Chapter 11
Motion of Mechanical Systems and Its Regulation(调节)

11.1 Introduction

实际的运动速度会出现波动

The actual motion of the input link depends on the mass distribution(分布) of the mechanism and the external forces acting on the mechanism.

Operating process of a machine:

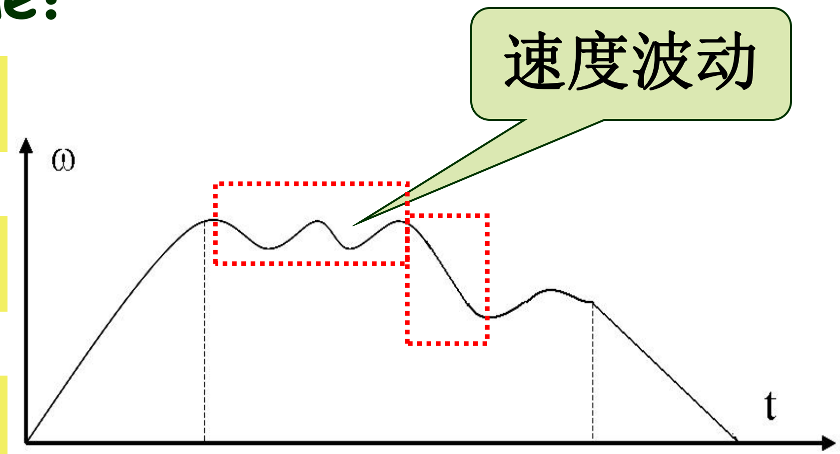
Starting phase(阶段)

↓

Steady working phase

↓

Stopping phase



速度波动
(1) periodic speed fluctuation 周期性速度波动
(2) aperiodic speed fluctuation 非周期性速度波动

11.2 Motion Equation of a Mechanical System

11.2.1 General Expression of the Equation of Motion

The mechanisms with one degree of freedom:
One independent parameter is enough.

整个机器的运动问题 → 某构件的运动问题

dE —— the kinetic energy of the system

dt —— the time interval(间隔)

dw—— the elementary(元) work done by all the external forces

按照功能之间的关系，可以列出：$dE = dw = Ndt$

N ——the sum of the instantaneous(瞬时) power(功率) of all the external forces.

For a planar mechanism consisting of *n* moving links, kinetic energy(动能) *E* of the system :

$$E=\sum_{i=1}^{n}E_i=\sum_{i=1}^{n}\left(\frac{1}{2}m_i v_{C_i}^2+\frac{1}{2}J_{C_i}\omega_i^2\right)$$

The sum of instantaneous powers N :

＋：Mi与ωi同向
—：Mi与ωi反向

$$N=\sum_{j=1}^{k}F_j v_j\cos\alpha_j\pm\sum_{i=1}^{}M_i\omega_i$$

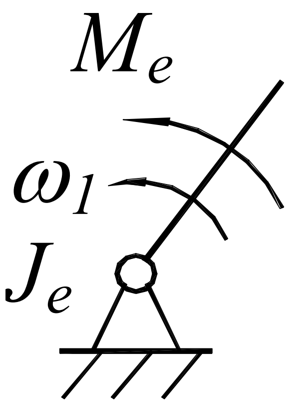
$$dE=Ndt$$

$$d\left[\sum_{i=1}^{n}\left(\frac{1}{2}m_i v_{C_i}^2+\frac{1}{2}J_{C_i}\omega_i^2\right)\right]=\left[\sum_{j=1}^{k}F_j v_j\cos\alpha_j+\sum_{i=1}^{n}M_i\omega_i\right]dt$$

The general equation of motion of a planar mechanism with the DOF=1 in differential(微分的) form.

