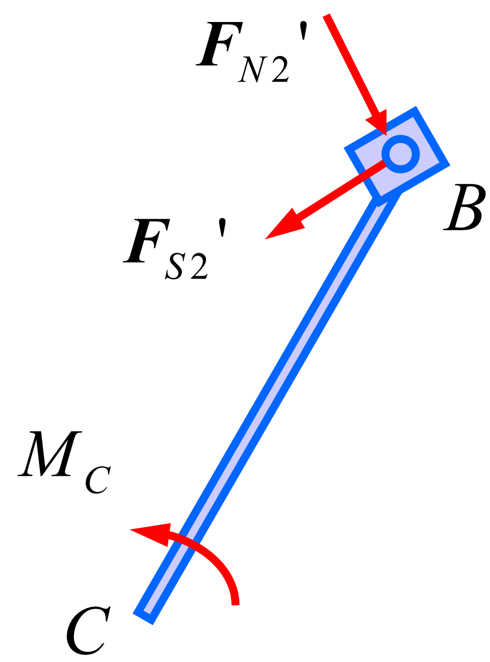
$$\Sigma M_C = 0 \qquad M_{C2} - F'_{N2} \cdot l \sin 60^\circ + F'_{s2} \cdot l \cos 60^\circ = 0$$

又 $F'_{s2} = F_{s2} = f_s F_{N2} = f_s F'_{N2}$

⇨ $M_{C2} = 49.61\mathrm{N \cdot m}$

⇨ 系统平衡时

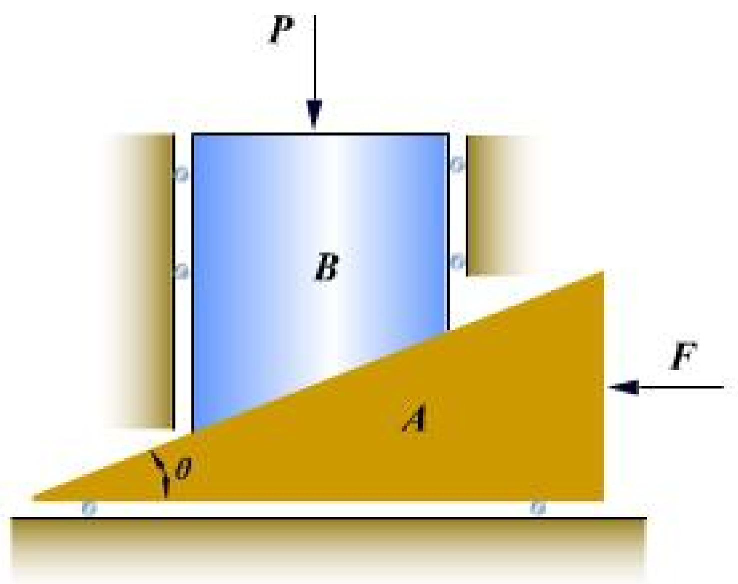
$$49.61\mathrm{N \cdot m} \le M_C \le 70.39\mathrm{N \cdot m}$$



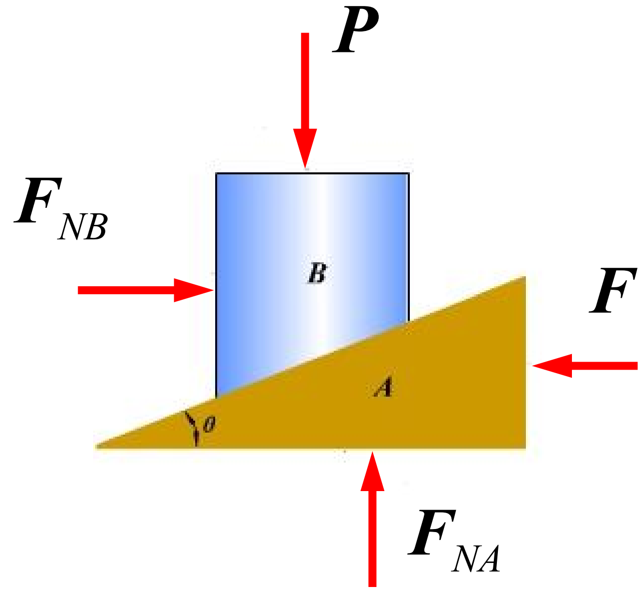
例4-7

已知：力$\boldsymbol{P}$，角θ，不计自重的A,B块间的静摩擦因数为f_s，其它接触处光滑；

求：使系统保持平衡的力$\boldsymbol{F}$的值.



