中科大2015年量子力学 真题答案

1. $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$，$\psi_n(x) = \sqrt{\frac{2}{a}}\cdot\sin\left(\frac{n\pi x}{a}\right)$

(1)、$\psi(x) = A\cdot\left\{\frac{1}{2}\left[\sin\left(\frac{5\pi x}{2a}+\frac{3\pi x}{2a}\right)-\sin\left(\frac{5\pi x}{2a}-\frac{3\pi x}{2a}\right)\right]-\frac{1}{2}\left[\sin\left(\frac{2\pi}{a}+\frac{2\pi x}{a}\right)+\sin\left(\frac{2\pi}{a}-\frac{2\pi x}{a}\right)\right]\right\}$

$= \frac{A}{2}\left(\sin\frac{8\pi x}{2a}-\sin\frac{2\pi x}{a}-\sin\frac{3\pi x}{a}-\sin\frac{2\pi x}{a}\right)$

$= \frac{A}{2}\left(\sin\frac{4\pi x}{a}-\sin\frac{3\pi x}{a}-2\sin\frac{2\pi x}{a}\right)$

$= \frac{1}{\sqrt{6}}\left[\psi_4(x)-\psi_3(x)-2\psi_1(x)\right]$

$\psi(x,t) = \frac{1}{\sqrt{6}}\left[\psi_4(x)\cdot\exp\left(-\frac{iE_4t}{\hbar}\right)-\psi_3(x)\cdot\exp\left(-\frac{iE_3t}{\hbar}\right)-2\psi_1(x)\cdot\exp\left(-\frac{iE_1t}{\hbar}\right)\right]$

设 $\omega = \frac{\pi^2\hbar}{2ma^2}$

则有 $\psi(x,t) = \frac{1}{\sqrt{6}}\left[\psi_4(x)\cdot\exp(-16i\omega t)-\psi_3(x)\exp(-9\cdot i\omega t)-2\psi_1(x)\cdot\exp(-i\omega t)\right]$

(2). $P(\text{基态}) = \left|\langle\psi_1(x)|\psi(x)\rangle\right|^2 = \frac{4}{6}$

(3)·测量结束后，波函数为 $\psi_1(x) = \sqrt{\frac{2}{a}}\cdot\sin\frac{2\pi x}{a}$

势垒右移动后，处于新基态的概率为 $P = \left|\langle\psi_1'(x)|\psi_1(x)\rangle\right|^2 = \left|\int_0^a\sqrt{\frac{1}{a}}\sin\frac{2\pi x}{a}\cdot\sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}dx\right|^2$

$= \left|\frac{4}{3\pi}\right|^2 = \frac{32}{9\pi^2}$