11.2.2 Dynamically Equivalent Model(等效动力学模型) of a Mechanical System

Suppose: Link 1 rotates about a fixed pivot and its angle of rotation φ_1



取构件1为等效构件（Equivalent Link），整个系统的功能关系可写成

$$d\left(\frac{1}{2}J_e\omega_1^2\right)=M_e\omega_1 dt$$

整个系统包含n个活动构件，系统总的功能关系也可写成

$$d\left[\sum_{i=1}^{n}\left(\frac{1}{2}m_i v_{C_i}^2+\frac{1}{2}J_{C_i}\omega_i^2\right)\right]=\left[\sum_{j=1}^{k}F_j v_j\cos\alpha_j\pm\sum_{i=1}^{n}M_i\omega_i\right]dt$$

$$d\left\{\frac{1}{2}\sum_{i=1}^{n}\left[m_i\left(\frac{v_{C_i}}{\omega_1}\right)^2+J_{C_i}\left(\frac{\omega_i}{\omega_1}\right)^2\right]\omega_1^2\right\}=\left[\sum_{j=1}^{k}F_j\left(\frac{v_j}{\omega_1}\right)\cos\alpha_j\pm\sum_{i=1}^{n}M_i\left(\frac{\omega_i}{\omega_1}\right)\right]\omega_1 dt$$

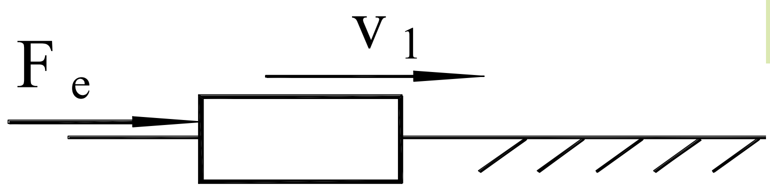
J_e: the equivalent moment of inertia(等效转动惯量)

$$J_e=\sum_{i=1}^{n}\left[m_i\left(\frac{v_{C_i}}{\omega_1}\right)^2+J_{C_i}\left(\frac{\omega_i}{\omega_1}\right)^2\right]$$

M_e: the equivalent moment of force(等效力矩)

$$M_e=\sum_{j=1}^{k}F_j\left(\frac{v_j}{\omega_1}\right)\cos\alpha_j\pm\sum_{i=1}^{n}M_i\left(\frac{\omega_i}{\omega_1}\right)$$

Similarly, a translating(平动的) member as an equivalent link



$$d\left(\frac{1}{2}m_e v_1^2\right)=F_e v_1 dt$$

m e

$$d\left[\sum_{i=1}^{n}\left(\frac{1}{2}m_i v_{C_i}^2+\frac{1}{2}J_{C_i}\omega_i^2\right)\right]=\left[\sum_{j=1}^{k}F_j v_j\cos\alpha_j\pm\sum_{i=1}^{n}M_i\omega_i\right]dt$$

$$d\left\{\frac{1}{2}\sum_{i=1}^{n}\left[m_i\left(\frac{v_{C_i}}{v_1}\right)^2+J_{C_i}\left(\frac{\omega_i}{v_1}\right)^2\right]v_1^2\right\}=\left[\sum_{j=1}^{k}F_j\left(\frac{v_j}{v_1}\right)\cos\alpha_j\pm\sum_{i=1}^{n}M_i\left(\frac{\omega_i}{v_1}\right)\right]v_1 dt$$

等效质量

the equivalent mass:

$$m_e=\sum_{i=1}^{n}\left[m_i\left(\frac{v_{C_i}}{v_1}\right)^2+J_{C_i}\left(\frac{\omega_i}{v_1}\right)^2\right]$$

等效力

the equivalent force:

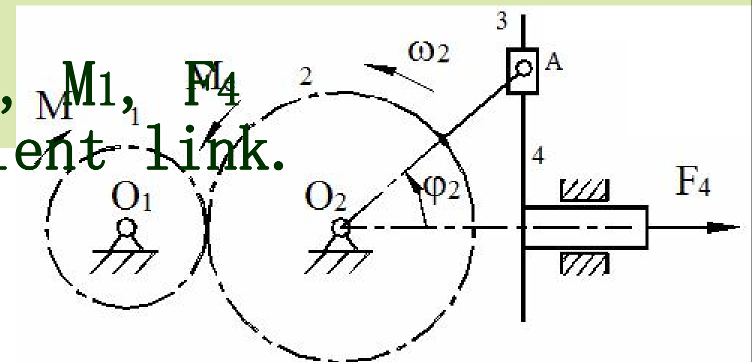
$$F_e=\sum_{j=1}^{k}F_j\left(\frac{v_j}{v_1}\right)\cos\alpha_j+\sum_{i=1}^{n}M_i\left(\frac{\omega_i}{v_1}\right)$$

Example 11-1

Given: Z_1, Z_2, J_1, J_2, m_3, m_4, $l = O_2A$, M_1, F_4

Find: Take the gear 2 as the equivalent link.