解： 取整体分析，画受力图



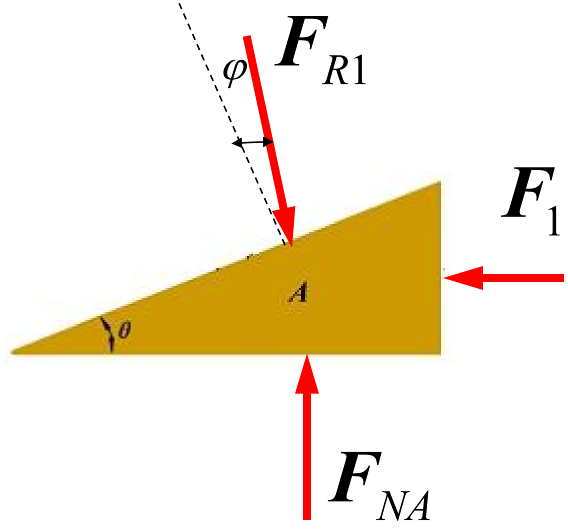
$$\Sigma F_y = 0 \quad F_{NA} - P = 0$$

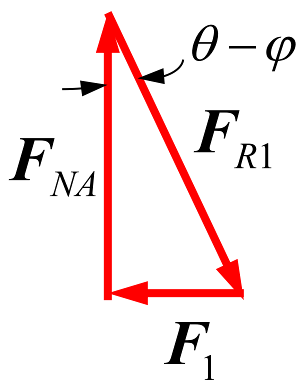
$$\Rightarrow F_{NA} = P$$

设力 $\boldsymbol{F}$ 小于 $\boldsymbol{F}_1$ 时，

楔块 A 向右运动

取楔块 A 分析 ，画受力图





$$F_1 = F_{NA}\tan(\theta-\varphi)$$

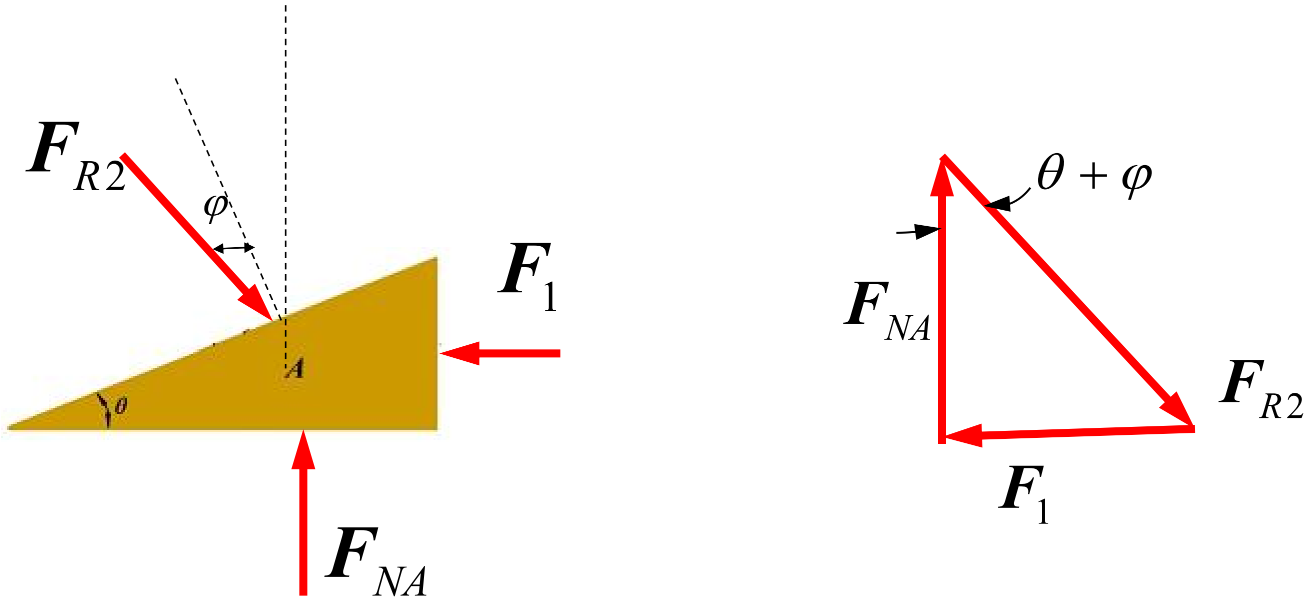
$$= P\tan(\theta-\varphi)$$

设力 F 大于 F_2 时，楔块 A 向左运动

取楔块 A 分析，画受力图

$$F_2 = F_{NA}\tan(\theta+\varphi) = P\tan(\theta+\varphi)$$