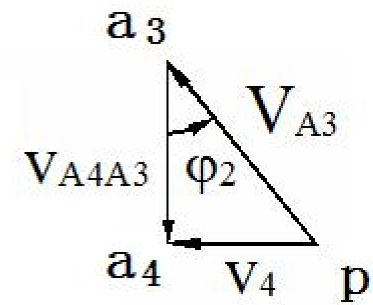
Calculate J_e and M_e.



Solution:

(1) Kinematic analysis of the mechanism

ω_2 ← The angular velocity of link



$$\omega_1 = \frac{Z_2}{Z_1}\omega_2 \qquad v_{A_3} = l\omega_2 \qquad v_4 = l\omega_2 \sin\varphi_2$$

(2) Calculation of Je （利用等效动能相等的原则来计算）

$$J_e = \left(\frac{\omega_1}{\omega_2}\right)^2 J_1 + J_2 + \left(\frac{v_{A_3}}{\omega_2}\right)^2 m_3 + \left(\frac{v_4}{\omega_2}\right)^2 m_4 = \left(\frac{Z_2}{Z_1}\right)^2 J_1 + J_2 + l^2 m_3 + l^2 m_4 \sin^2\varphi_2$$

$$J_e = \sum_{i=1}^{n}\left[m_i\left(\frac{v_{C_i}}{\omega_1}\right)^2 + J_{C_i}\left(\frac{\omega_i}{\omega_1}\right)^2\right]$$

(3) Calculation of Me: （利用等效功相等的原则来计算）

$$M_e = \left(\frac{\omega_1}{\omega_2}\right)M_1 - \frac{v_4}{\omega_2}\cdot\cos 180^\circ \cdot F_4 = \frac{z_2}{z_1}M_1 - F_4 \cdot l \cdot \sin\varphi_2$$

$$M_e = \sum_{j=1}^{k} F_j\left(\frac{v_j}{\omega_1}\right)\cos\alpha_j \pm \sum_{i=1}^{n} M_i\left(\frac{\omega_i}{\omega_1}\right)$$

求解等效力（矩）的时候，还可用速度多边形杠杆法。

作机构的转向速度多边形，将等效力（矩）和被代替的力（矩）平移到其作用点的影像上。

↓

使二者对极点的力矩大小相等，方向相同，即可求出等效力（矩）。

注意：等效力（矩）是假想力（矩），并非被代替的已知给定力的合力（矩）。

~~11.3 Solution of the Motion Equation of a Mechanical System~~

11.4 Periodic Speed Fluctuation and its Regulation

11.4.1 Condition for a Periodic Steady working State

循环稳定运行的条件：

$$\int_{\varphi}^{\varphi+\varphi_T} M_d d\varphi = \int_{\varphi}^{\varphi+\varphi_T} M_r d\varphi$$

φ_T ——the period of the system, 循环周期

M_d ——the driving moment, 驱动力矩

M_r ——the resistant moment, 阻力矩

11.4.2 Coefficient of Speed Fluctuation

$$\delta = \frac{\omega_{\max} - \omega_{\min}}{\omega_m}$$ ----Coefficients of Speed Fluctuation 机器运转速度不均匀系数

$$\omega_m = \frac{\omega_{\max} + \omega_{\min}}{2}$$ ----average angular speed 平均角速度

$$\omega_{\max}^2 - \omega_{\min}^2 = 2\delta\omega_m^2$$

不同机器的[δ]不同

Table 11-1
Allowable Coefficients of Speed Fluctua n
[δ]