$$\Rightarrow \quad P\tan(\theta-\varphi) \le F \le P\tan(\theta+\varphi)$$



例4-8

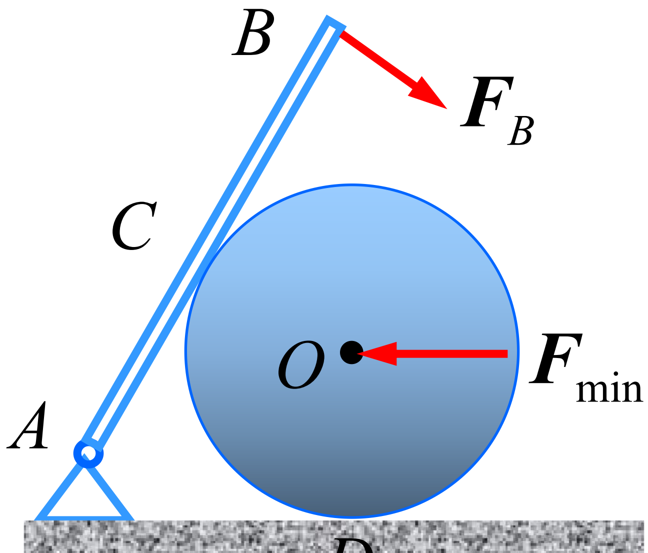
已知：均质轮重 $P=100\text{N}$，杆无重，r, l，$\theta=60^\circ$ 时，

$$AC=CB=\frac{l}{2};\quad F_B=50\text{N},\quad f_C=0.4$$ （杆，轮间）

求：若要维持系统平衡

（1）$f_D=0.3$（轮，地面间），轮心 O 处水平推力 $F_{\min}$

（2）$f_D=0.15$（轮，地面间），轮心 O 处水平推力 $F_{\min}$.



解： F 小于某值，轮将向右滚动.

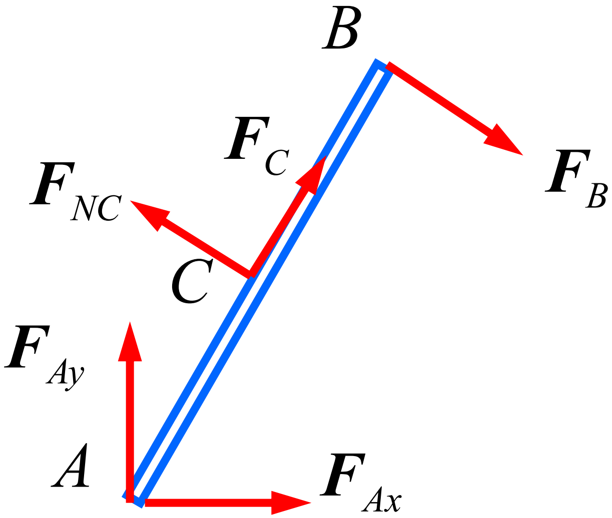
C, D 两处有一处摩擦力达最大值，系统即将运动.