Machines	[δ]	Machines	[δ]
Rock crusher	1/5~1/20	Pump(泵), blower(风机)	1/30~1/50
Punching machine	1/7~1/10	Paper machine	1/40~1/50
Rolling mill (轧钢机)	1/10~1/25	Spinning(纺纱) machine	1/60~1/100
Automobile	1/20~1/60	DC generator	1/100~1/200
Machine tools	1/30~1/40	AC generator (交流发电机)	1/200~1/300

分析机器运转速度不均匀系数 δ 与哪些因素有关：

机器的最大动能：
$$\frac{1}{2}J\omega_{max}^{2}=W_{max}+\frac{1}{2}J_0\omega_0^2$$

驱动力矩与阻力矩综合后所作的最大功

机器的最小动能：
$$\frac{1}{2}J\omega_{min}^{2}=W_{min}+\frac{1}{2}J_0\omega_0^2$$

驱动力矩与阻力矩综合后所作的最小功

$$\frac{1}{2}J\left(\omega_{max}^2-\omega_{min}^2\right)=W_{max}-W_{min}$$

$$\omega_{max}^2-\omega_{min}^2=2\delta\omega_m^2$$

$$J\delta\omega_m^2=W_{max}-W_{min}=\Delta W$$

The maximum increment of work
功的最大增量，最大盈亏功

$$\delta=\frac{\Delta W_{max}}{\omega_m^2 J}$$

如何减小机器的运转不均匀系数：增加机器的转动惯量

Add a flywheel 飞轮

（具有大转动惯量的零件）

11.4.3 Calculate of moment of inertia of a Flywheel

$$\delta = \frac{\Delta W_{max}}{\omega_m^2 J}$$

$$\delta < [\delta]$$

$$J_F = \frac{\Delta W_{max}}{\omega_m^2[\delta]} - J_c$$

除飞轮外其它构件的转动惯量，其值相对于飞轮来说较小。

$$J_F \approx \frac{\Delta W_{max}}{\omega_m^2[\delta]}$$

分析：

1）若过分要求较小的[δ]，会使J_F激增，飞轮过于笨重。

2）J_F与ω_m^2成反比，可将飞轮安装在ω_m较高的轴上。

3）ΔW_{max}越大，飞轮的J_F越大

Example 11-4

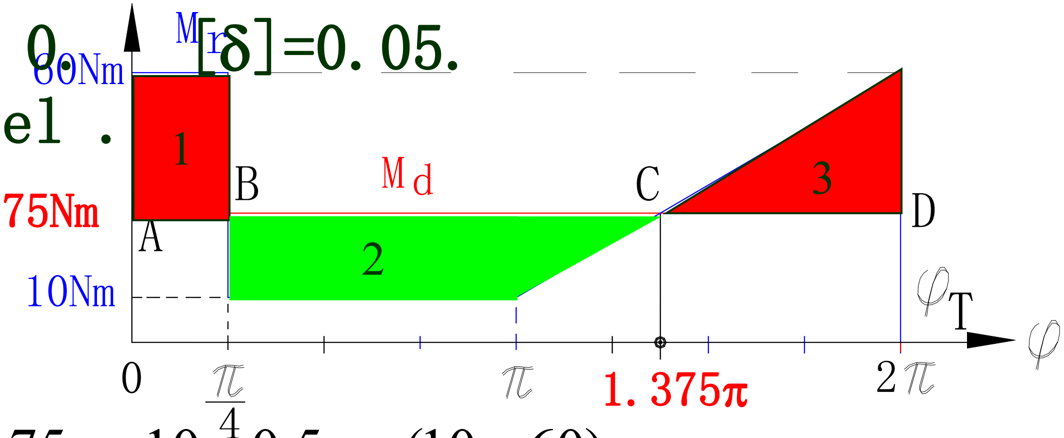
M_r is given as shown in Fig.11-4, $\varphi_T = 2\pi$. M_d is constant.

n_m =1000 r/min. J_C = 0. [δ]=0.05.

Find J_F of the flywheel .