(1) 先设 C 处摩擦力达最大值，取杆与轮.

$$\Sigma M_A = 0 \quad F_{NC} \cdot \frac{l}{2} - F_B \cdot l = 0$$

$$F_{NC} = 100\text{N}$$

$$F_C = F_{C\max} = f_C F_{NC} = 40\text{N}$$



$$\Sigma M_O = 0 \quad F_C' \cdot r - F_D \cdot r = 0$$

$$\Sigma F_x = 0 \quad F_{NC}' \sin 60^\circ - F_C' \cos 60^\circ - F_{\min} - F_D = 0$$

$$\Sigma F_y = 0 \quad F_{ND} - P - F_{NC}' \cos 60^\circ - F_C' \sin 60^\circ = 0$$

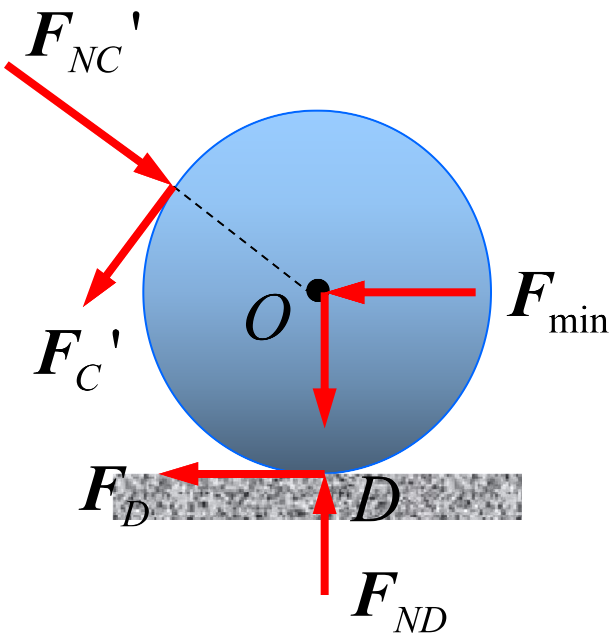
$$F_{NC}' = F_{NC} = 100\text{N}$$

$$F_D = F_C' = 40\text{N} \quad F_{\min} = 26.6\text{N}$$

$$F_{ND} = 184.6\text{N}$$

当$f_s = 0.3$时，$F_{D\max} = f_s F_{ND} = 55.39\text{N}$

当$f_s = 0.15$时，$F_{D\max} = f_s F_{ND} = 27.59\text{N}$



当$f_s = 0.3$时，$F_D = 40\text{N} < F_{D\max}$，$D$ 处无滑动

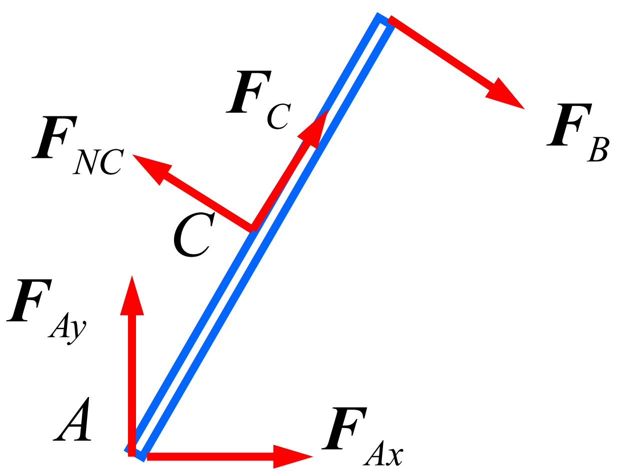
$\Longrightarrow$ $F_{\min} = 26.6\text{N}$

当$f_s = 0.15$时，$F_D = 40\text{N} > F_{D\max}$，$D$ 处有滑动

(2)　D 处摩擦力达最大值，取杆与轮.

$$\Sigma M_A = 0 \qquad F_{NC} \cdot \frac{l}{2} - F_B \cdot l = 0$$

$\Longrightarrow$ $F_{NC} = 100\text{N}$ 不变



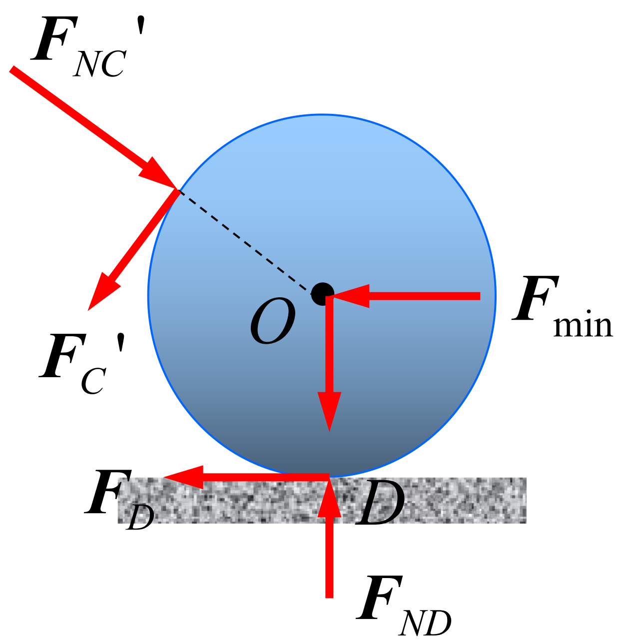
但 $F_C \neq F_{C\max} = f_C F_{NC}$

对轮 $\Sigma M_O = 0 \quad F'_C \cdot r - F_D \cdot r = 0$

$\Sigma F_x = 0 \quad F'_{NC} \sin 60^\circ - F'_C \cos 60^\circ - F_{\min} - F_D = 0$

$\Sigma F_y = 0 \quad F_{ND} - P - F'_{NC} \cos 60^\circ - F'_C \sin 60^\circ = 0$

$F_D = f_D F_{ND}$



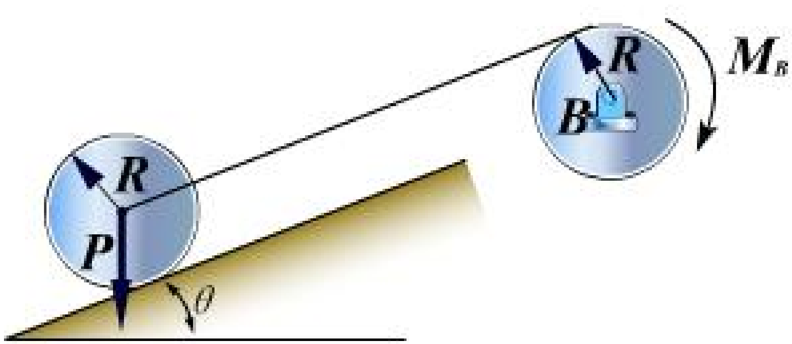
当 $f_D = 0.15$ 时，解得 $F_{ND} = 172.4\text{N}$

$F_D = F_C = f_D F_{ND} = 25.86\text{N}$

C 处无滑动 $F_{\min} = 47.81\text{N}$.

例4-9

已知：P，R，θ，δ；



求：（1）使系统平衡时，力偶矩M_B；

（2）圆柱O匀速纯滚动时，静滑动摩擦系数的最小值.