Solution:

(1) Find M_d



$$2\pi M_d = \int_0^{2\pi} M_r d\varphi$$

$$= 0.25\pi \times 60 + 0.75\pi \times 10 + 0.5\pi \times (10 + 60)$$

$\varphi_C = 1.375\pi$

(2) Find the position where ω_{max} and ω_{min} appear

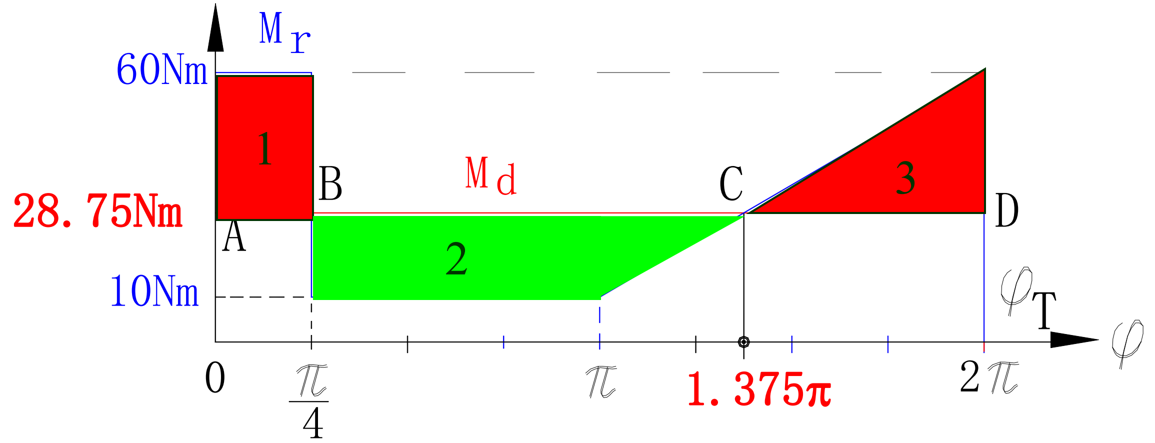
面积1代表亏功，即阻力所作的功大于驱动力的，B点的转速具有局部极小值；

面积2代表盈功，即驱动力所作的功大于阻力的，C点的转速具有局部极大值.

$$W_1 = (28.75 - 60) \times \frac{\pi}{4} = -7.8125\pi = -24.54(Nm)$$

$$\Delta W_{max} = W_2 = \frac{1}{2} \times [(1.375\pi - \frac{\pi}{4}) + (\pi - \frac{\pi}{4})] \times (28.75 - 10) = 17.578\pi = 55.22(Nm)$$

$$W_3 = -(55.22 - 24.54) = -30.68(Nm)$$



(3) Find the maximum increment of work（最大盈亏功）

$$\Delta W_{\max} = E_{\max} - E_{\min} = W_2 = 55.22Nm$$

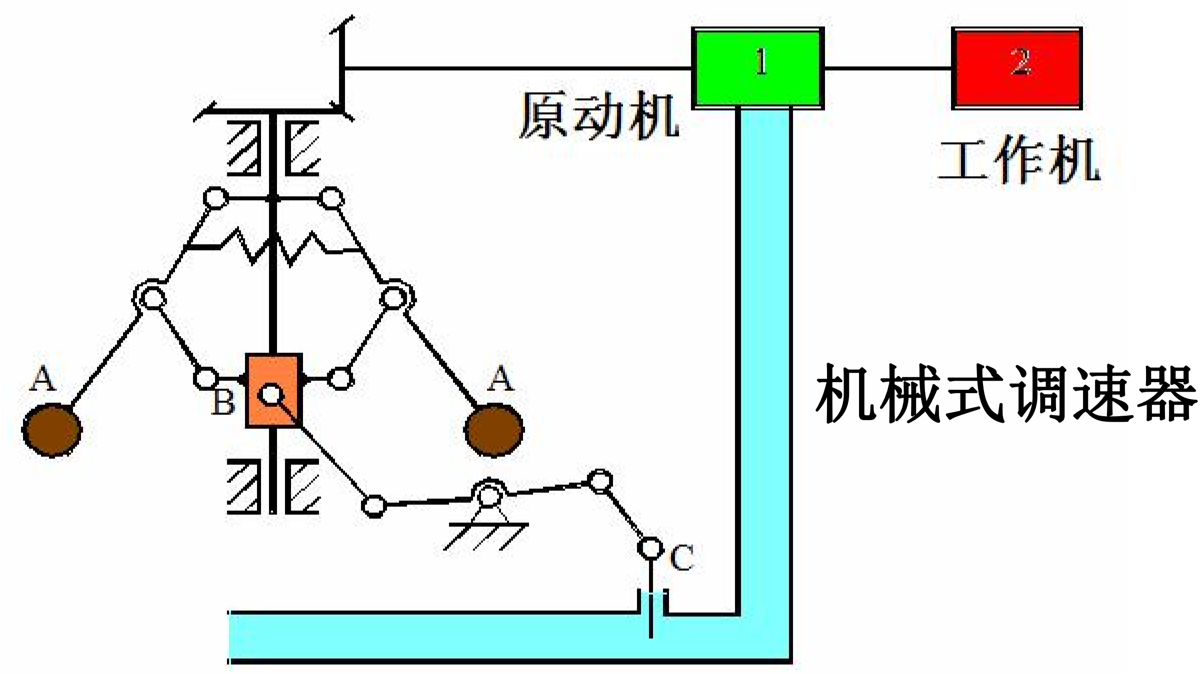
(4) Calculate JF of the flywheel.