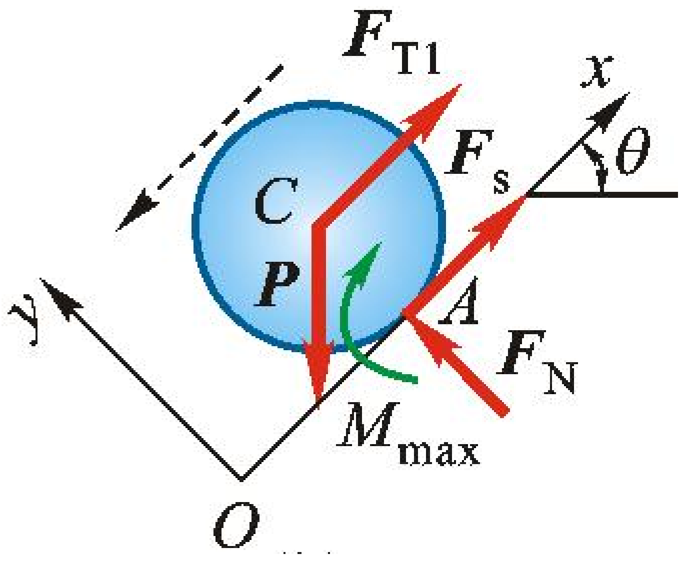
解： （1）设圆柱O 有向下滚动趋势，取圆柱O

$$\Sigma M_A = 0$$

$$P\sin\theta \cdot R - F_{T1} \cdot R - M_{max} = 0$$

$$\Sigma F_y = 0 \quad F_N - P\cos\theta = 0$$

又 $M_{max} = \delta F_N$



$$\Rightarrow \quad F_{T1} = P(\sin\theta - \frac{\delta}{R}\cos\theta)$$

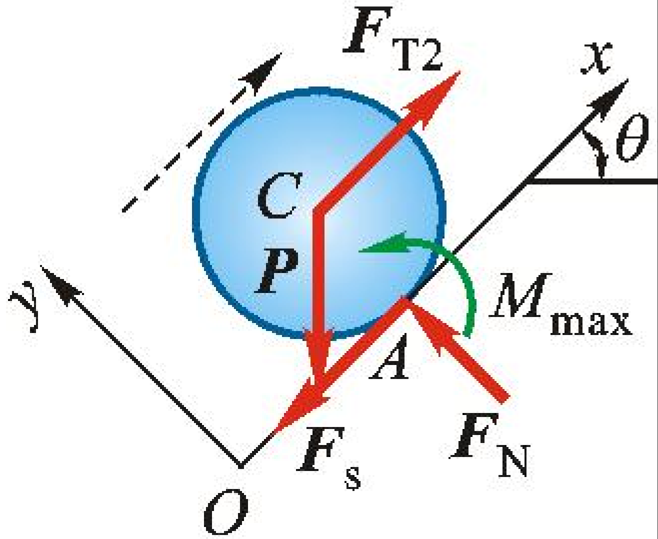
设圆柱O有向上滚动趋势，取圆柱O

$$\Sigma M_A = 0$$

$$P\sin\theta \cdot R - F_{T2} \cdot R + M_{max} = 0$$

$$\Sigma F_y = 0 \qquad F_N - P\cos\theta = 0$$

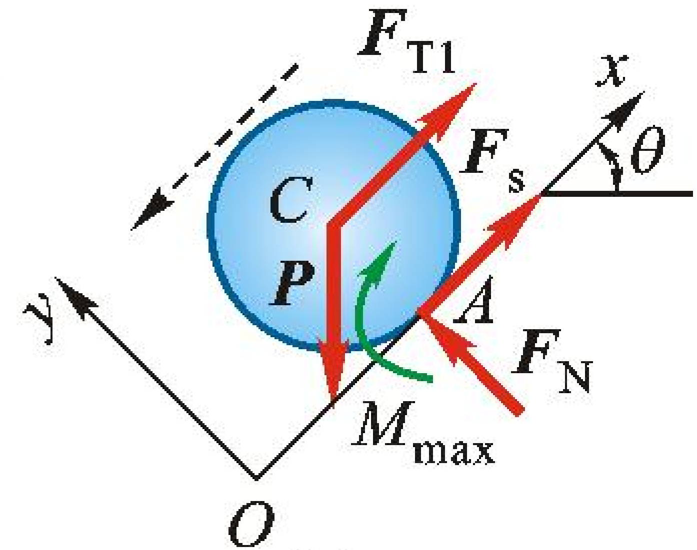
又 $M_{max} = \delta F_N$



$$\Rightarrow \quad F'_{\mathrm{T}\max} = P(\sin\theta + \frac{\delta}{R}\cos\theta) \qquad F_{\mathrm{s}} \le f_{\mathrm{s}} F_{\mathrm{N1}} = f_{\mathrm{s}} P\cos\theta$$

系统平衡时 $P(R\sin\theta - \delta\cos\theta) \le M_B \le P(R\sin\theta + \delta\cos\theta)$

（2）设圆柱 O 有向下滚动趋势.



$$\Sigma M_C = 0 \qquad F_s \cdot R - M_{\max} = 0$$

$$\Sigma F_y = 0 \qquad F_{\mathrm{N}} - P\cos\theta = 0$$

又 $M_{\max} = \delta F_{\mathrm{N}}$

$$\Rightarrow \quad F_{\mathrm{s}} = \frac{\delta}{R} P\cos\theta$$

只滚不滑时，应有 $F_s \le f_s F_{\mathrm{N}} = f_s P\cos\theta$ 则 $f_{\mathrm{s}} \ge \dfrac{\delta}{R}$

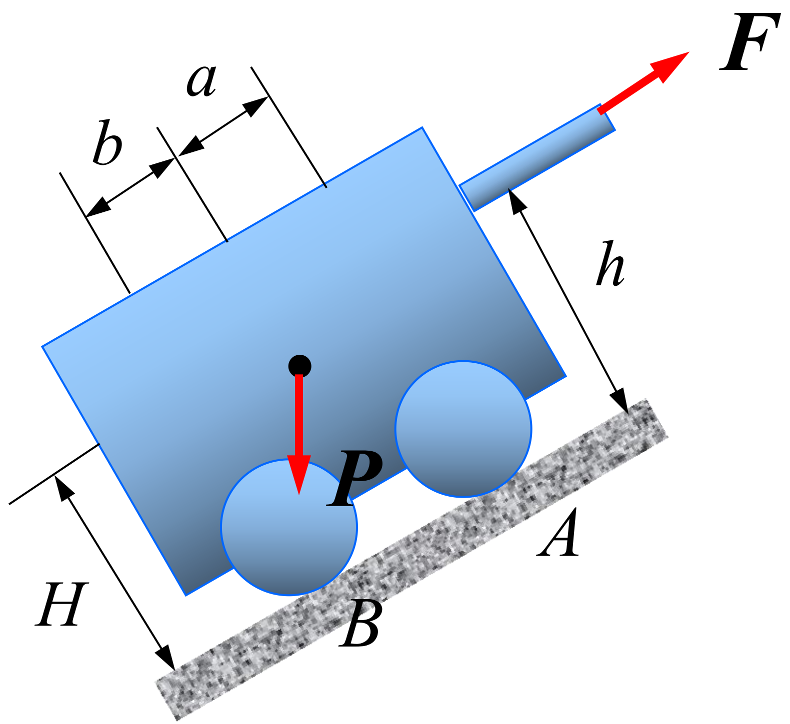
同理，圆柱 O 有向上滚动趋势时得 $f_{\mathrm{s}} \ge \dfrac{\delta}{R}$

圆柱匀速纯滚时，$f_{\mathrm{s}} \ge \dfrac{\delta}{R}$.

例4-10

已知：拖车总重 P，车轮半径 R, δ, θ，其他尺寸如图；

求：拉动拖车最小牵引力 F（F 平行于斜坡）.



解： 取整体

$\Sigma F_x = 0$ $F - F_{As} - F_{Bs} - P\sin\theta = 0$ （1）

$\Sigma F_y = 0$ $F_{AN} + F_{BN} - P\cos\theta = 0$ （2）

$\Sigma M_B = 0$

$F_{AN}(a+b) - Fh - P\cos\theta \cdot b + P\sin\theta \cdot H + M_A + M_B = 0$ （3）