$$J_F \geq \frac{\Delta W_{\max}}{\omega_m^2[\delta]} = \frac{55.22}{\left(\frac{1000 \times 2\pi}{60}\right)^2 \times 0.05} = 0.1(kgm^2)$$

11.5 Introduction to Aperiodic(非周期性的) Speed Fluctuation and its Regulation

The resistance to a machine may change irregularly(不规则地) or randomly（随机地). This results in the so-called aperiodic speed fluctuation.（非周期性的速度波动）

speed regulator (调速器) centrifugal governor(离心调节)



Chapter 11

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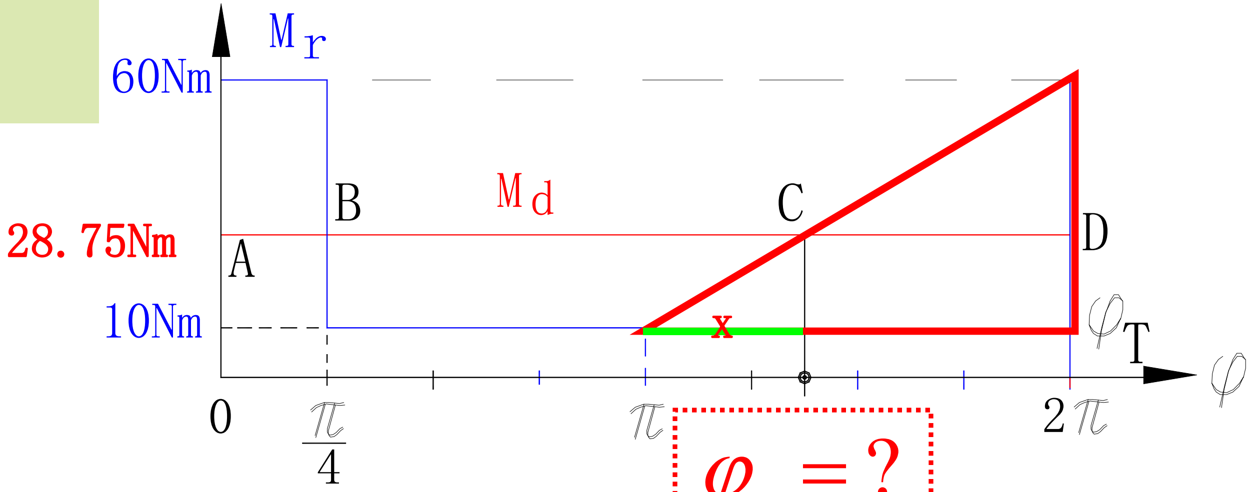

$\varphi_c = ?$

$$\frac{28.75-10}{x} = \frac{60-10}{2\pi-\pi}$$

$$x = \frac{18.75}{50} \times \pi = 0.375\pi$$

$$\varphi_c = \pi + 0.375\pi = 1.375\pi$$