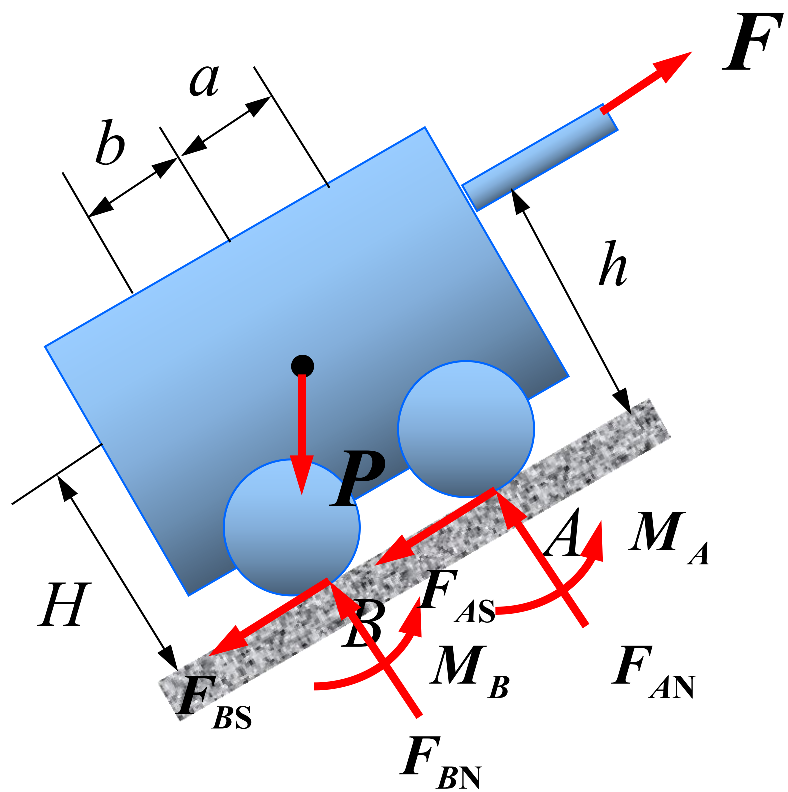
$M_A = \delta F_{AN}$ （4）

$M_B = \delta F_{BN}$ （5）

能否用 $F_{As} = f_s F_{AN}$ ，

$F_{Bs} = f_s F_{BN}$

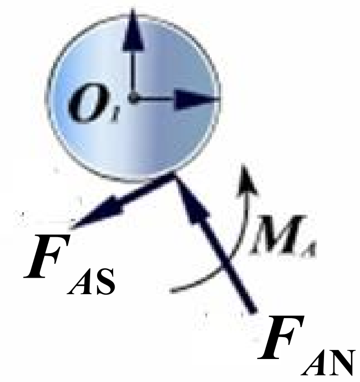
作为补充方程?



取前、后轮

$$\Sigma M_{O1} = 0 \qquad M_A - F_{As}R = 0 \qquad (6)$$

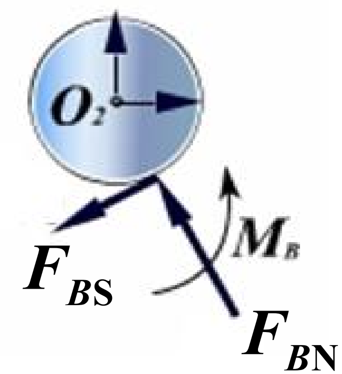
$$\Sigma M_{O2} = 0 \qquad M_B - F_{Bs}R = 0 \qquad (7)$$



七个方程联立解得 $F_{\min} = P(\sin\theta + \dfrac{\delta}{R}\cos\theta)$

若 $\theta = 90^{\circ}$， 则 $F_{\min} = P$， 意味什么？

若 $\theta = 0^{\circ}$ ， 则 $F_{\min} = \dfrac{\delta}{R}P$, 意味什么？



若拖车总重量 $P = 40\text{kN}$，车轮半径 $R = 440\text{mm}$ ，在水平路上行驶（$\theta = 0$），$\delta = 4.4\text{mm}$

$$F_{\min} = \frac{\delta}{R}P = \frac{4.4 \times 40}{440} = 0.4\text{kN}$$ 牵引力为总重的1%。